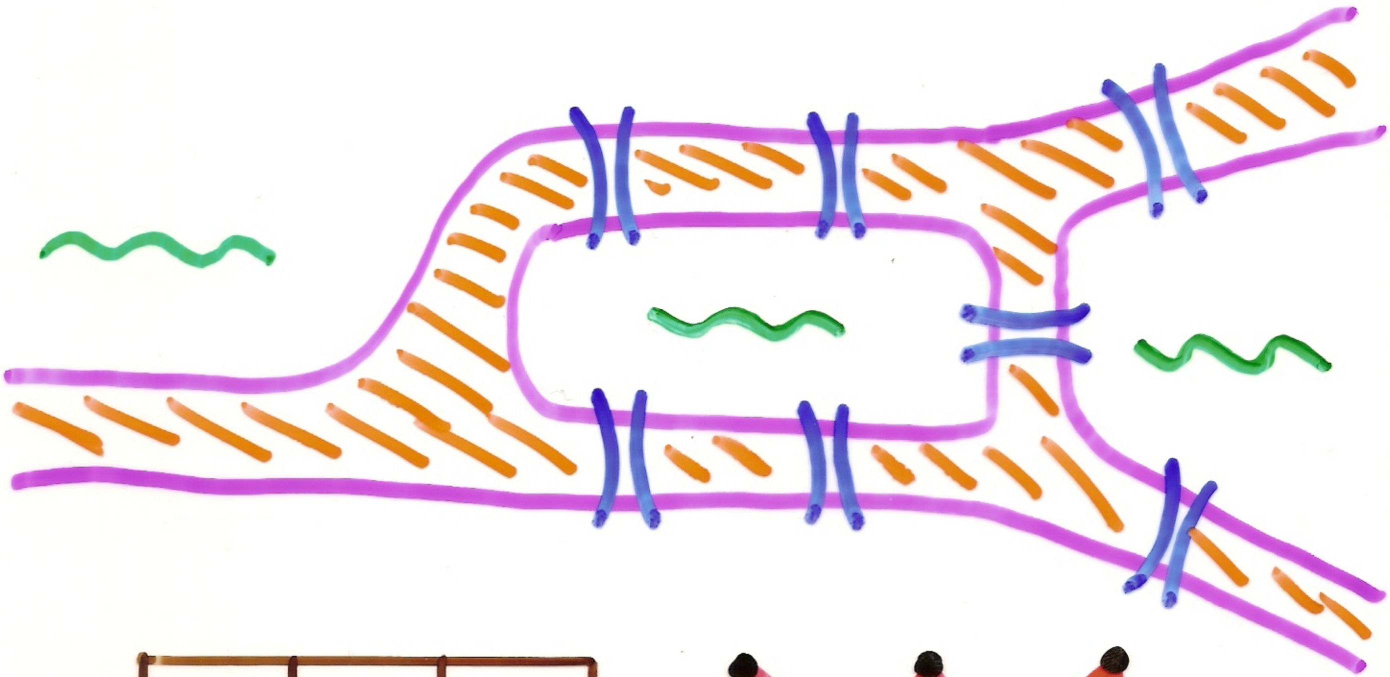
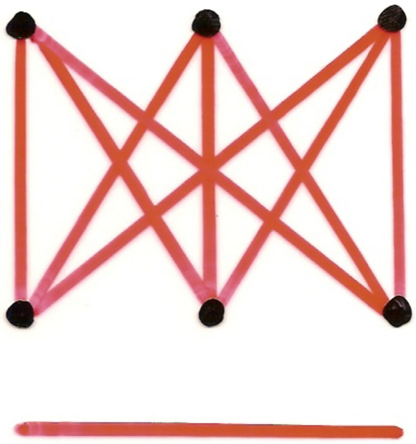
A Millennium of Mathematical Puzzles

Robin Wilson



4	9	2
3	5	7
8	1	6



Some observations

- Many puzzles recur throughout history

- One person's recreational puzzle is another's serious mathematics, and *vice versa*

- Recreational mathematics is a good vehicle for teaching serious mathematical ideas

- Some problems can be most easily solved with a good choice of notation, or the use of a good diagram

Rhind papyrus, Problem 79 (1650 BC)

Houses	7
Cats	49
Rice	343
Wheat	2401
Hekat	16807
	19607

Fibonacci, Liber Abaci (1202 AD)

7 old women are going to Rome
each has 7 mules
each mule carries 7 sacks
each sack contains 7 loaves
each loaf has 7 knives
each knife has 7 sheaths
What is the total number of things?

Nursery rhyme

As I was going to St Ives
I met a man with 7 wives,
Each wife had 7 sacks,
Each sack had 7 cats,
Each cat had 7 kits,
Kits, cats, sacks and wives,
How many were going to St Ives?

River-Crossing Puzzles

Alcuin of York (9th century):

Wolf, goat and cabbage

Lewis Carroll (19th century):

Fox, goose and a bag of corn

× Fox and goose / Goose and corn

FGC ———— ·

1. Take goose across – leave it there
   return alone F&C ———— G

2. Take corn across – leave it there
   return with goose FG ———— C

3. Take fox across – leave it there
   return alone G ———— FC

4. Take goose across · ———— FGC

A Variation

- A Queen (195 lb), her son (90 lb) and daughter (165 lb) are imprisoned at the top of a high tower.
- Outside is a pulley with a rope over it, with a basket at each end (of equal weight).
- There is a weight in the room (75 lb).
- The descending basket must not be more than 15 lb heavier than the ascending one.
- How can they escape?

What if there are also the Queen's pig (60 lbs), dog (45 lbs) and cat (30 lbs)?

The Greek Anthology, Problem 126

This tomb holds Diophantus.
Ah, how great a marvel!
The tomb tells scientifically the measure
of his life.

God granted to him to be a boy
for the sixth part of his life,
and adding a twelfth part to this.

He clothed his cheeks with down;

He lit him with the light of wedlock after a
seventh part, and five years after his marriage
He granted him a son.

Alas! Late-born wretched child;
after attaining the measure
of half his father's life, chill Fate took him.

After consoling his grief by this science of
numbers for four years, he ended his life.

How old was Diophantus?

Diophantus spent $1/6$ of his life in childhood, $1/12$ in youth, and $1/7$ more as a bachelor.

Five years after his marriage there was a son who died four years before his father at $1/2$ his father's final age.

x = Diophantus's age :

$$\left(\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x\right) + 5 + \frac{1}{2}x + 4 = x,$$

so $x = 84$ years

A problem involving ages

The combined ages of Mary and Ann are 44 years, and Mary is twice as old as Ann was when Mary was half as old as Ann will be when Ann is three times as old as Mary was when Mary was three times as old as Ann.

How old is Mary?

Dudeney: Amusements in Mathematics, 1917.

The Greek Anthology, Problem 132

This is Polyphemus the brazen Cyclops,
and as if on him someone made
an eye, a mouth, and a hand,
connecting them with pipes.

He looks quite as if he were dripping water
and seems also to be spouting it
from his mouth.

None of the spouts is irregular;
that from his hand when running
will fill the cistern in three days only,
that *from his eye in one day*,
and *from his mouth in two-fifths of a day*.

Who will tell me the time it takes
when all three are running?

Fibonacci (Leonardo of Pisa)

Liber abaci (1202)

Book of squares

Hindu-Arabic numerals

Liber Abaci — two problems

There is a tree, $^1/_4$ and $^1/_3$ of which lie below ground; 21 palmi. How tall is the tree?

Guess 12, giving $3 + 4 = 7$ (instead of 21)

– scale up to give $3 \times 12 = 36$.

If a lion eats a sheep in 4 hours, a leopard eats it in 5 hours, and a bear eats it in 6 hours, how long would they all take together?

In one hour, they eat

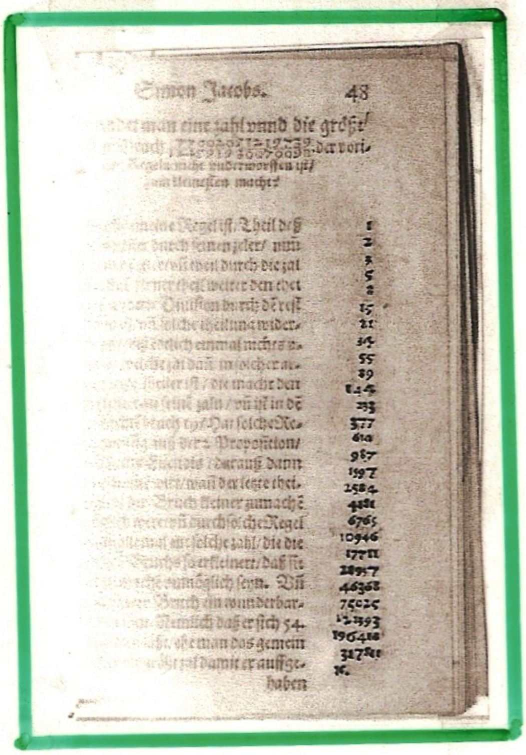
$$\frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{37}{60} \text{ sheep,}$$

so they take $\frac{60}{37} = 1\frac{23}{37}$ hours.

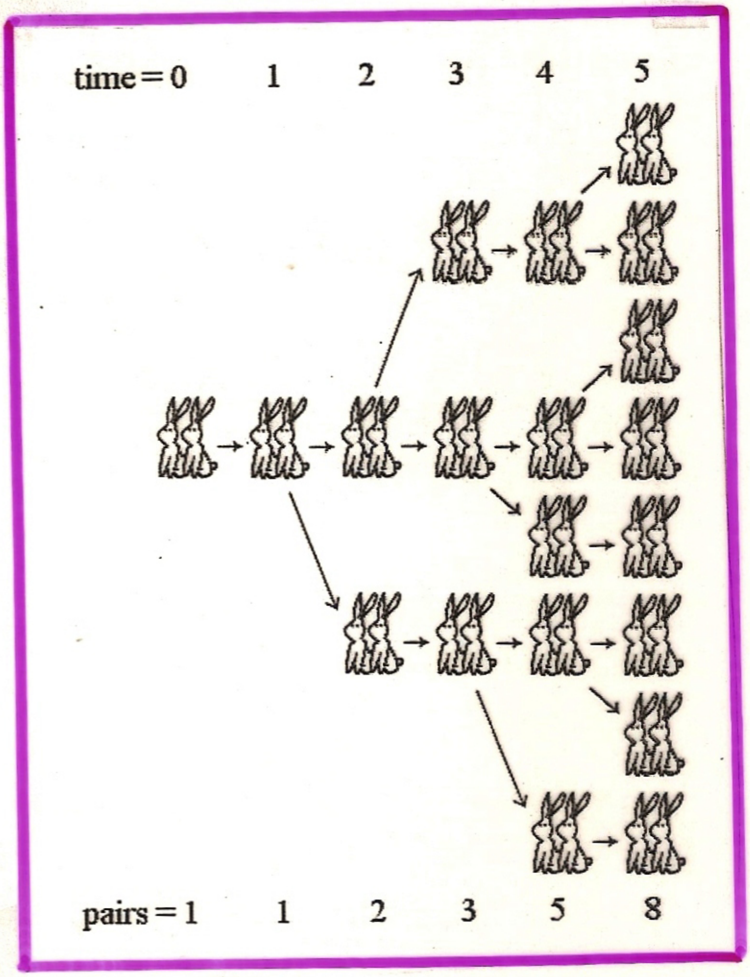
Fibonacci and the rabbits

How many pairs of rabbits can be bred from one pair in a year?

- each month they produce another pair
- in their second month each new pair can breed



Simon Jacobs. 48



1, 1, 2, 3, 5, 8, 13, 21,

34, 55, 89, 144, 233,

377

The problem of the birds

If I buy 3 sparrows for a penny,

turtle-doves 2 for a penny,

and doves for two pence –

and spend 30 pence for 30 birds,

how many of each kind do I buy?

We have: $s + t + d = 30$

$s/3 + t/2 + 2d = 30$,

so: $2s + 3t + 12d = 180$.

Eliminating s: $t + 10d = 120$, so $10 \mid t$.

So: $t = 10$, $d = 11$, $s = 9$ ✓

or $t = 20$, $d = 10$, $s = 0$ ✗

'Russian' multiplication

$23 \times 89 = 2047$

23	89
11	178
5	356
~~2~~	~~712~~
1	1424
	2047

89	23
~~44~~	~~46~~
~~22~~	~~92~~
11	184
5	368
~~2~~	~~736~~
1	1472
	2047

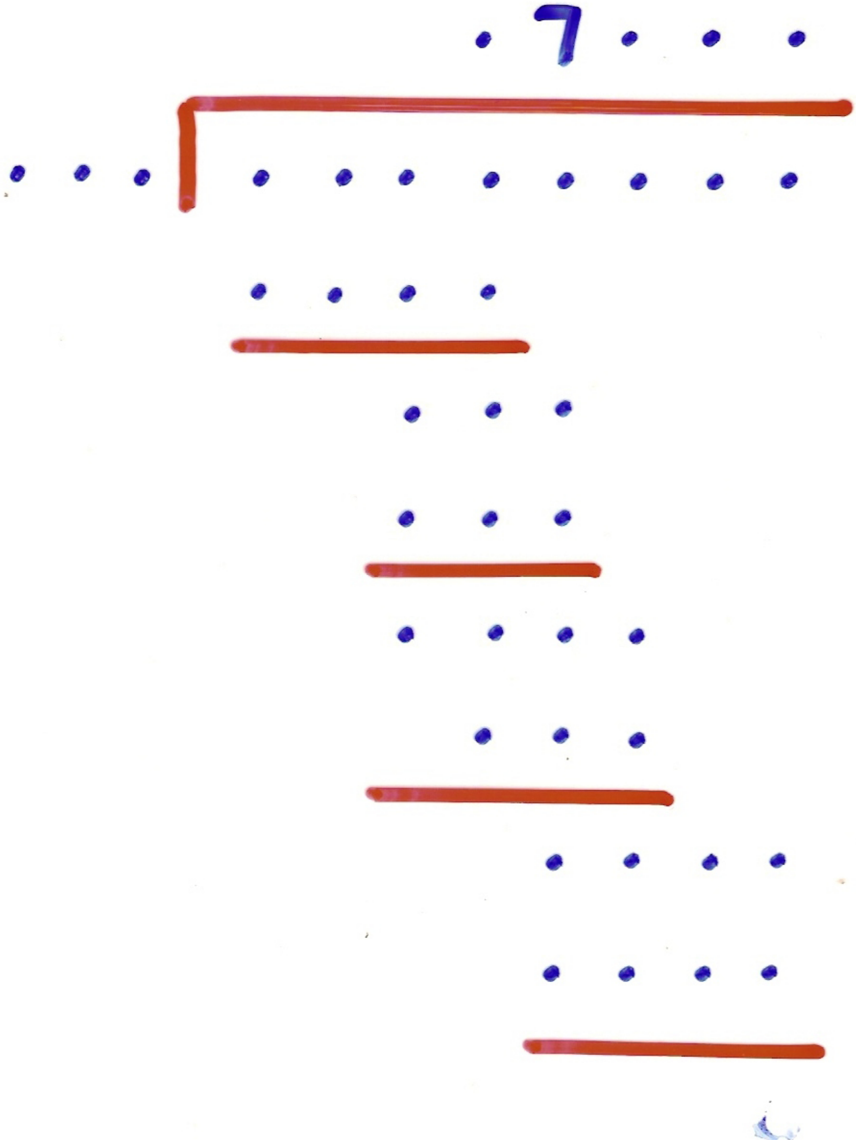
Two addition sums

	•	•	•	
	•	•	•	(reverse)
	•	•	•	(subtract)
	•	•	•	(reverse)
•	•	•	•	(add)

£9	16s	5d	
£5	16s	9d	(reverse)
£3	19s	8d	(subtract)
£8	19s	3d	(reverse)
£12	18s	11d	(add)

A division sum

$$
\begin{array}{r|cccccccc}
 & & & & 9 & 7 & 8 & 0 & 9 \\
\hline
124 & 1 & 2 & 1 & 2 & 8 & 3 & 0 & 6 \\
 & 1 & 1 & 1 & 6 & & & & \\
\cline{2-5}
 & & & 9 & 6 & 8 & & & \\
 & & & 8 & 6 & 8 & & & \\
\cline{4-6}
 & & & 1 & 0 & 0 & 3 & & \\
 & & & & 9 & 9 & 2 & & \\
\cline{4-7}
 & & & & & 1 & 1 & 1 & 6 \\
 & & & & & 1 & 1 & 1 & 6 \\
\cline{6-9}
 & & & & & & & & 0
\end{array}
$$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Lewis Carroll's money problem

A customer bought goods in a shop

to the amount of 7s. 3d.

The only money he had was

a half-sovereign, a florin, and a sixpence:

so he wanted change.

The shopman only had

a crown, a shilling, and a penny.

But a friend happened to come in,

who had a double-florin, a half-crown,

a fourpenny-bit, and a threepenny bit.

Could they manage it?

Start:	C	10/-	2/-	6d		12/6
	S	5/-	1/-	1d		6/1
	F	4/-	2/6	4d	3d	7/1
End:	C	5/-	3d			5/3
	S	10/-	2/-	1/-	4d	13/4
	F	4/-	2/6	6d	1d	7/1

A problem involving speeds

If I drive from Oxford to Cambridge

at 40 miles per hour

and then from Cambridge to Oxford

at 60 miles per hour,

what is my average speed

for the whole journey?

If d is the one-way distance,

then $\text{time1} = d/40$

$\text{time2} = d/60$

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{2d}{\frac{d}{40} + \frac{d}{60}} = \frac{2d}{5d/120} = 48 \text{ mph.}$$

1. (1) Babies are illogical.

(2) Nobody is despised who can manage a crocodile.

(3) Illogical persons are despised.

⇒ Babies cannot manage crocodiles.

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2. (1) No kitten that loves fish is unteachable.

(2) No kitten without a tail will play with a gorilla.

(3) Kittens with whiskers always love fish.

(4) No teachable kitten has green eyes.

(5) No kittens have tails unless they have whiskers.

⇒ No kitten with green eyes will play with a gorilla.

A problem in logic

Pamela Potter's pease pottage is putrid
provided that
Pablo Picasso painted potted palms.

Either Pablo Picasso painted potted palms,
or Peter Piper did not pick a peck
of pickled peppers.

There are two possibilities:
either Peter Piper picked a peck of pickled peppers or else it is impossible that both Pablo Picasso did not paint potted palms
and that Pamela Potter's porridge is putrid.

Is Pamela's porridge putrid?

Solution: p = *Pamela's porridge is putrid*

q = *Pablo Picasso painted potted palms*

r = *Peter Piper picked pickled pepper*

(1) $q \rightarrow p$ (2) q or $(\sim r)$ (3) r or $\sim [(\sim q) \,\&\, (\sim p)]$

If p is false, then by (1) q is false;

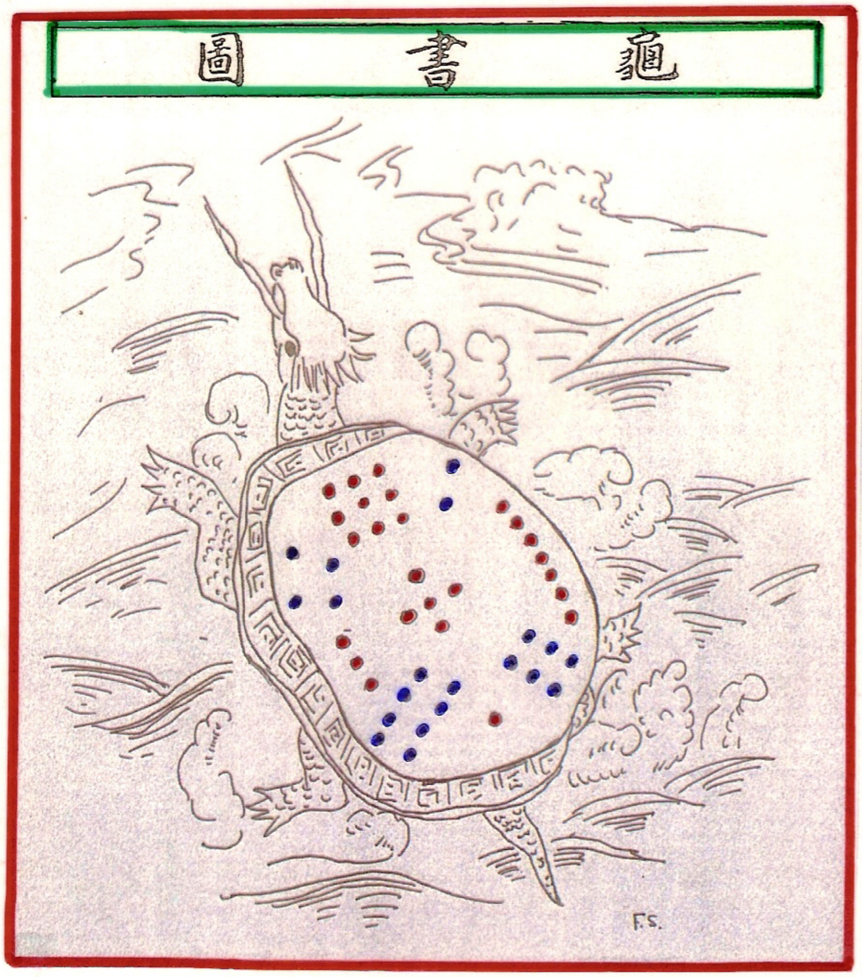
by (2), r is false; and (3) is then contradictory.

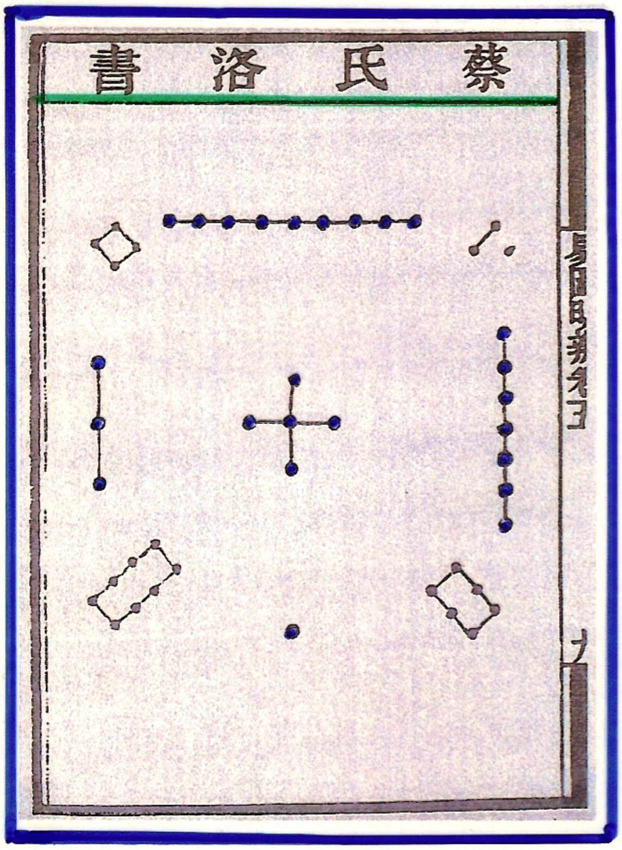
So *p is true*: *Pamela's porridge is putrid*

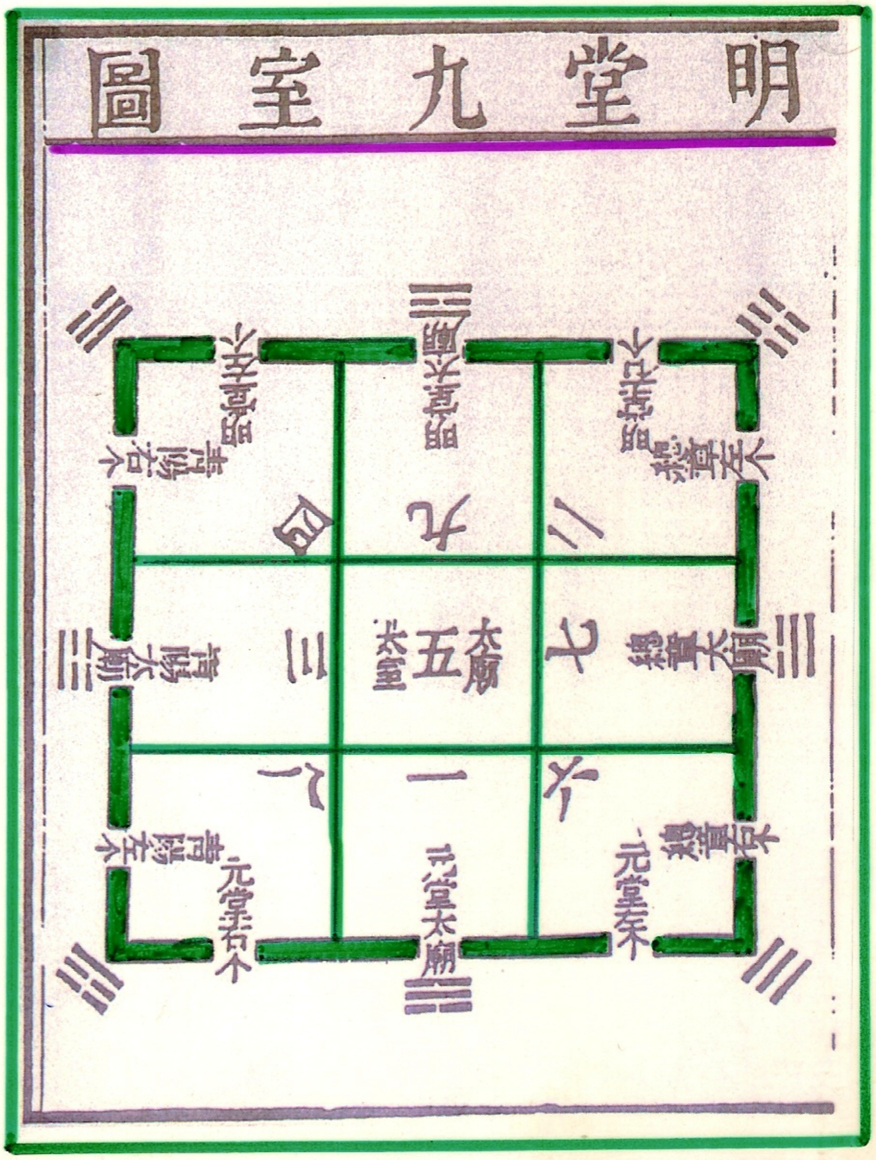
Lo-Shu

Magic Square

4	9	2
3	5	7
8	1	6

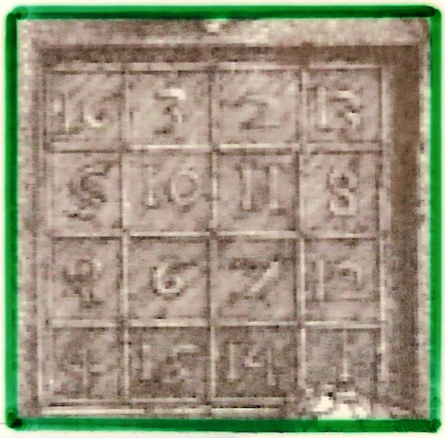


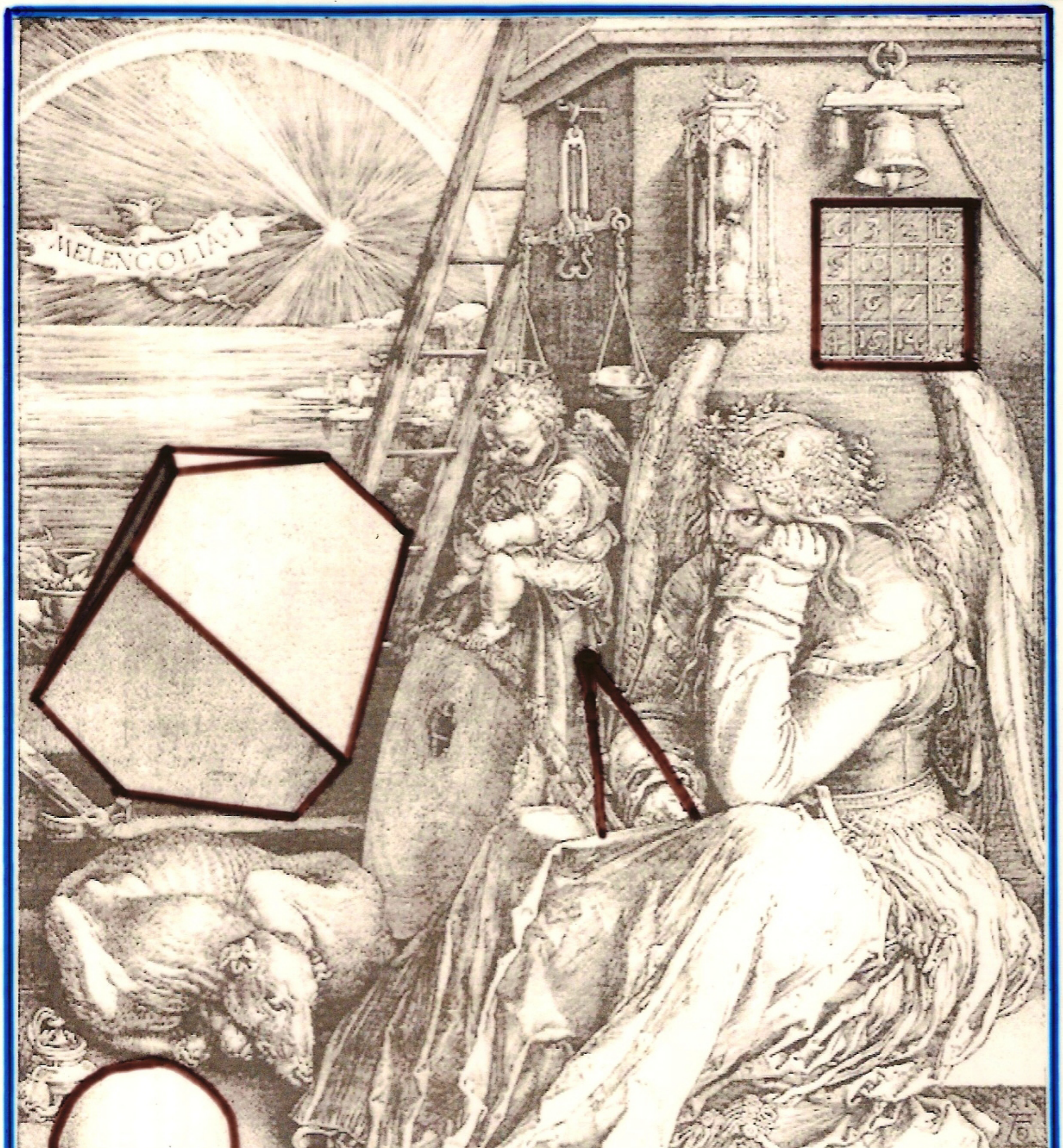




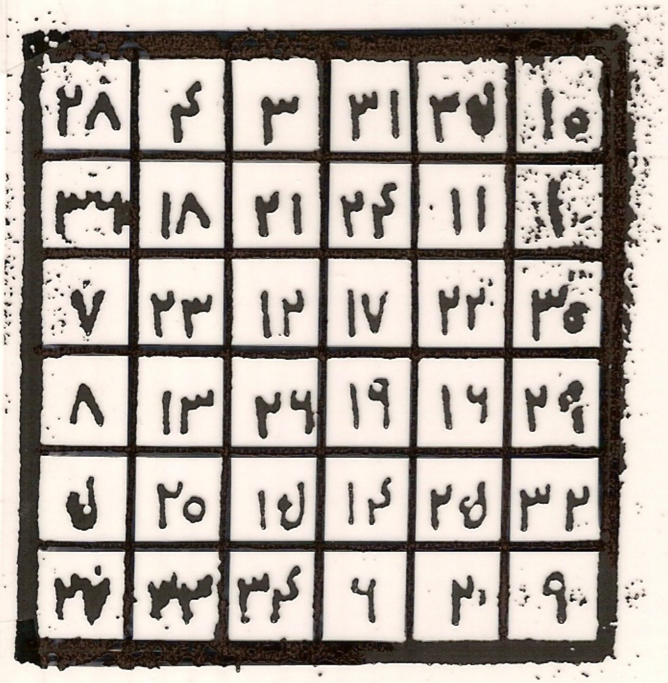
Dürer's 'Melencolia'

(1514)





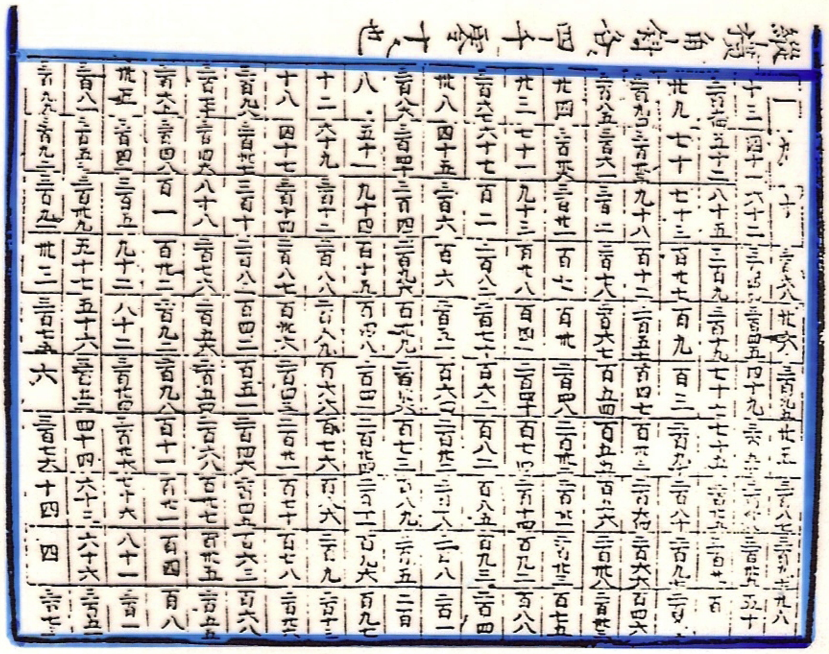
An Arabic Magic Square



Iron plate found at Xian

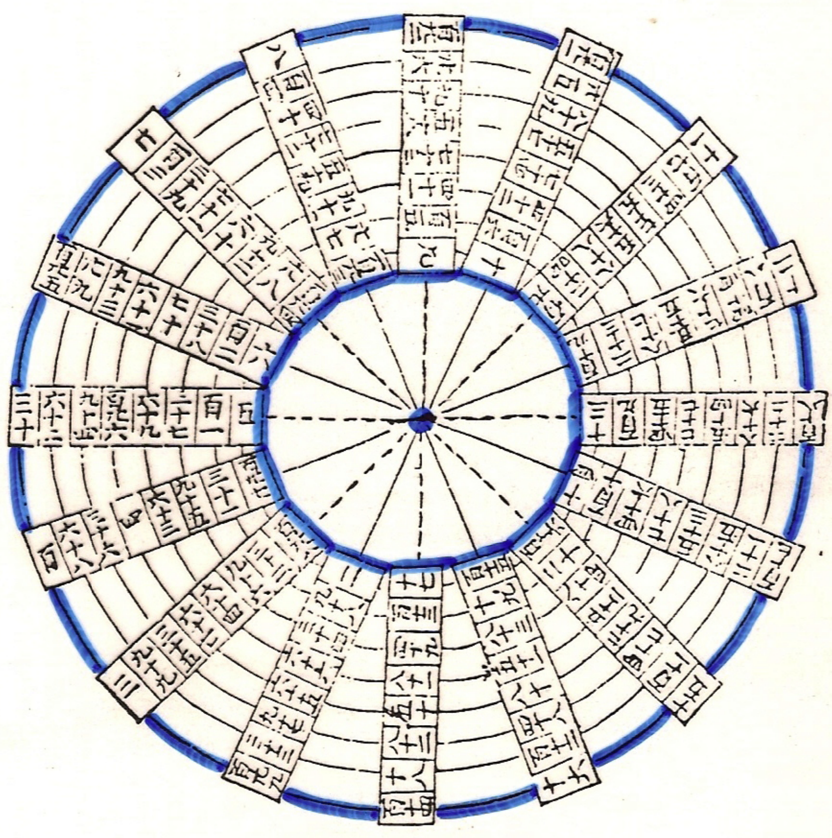
28	4	3	31	35	10
36	18	21	24	11	1
7	23	12	17	22	30
8	13	26	19	16	29
5	20	15	14	25	32
27	33	34	6	2	9

Magic squares in Japan (17th cent.)



MAGIC SQUARES IN JAPAN

Half of a magic square as given in Hoshino Sanenobu's *Kō-ko-gen Shō*, 1673



MAGIC CIRCLE OF 129 NUMBERS

From Muramatsu Kudayū Mosei's *Mantoku Jinkō-ki*, 1665. The numbers in each radius add to 524, or 525 with the center 1

1275 :

Yang Hui :

9 × 9 =
nine
3×3 magic
squares

31	76	13	36	81	18	29	74	11
22	40	58	27	45	63	20	38	56
67	4	49	72	9	54	65	2	47
30	75	12	32	77	14	34	79	16
21	39	57	23	41	59	25	43	61
66	3	48	68	5	50	70	7	52
35	80	17	28	73	10	33	78	15
26	44	62	19	37	55	24	42	60
71	8	53	64	1	46	69	6	51

Magic square of al-Antaakii (d. 987)

62	2	222	220	8	10	214	213	212	16	18	206	204	24	64
126	78	26	198	196	32	11	189	207	34	190	188	40	80	100
128	122	94	42	182	7	35	173	183	203	180	48	96	104	98
50	124	118	110	3	31	51	165	167	179	199	112	108	102	176
52	70	120	201	75	159	155	153	83	87	79	25	106	156	174
54	72	205	181	141	95	135	133	103	99	85	45	21	154	172
170	209	185	169	145	125	111	121	107	101	81	57	41	17	56
211	187	171	163	149	129	109	113	117	97	77	63	55	39	15
168	9	33	49	69	89	119	105	115	137	157	177	193	217	58
60	82	5	29	65	127	91	93	123	131	161	197	221	144	166
66	142	90	1	147	67	71	73	143	139	151	225	136	84	160
158	140	92	114	223	195	175	61	59	47	27	116	134	86	68
152	88	130	184	44	219	191	53	43	23	46	178	132	138	74
76	146	200	28	30	194	215	37	19	192	36	38	186	148	154
162	224	4	6	218	216	12	13	14	210	208	20	22	202	164

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62	2	222	220	8	10	214	213	212	16	18.	206	204	24	64
126	78	26	198	196	32	11	189	207	34	190	188	40	80	100
128	122	94	42	182	7	35	173	183	203	180	48	96	104	98
50	124	118	110	3	31	51	165	167	179	199	112	108	102	176
52	70	120	201	75	159	155	153	83	87	79	25	106	156	174
54	72	205	181	141	95	135	133	103	99	85	45	21	154	172
170	209	185	169	145	125	111	121	107	101	81	57	41	17	56
211	187	171	163	149	129	109	113	117	97	77	63	55	39	15
168	9	33	49	69	89	119	105	115	137	157	177	193	217	58
60	82	5	29	65	127	91	93	123	131	161	197	221	144	166
66	142	90	1	147	67	71	73	143	139	151	225	136	84	160
158	140	92	114	223	195	175	61	59	47	27	116	134	86	68
152	88	130	184	44	219	191	53	43	23	46	178	132	138	74
76	146	200	28	30	194	215	37	19	192	36	38	186	148	154
162	224	4	6	218	216	12	13	14	210	208	20	22	202	164

Benjamin Franklin's magic square

16 × 16

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

sum of any row or column = 2056

sum of any half-row or half-column = 1028

sum of four corners

+ sum of four central squares = 1028

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16 × 16

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
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60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
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60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

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Sum of four corners

+ sum of four central squares = 1028

Latin Squares

1	2	3
2	3	1
3	1	2

A	B	C	D
C	D	A	B
D	C	B	A
B	A	D	C

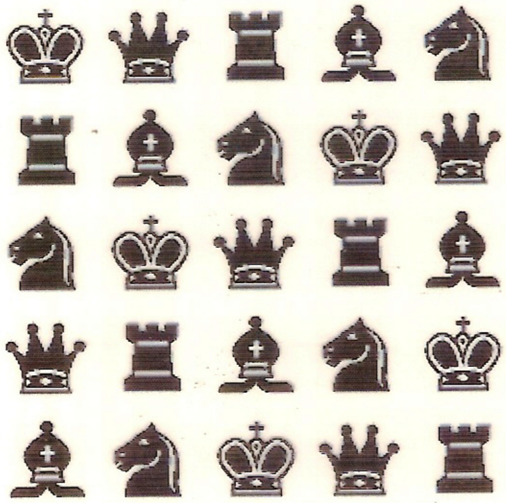
L	A	T	I	N
T	I	N	L	A
N	L	A	T	I
A	T	I	N	L
I	N	L	A	T

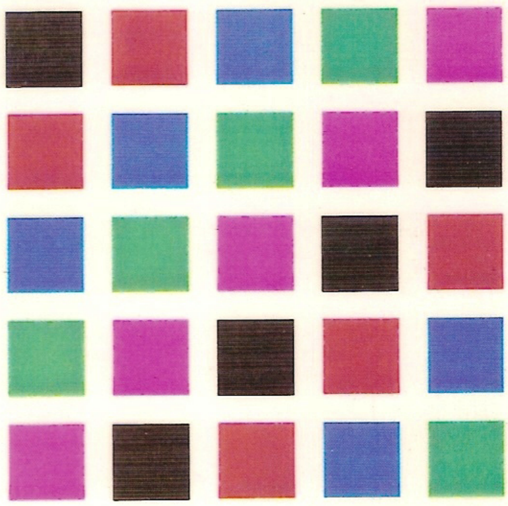
Orthogonal Latin squares

Ozanam (1725): Récréations math...

J ♦	Q ♥	K ♠	A ♣
Q ♠	J ♣	A ♦	K ♥
K ♣	A ♠	J ♥	Q ♦
A ♥	K ♦	Q ♣	J ♠

Orthogonal 5×5 Latin squares





Euler's 36 officers problem (1782)

Arrange 36 officers, one of each of six ranks and one of each of six regiments, in a square array, so that each row and column has one officer of each rank and one of each regiment.

Euler: for the $n \times n$ problem:

- cannot be done (?) for

  $n = 6, 10, 14, 18, 22, \ldots$

- can be done in all other cases

G. Tarry (1900): Euler correct for $n = 6$

Bose, Shrikande and Parker (~1960):

Euler wrong for all $n = 10, 14, 18, \ldots$

"Here we go then – another day, another Su doku"

A Latin square

8	5	6	7	2	3	9	4	1
2	7	4	9	1	6	3	8	5
3	1	9	4	8	5	2	6	7
9	8	3	6	5	1	7	2	4
5	4	1	2	3	7	8	9	6
6	2	7	8	9	4	5	1	3
7	3	2	1	4	8	6	5	9
4	6	8	5	7	9	1	3	2
1	9	5	3	6	2	4	7	8

POLYGONAL

7								1
		4		8		5		
6								8
			2		3			
				1				
	9						4	
3		1				2		5

SAMURAI SUDOKU

9	5					4		3
4					5		7	
		2		4		8		9
			7				3	
		9		3		2		
	6				8			
1		8		5		7		
	9		4					5
5		6					1	2

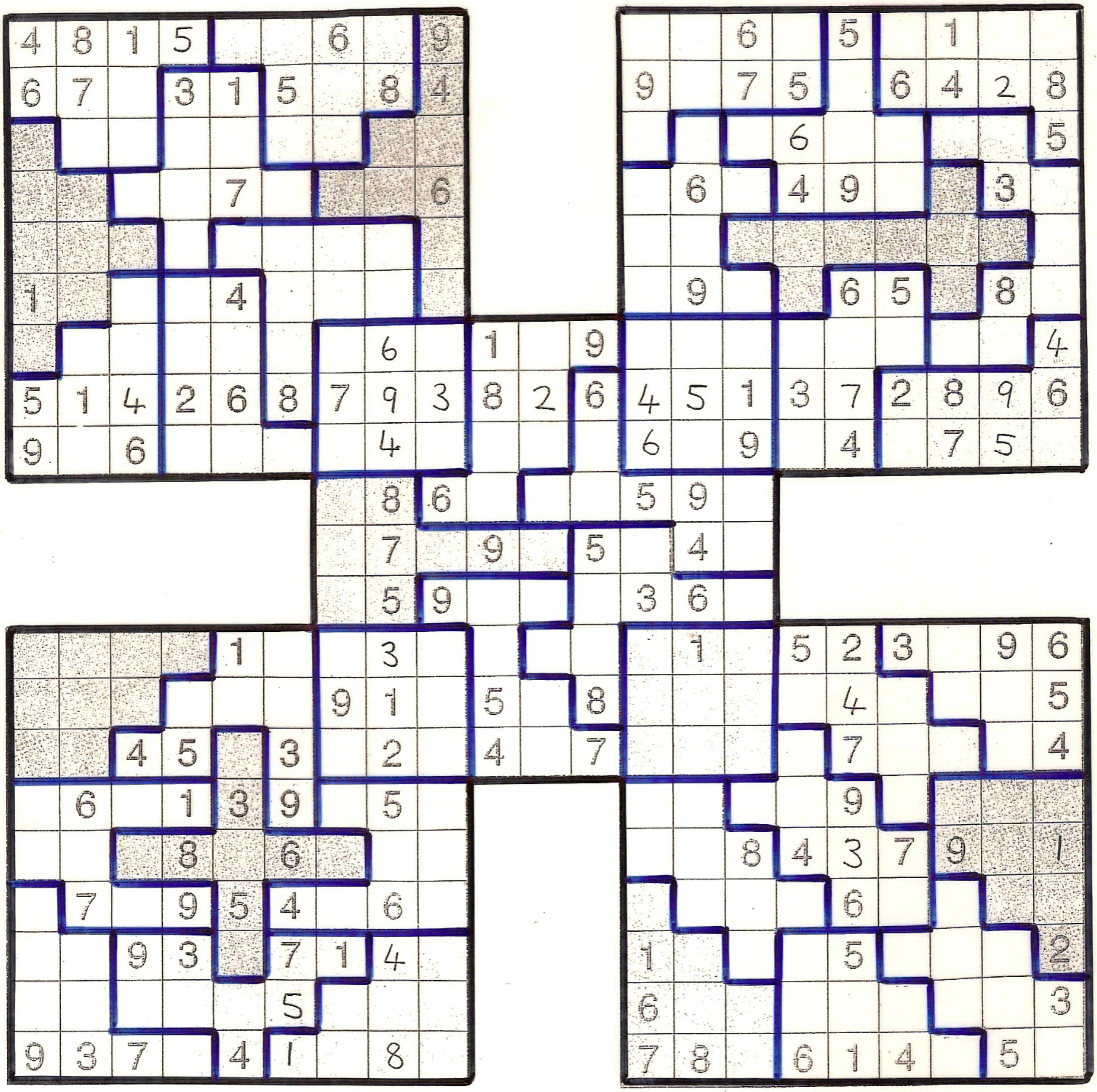
	6						2	
4		2			6			1
	9		3	8		7		
		4		1			3	
		8	5		9	2		
	5			4		6		
		1		2	7		7	
9			8			4		5
	4						8	

7								1
		5				9		
	1	2	6				4	
		9	7					
				1				
					3	6		
	7				8	3	9	
		3				4		
6								7

	6						7	
7		9			1			3
	2		8	3		6		
		8		4			3	
		5	9		2	7		
	9			1		2		
		6		7	4		5	
4			2			3		8
	1						6	

3	9					7		4
4					7		2	
		7		5		3		9
			8				7	
		9		3		2		
	2				5			
1		3		6		5		
	8		5					7
2		4					3	6

SAMURAI POLYGONAL



Orthogonal sudoku

				8	5	◇		
	♡			7 □	6	1		
		6 #				♠		⋆
			6		3 □	8 ⋆	5	
6	4 ⋆			♠			1	♣
3	8		1 ♡		2 ♣	□		6
♠	3	♡	5 □		⋆	9 △		4
			7	3			6	
		♣		△	4	7		8

The Dion Cube®

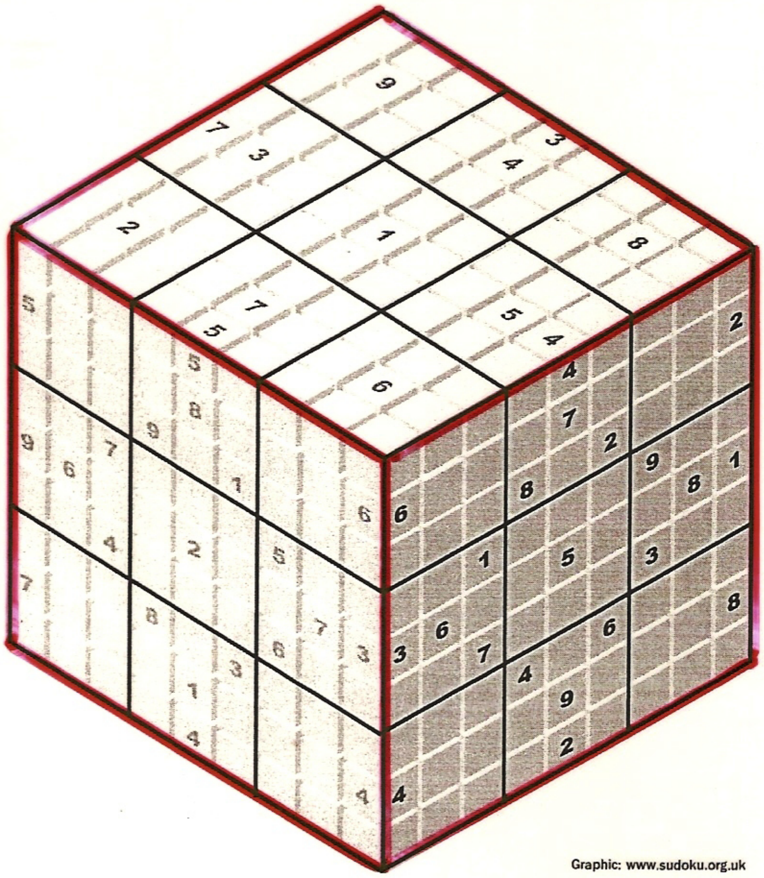
The ultimate sudoku puzzle only available from www.sudoku.org.uk

Just like a normal sudoku there is only one solution to this puzzle. Don't think of each slice as a two-dimensional puzzle, remember that "underneath" those blank squares is another set of nine numbers that also follows the sudoku rules.

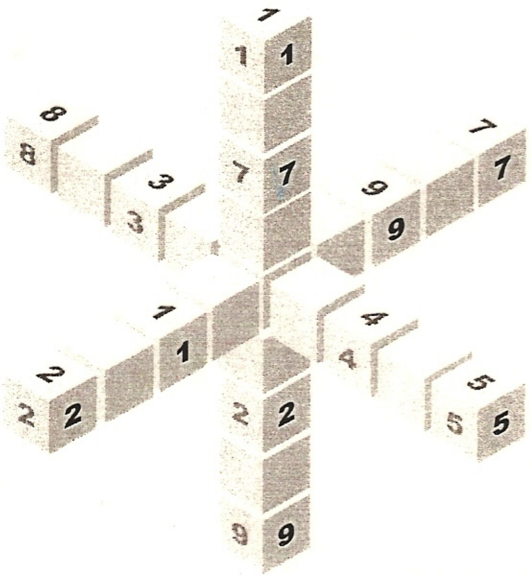
You know how to solve a normal two-dimensional sudoku: each row, column and 3x3 box must contain the numbers 1-9 without any repeats. The same goes for the Dion Cube, but now you will have to take another dimension into account. For example, if you look from the "top" face and the number 5 right at the front centre position in the illustration, this 5 is true for the row and column and box of the top "slice" and also for the vertical column below it. It has to be true for the row, column and box of the front face and similarly for the vertical slice at that position. If you want to be pedantic, it would also be true if you were looking at it from the other direction.

The puzzle is set out on the following page as nine slices, top down, showing all the clues contained within the puzzle.

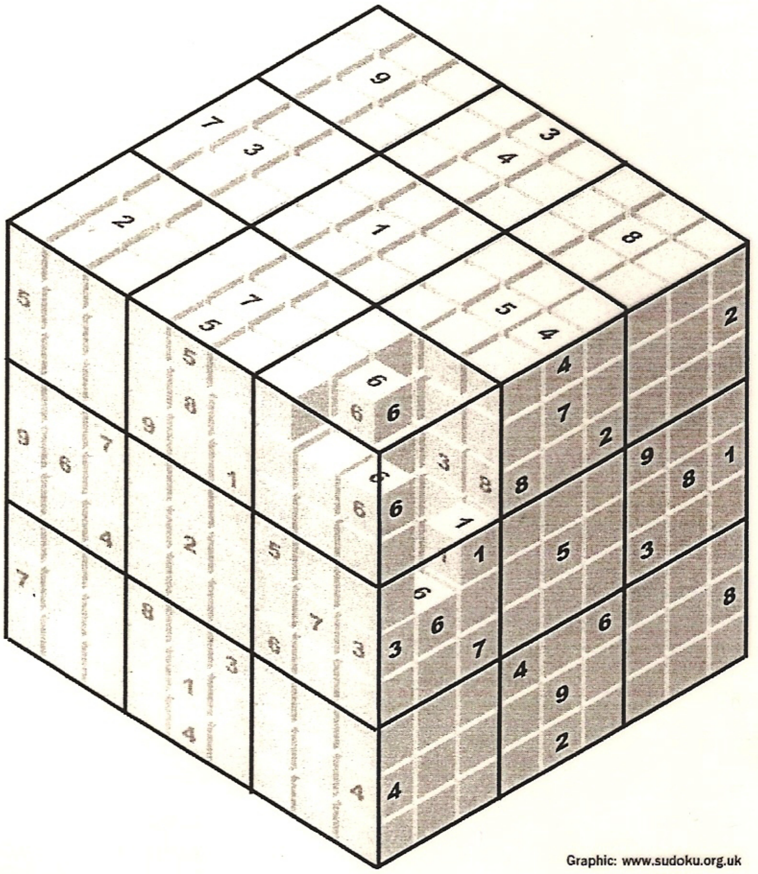
You will need plenty of worksheets, downloadable from www.sudoku.org.uk.



Graphic: www.sudoku.org.uk

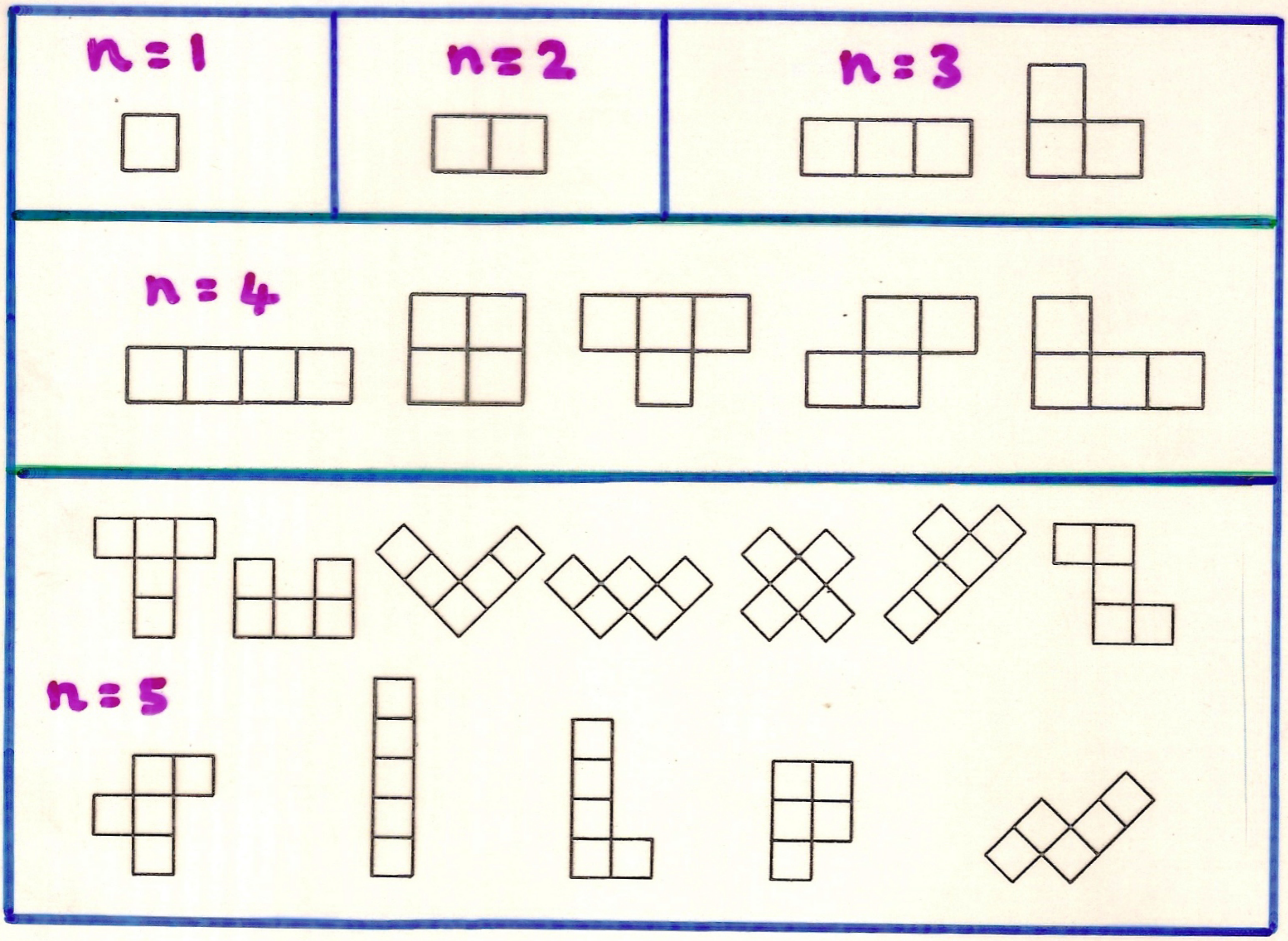


If you examine the puzzle on the next page you will see that on slice five, at the very centre is the number 6. It's hidden right at the centre of this illustration but it must be the only 6 in this part of the puzzle.



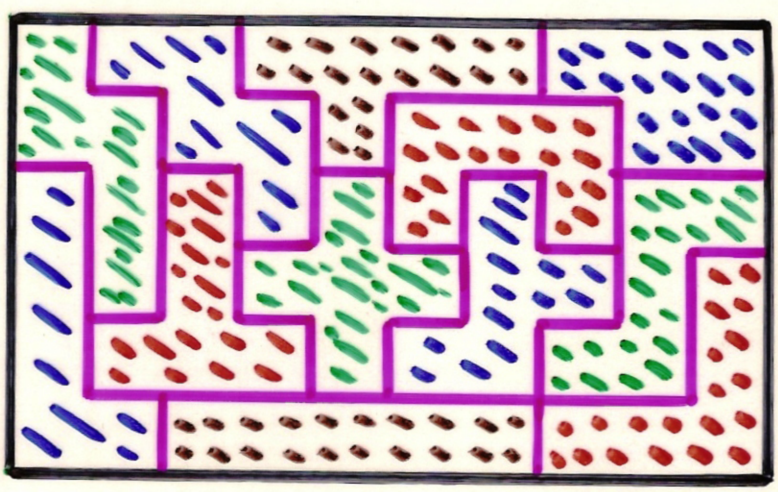
Graphic: www.sudoku.org.uk

Polyominoes [n-ominoes]



6 × 10

[2339 ways]

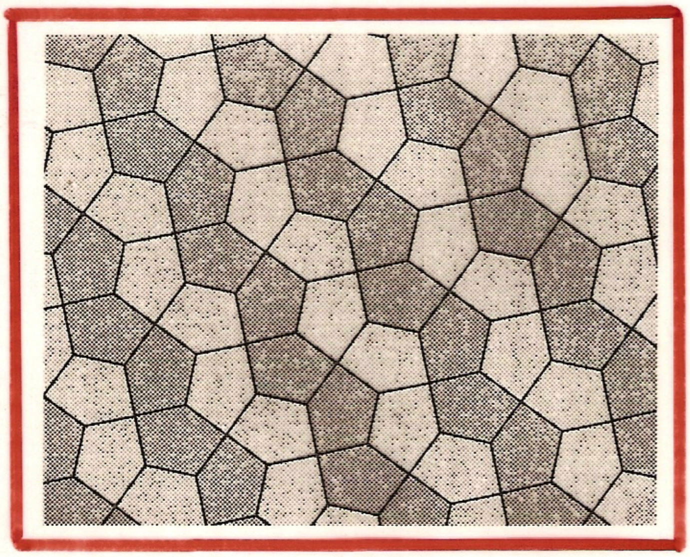


5 × 12 ?

4 × 15 ?

3 × 20 ?

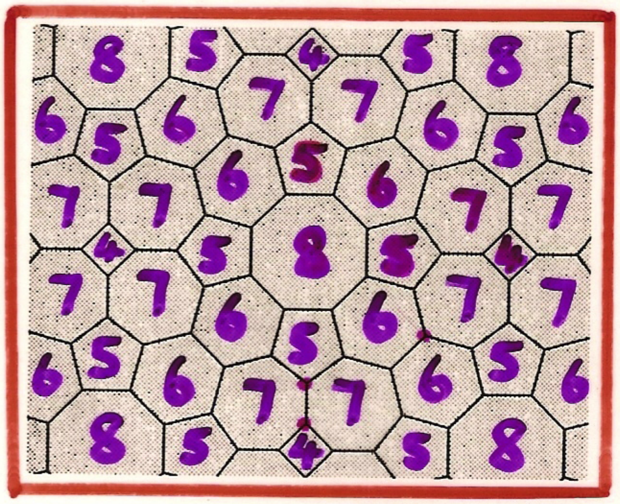
Arthur C. Clarke : Imperial Earth

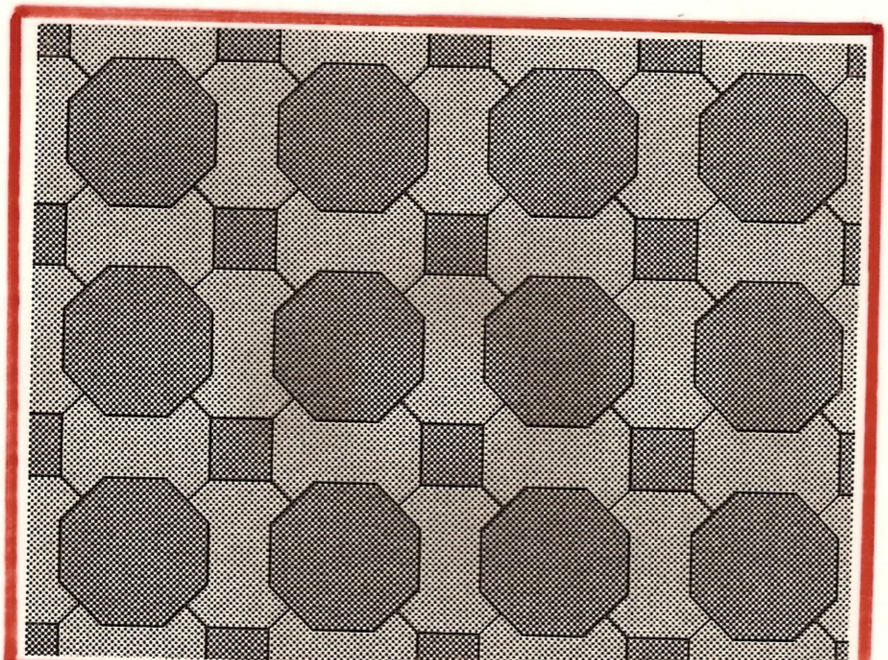


Cairo pavement

(pentagons)

Colorado floor pattern

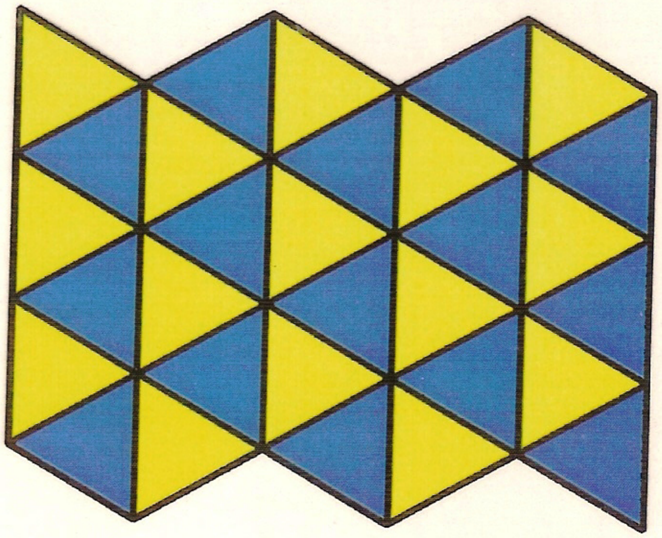


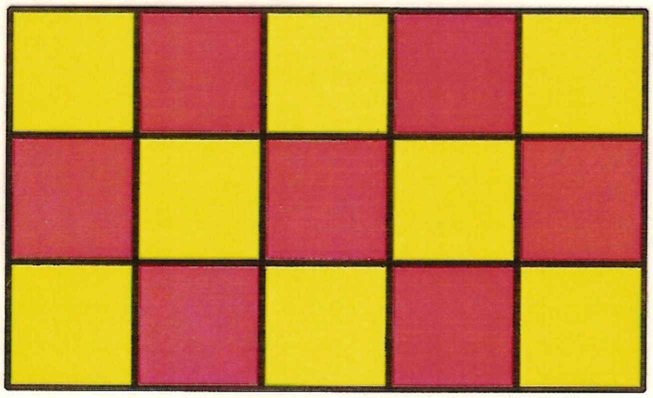


Bangkok pavement

The three regular tilings

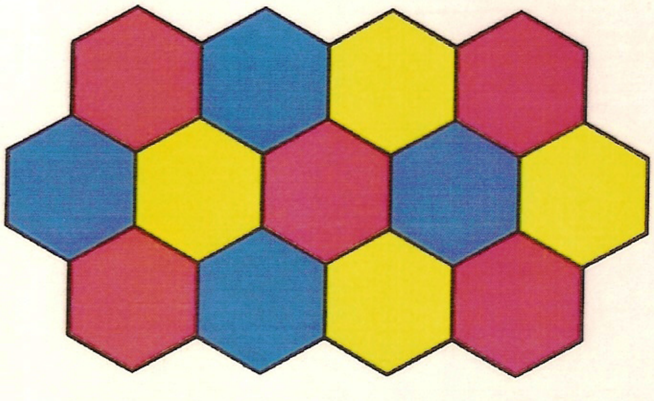
triangular



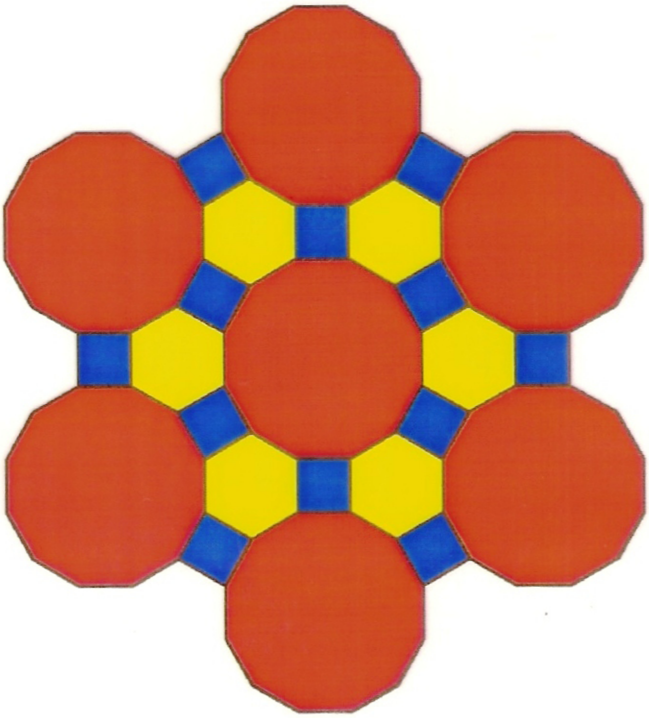


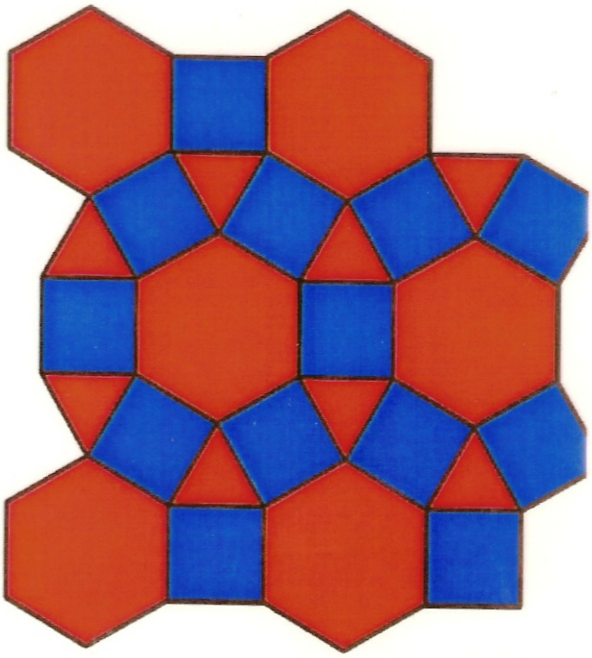
square

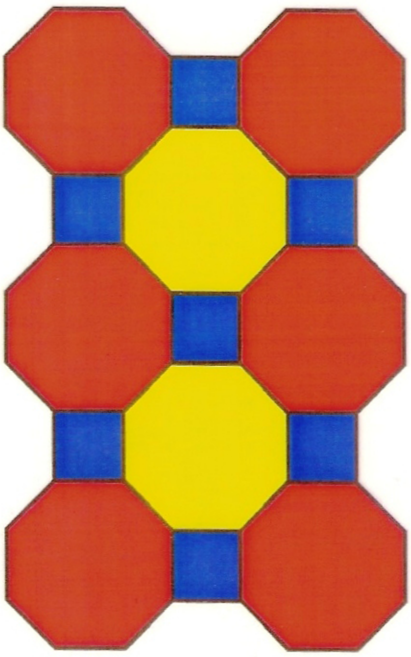
hexagonal

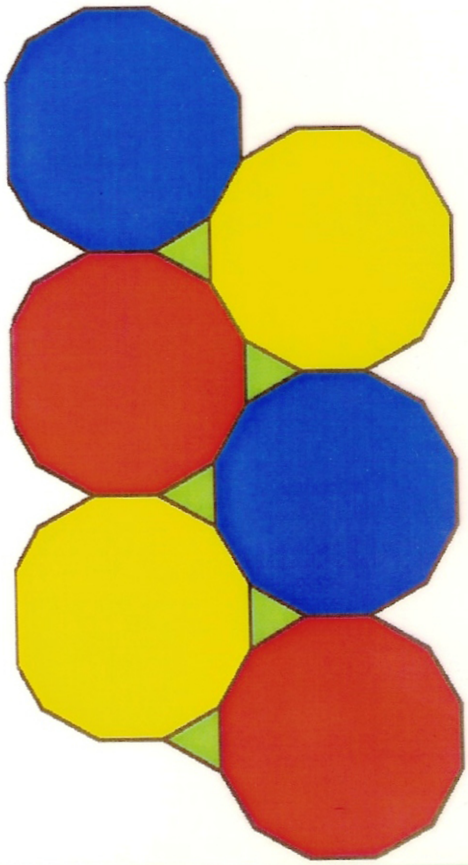


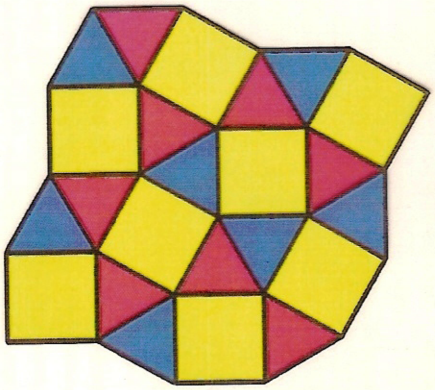
Semi-regular tilings



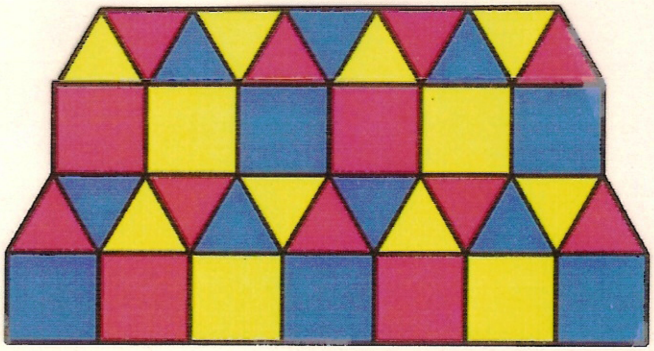




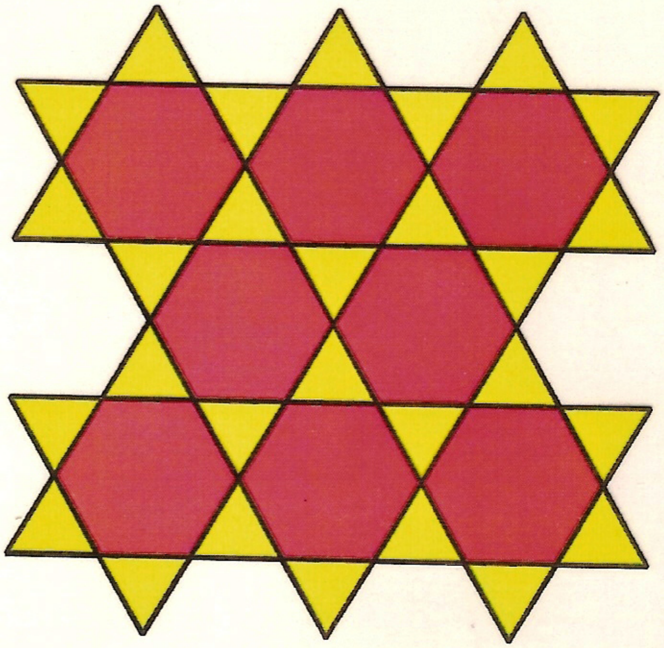




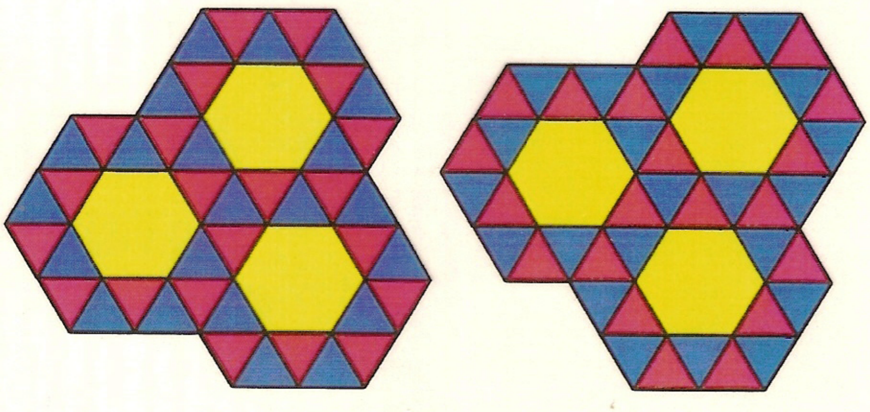
3.3.4.3.4



3.3.3.4.4



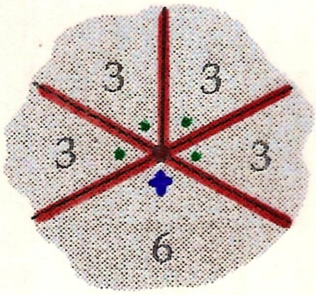
3.6.3.6



3.3.3.3.6

enantiomorphs

Fitting together polygons



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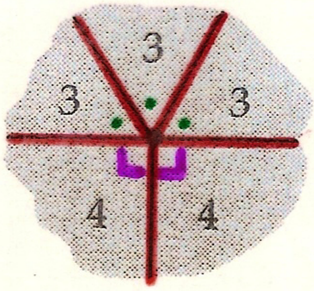

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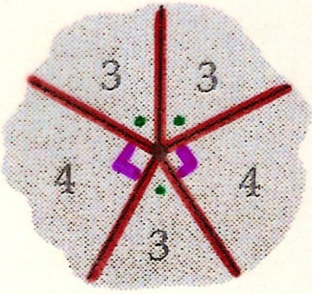

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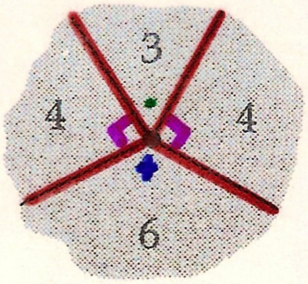

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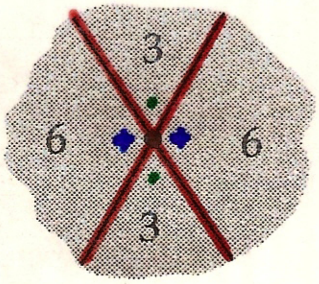
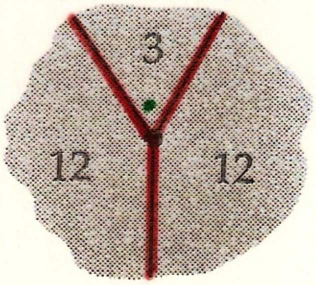
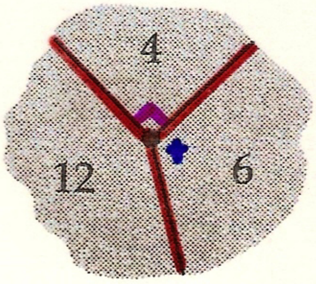


3.6.3.6



3.12.12



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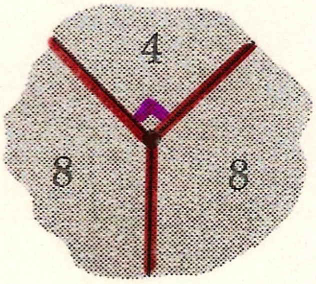
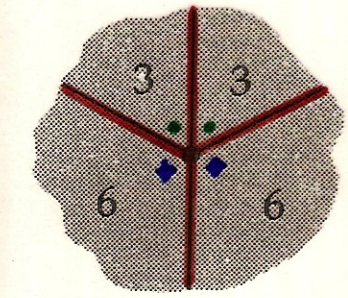


4.8.8



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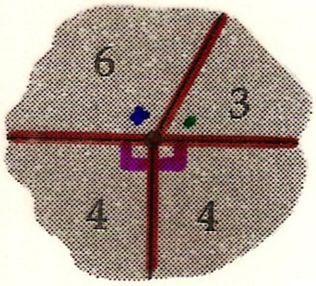

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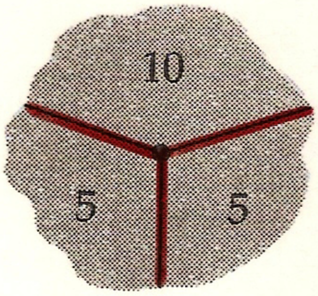
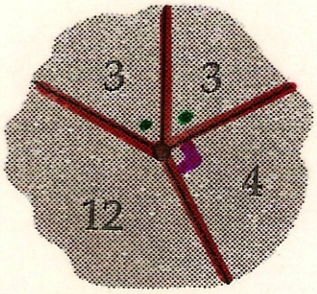
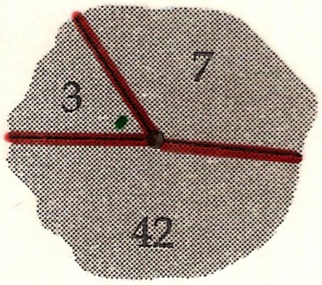


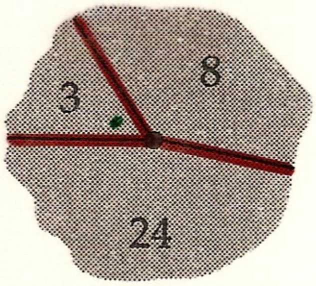
5.5.10



3.3.4.12



3.7.42



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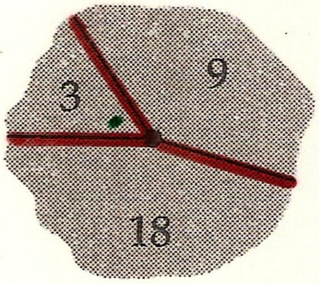
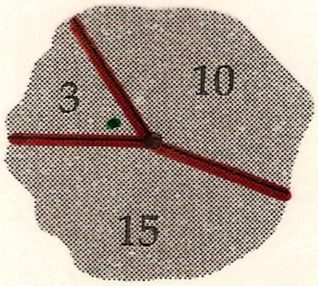
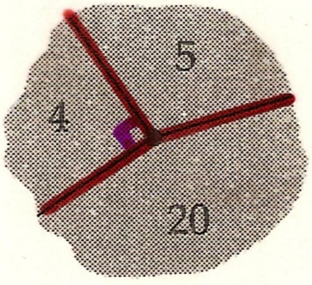


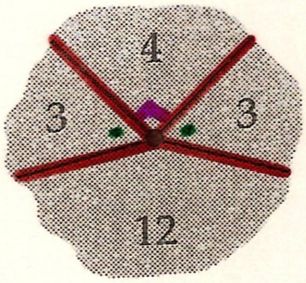
3.9.18



3.10.15

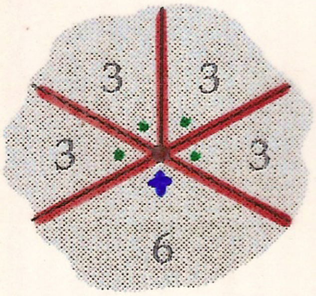


4.5.20



3.4.3.12

Fitting together polygons



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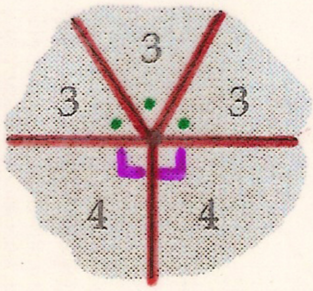

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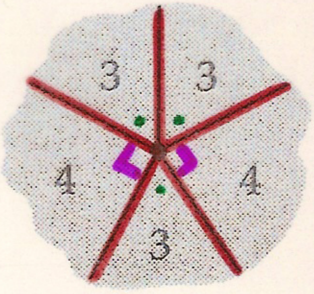

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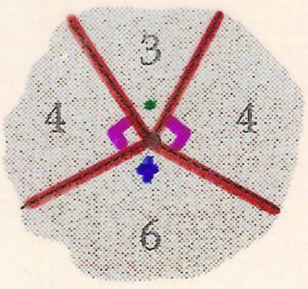
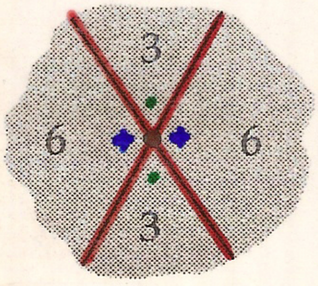


3.4.6.4



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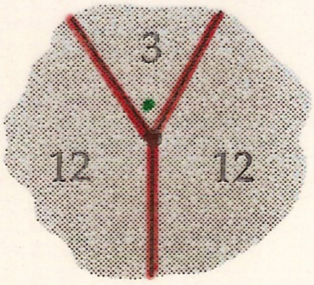
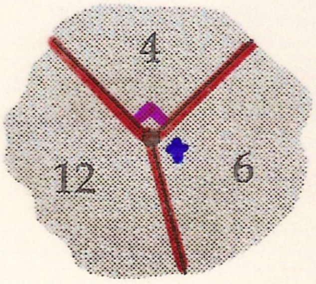
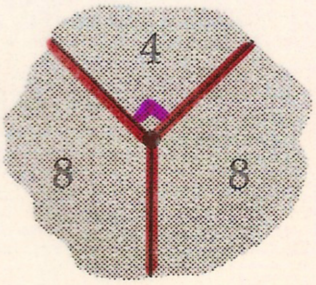


3.12.12



4.6.12



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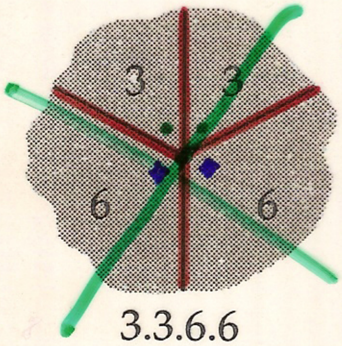

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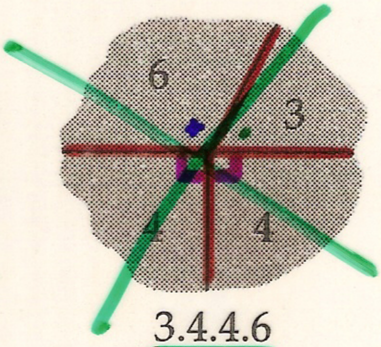

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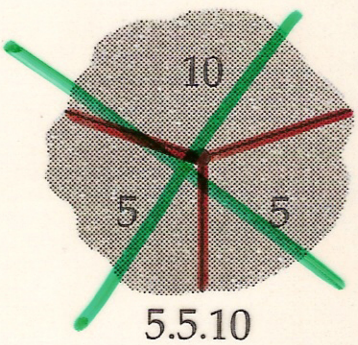
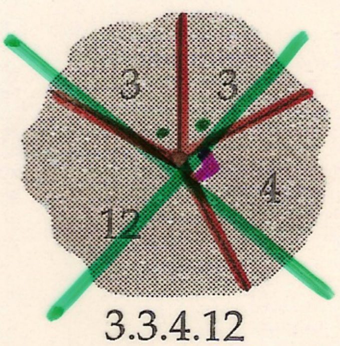
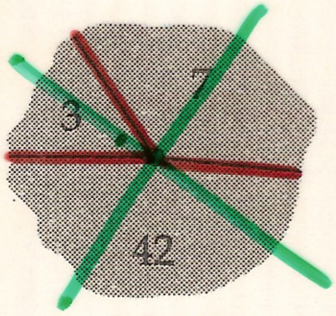


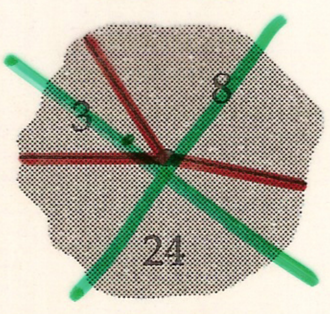
5.5.10



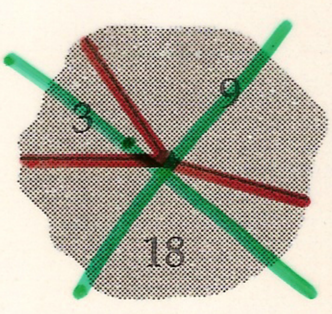
3.3.4.12



3.7.42



3.8.24



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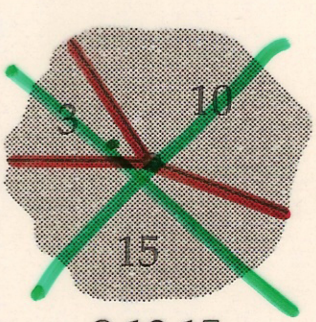

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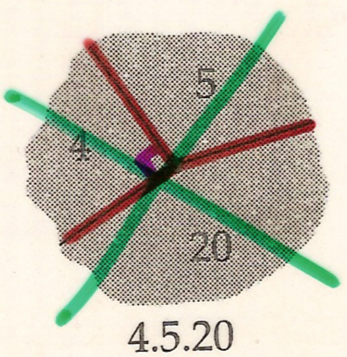
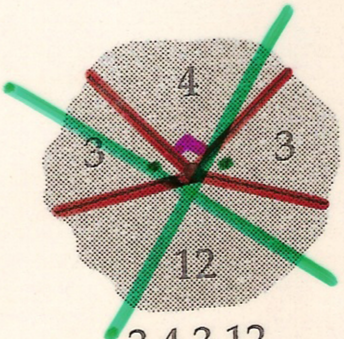


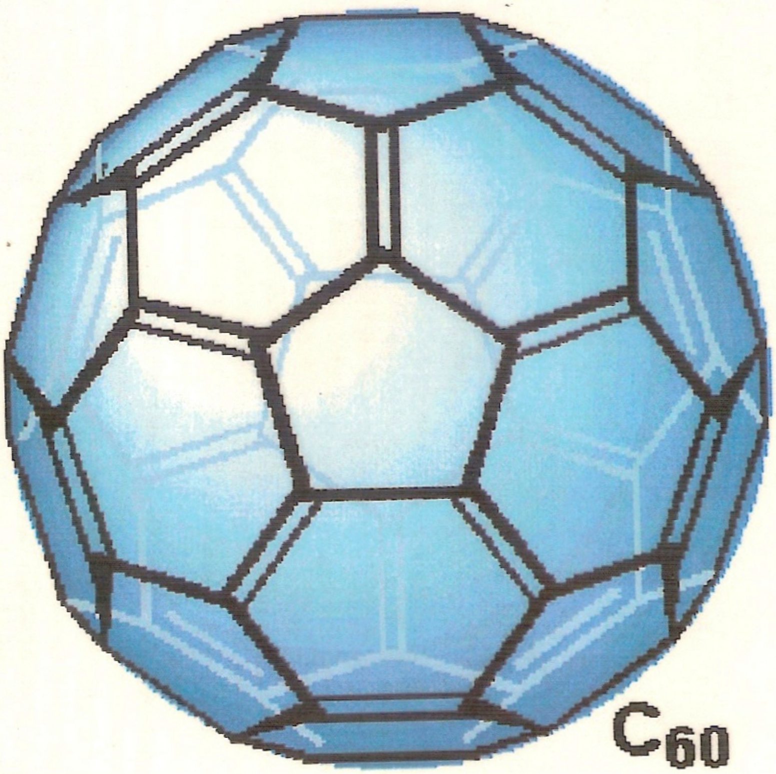
4.5.20



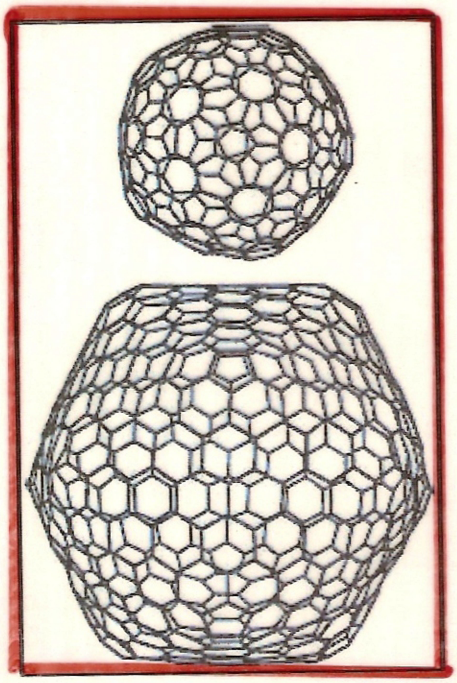
3.4.3.12

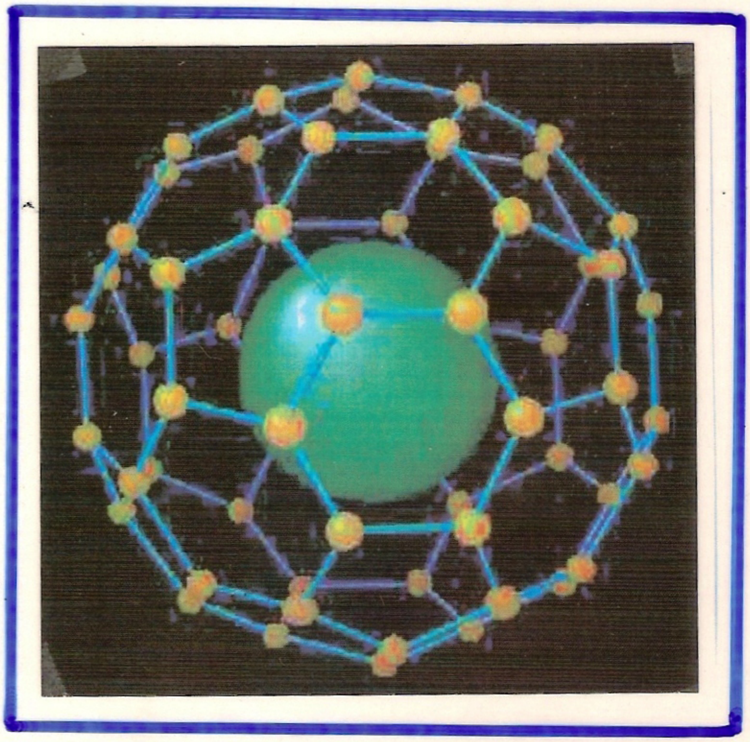
Fullerenes

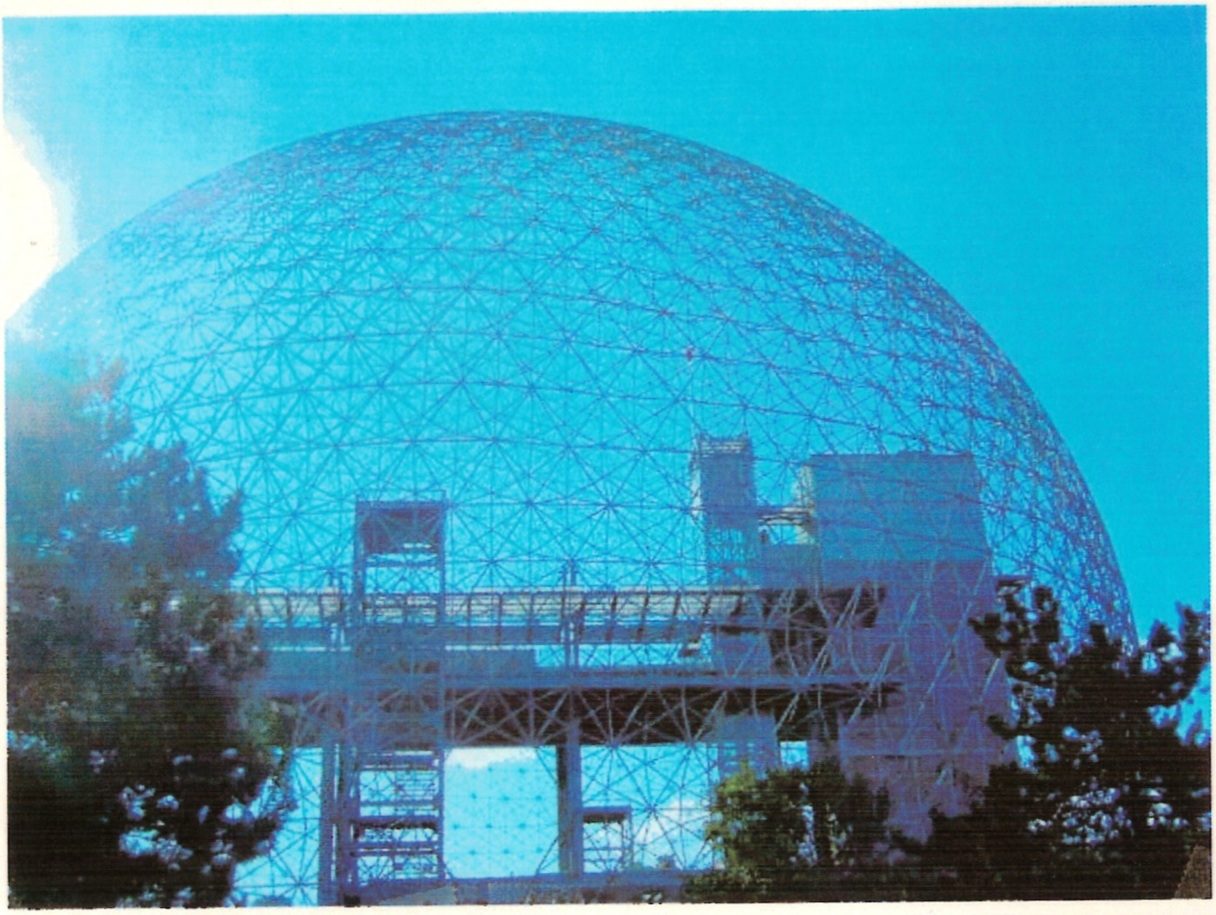
(Buckyballs)



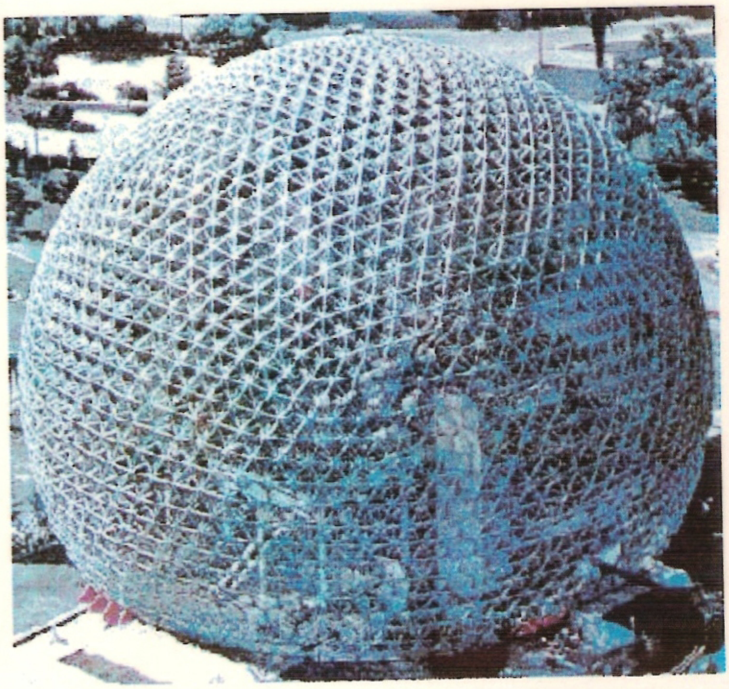
truncated icosahedron

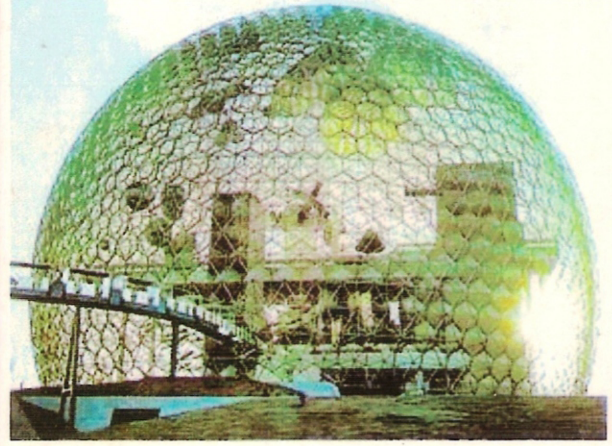






geodesic dome





Expo 67

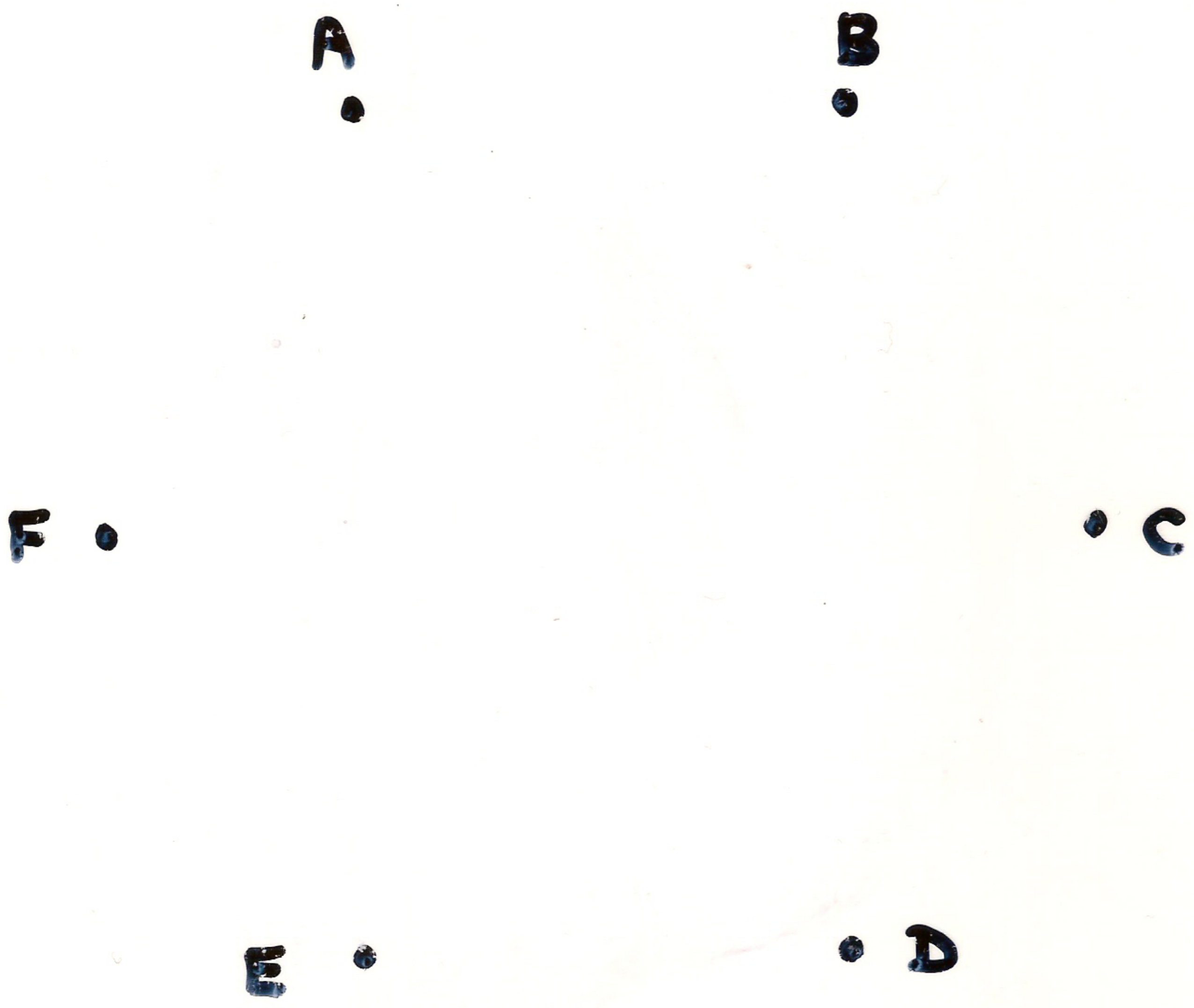
Montreal

Six People at a Party

– there are **always**

3 mutual acquaintances

or 3 mutual non-acquaintances



acquaintances

non-acquaintances

People at a party

A group of 100 people meet at a party.

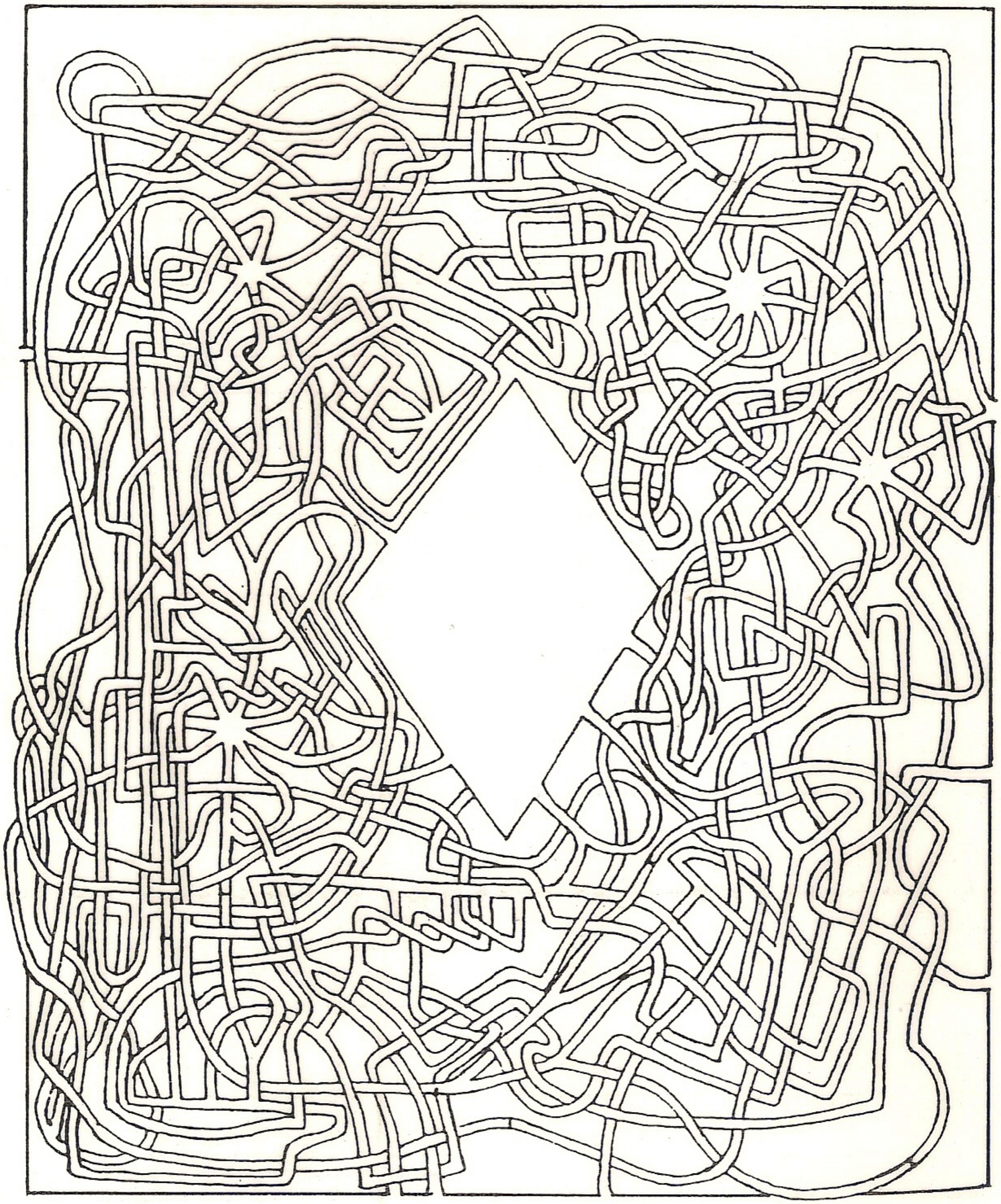
Each of them shakes hands with several other people.

There must be at least two people who have shaken hands with the same number of people. Why?

If the numbers of handshakes are all different, they must be:

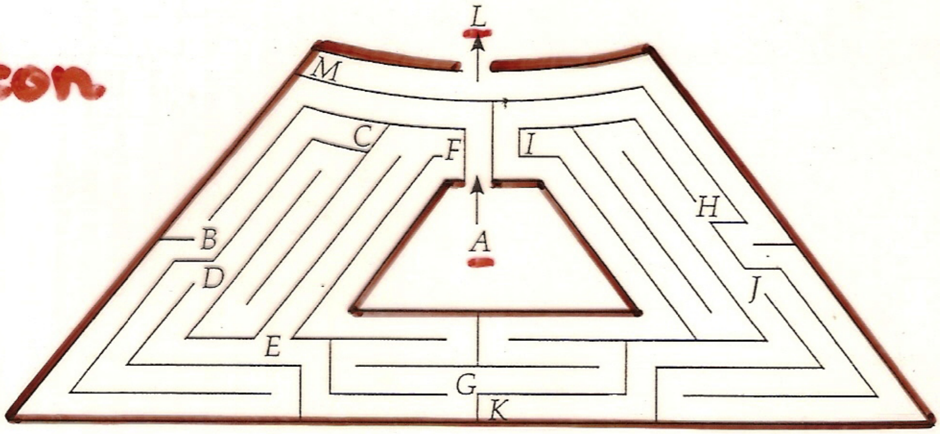
99 98 97 ... 3 2 1 0

these conflict

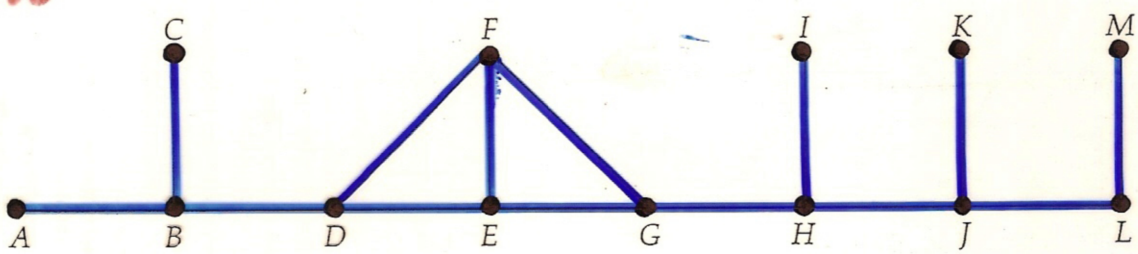


Tracing Mazes

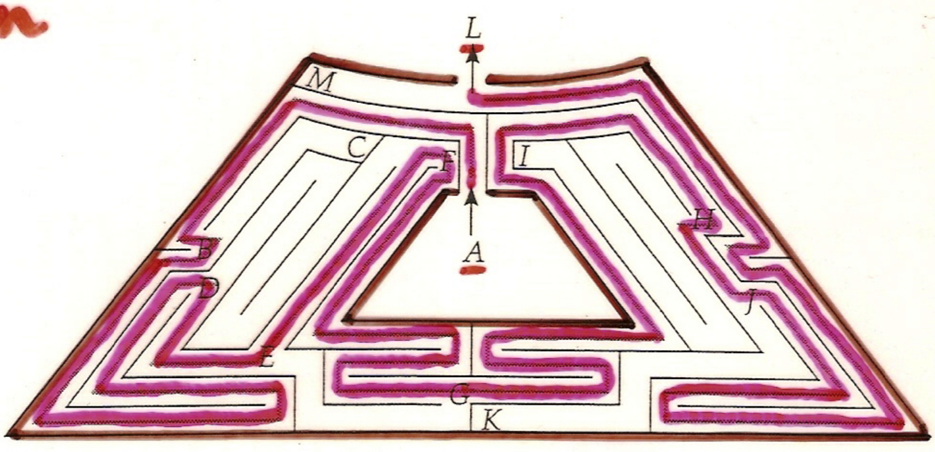
Hampton Court Maze

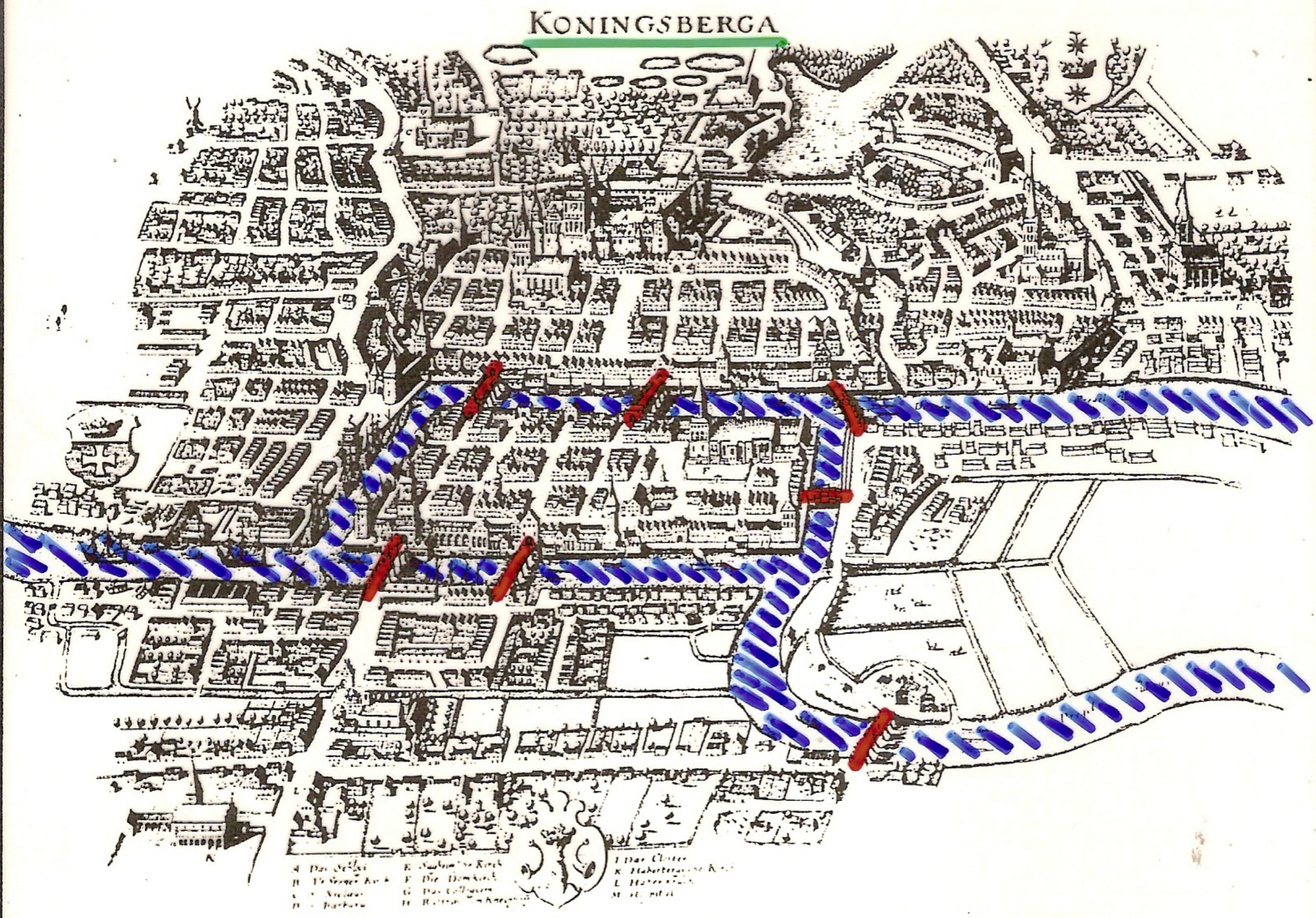


Graph



Solution

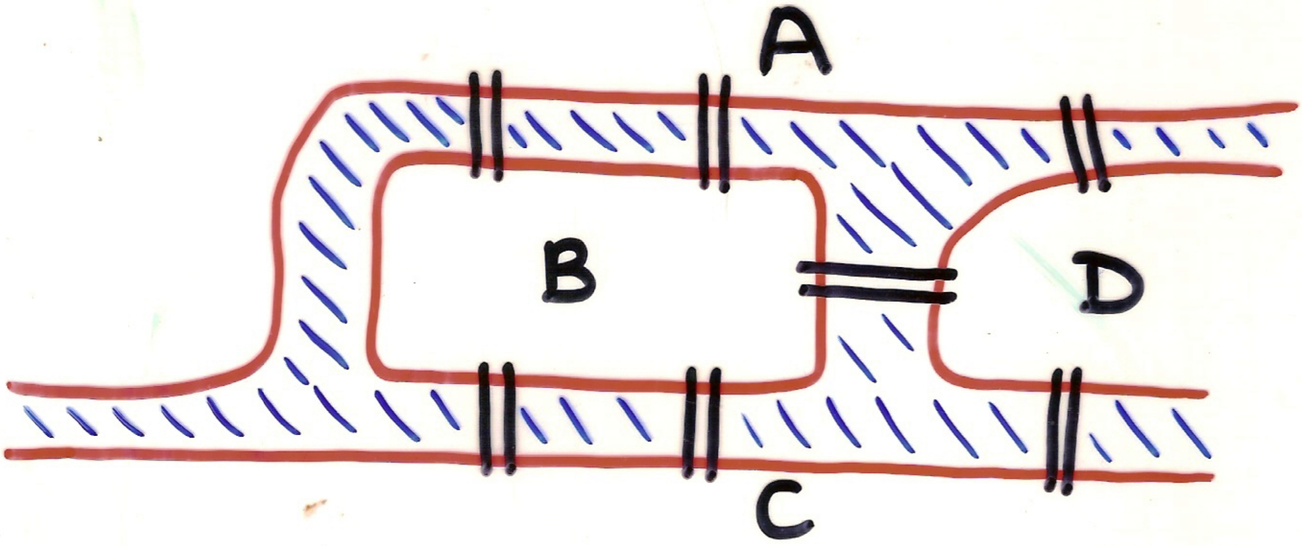




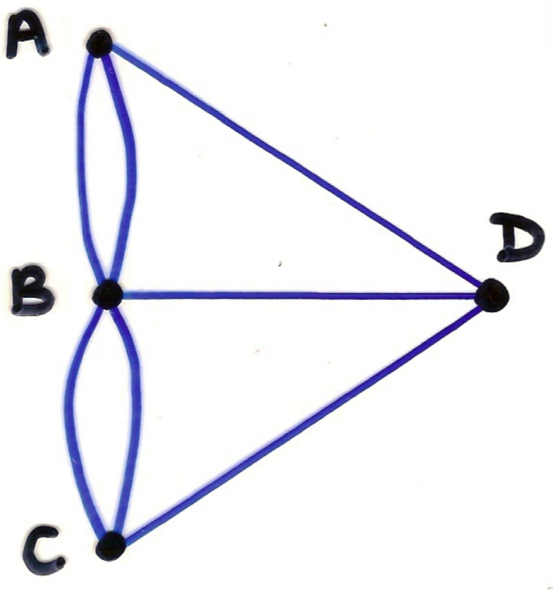
MAP OF KÖNIGSBERG

from M. Zeiller, Topographia Prussiae et Pomerelliae, Frankfurt, c. 1650.

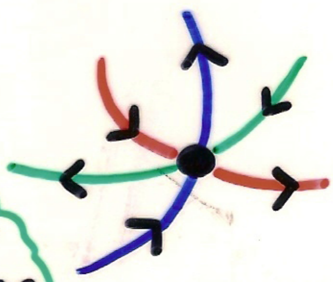
The BRIDGES of KÖNIGSBERG



Solved by EULER in 1736



whenever you enter a vertex, you must be able to leave it

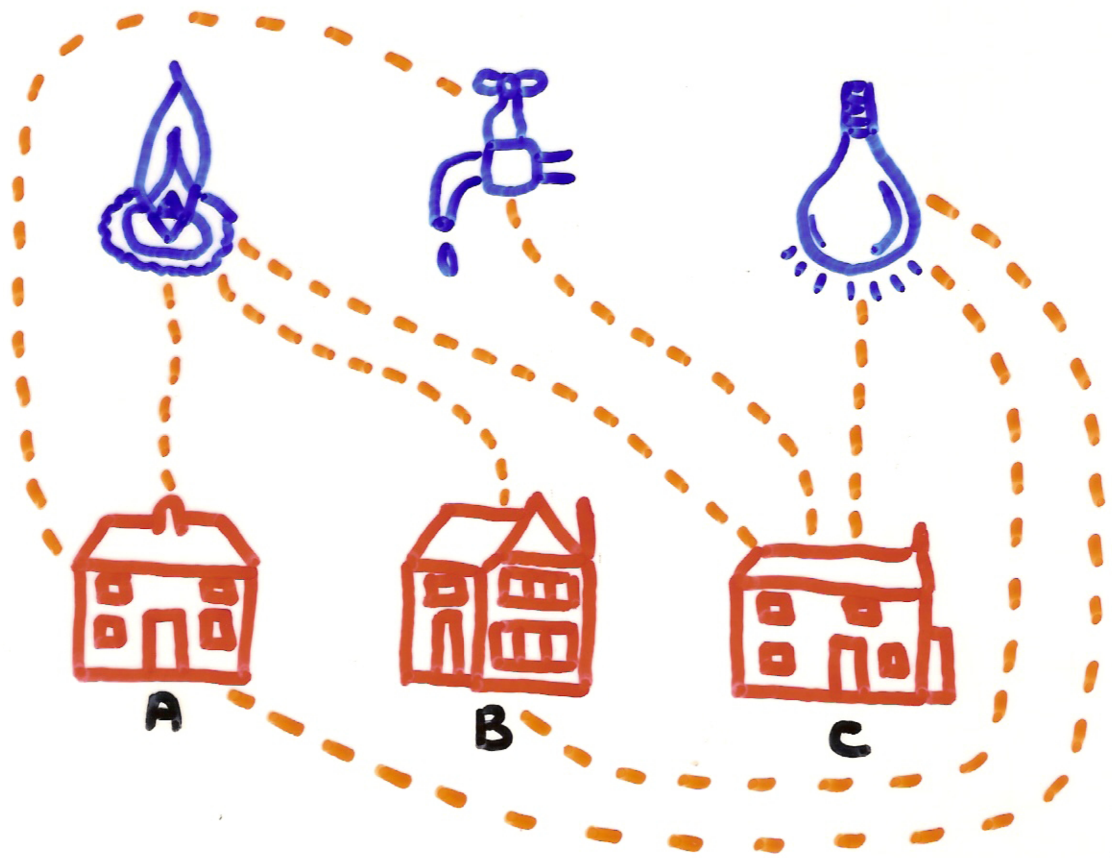


So 0 or 2 odd degrees.

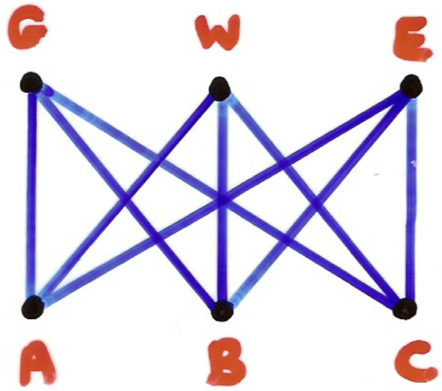
even degree

Königsberg has degrees 3, 3, 3, 5, and so is impossible.

Gas, Water and Electricity

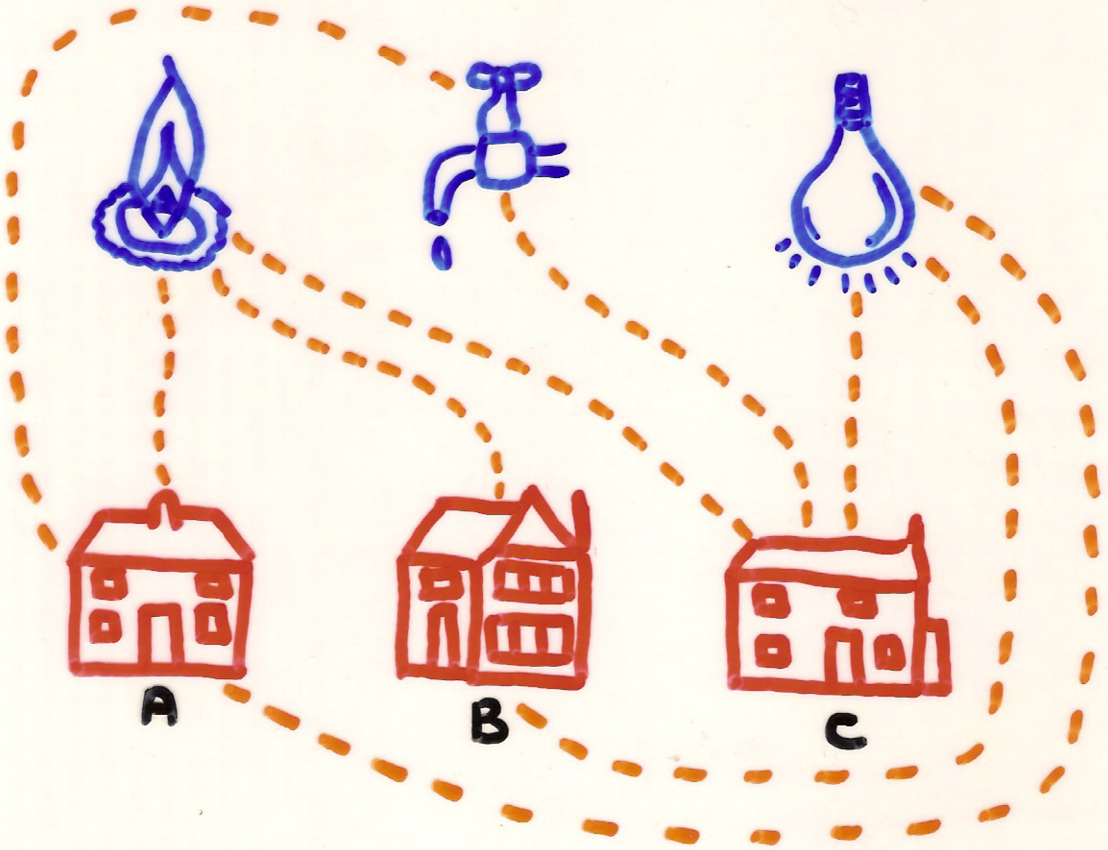


Can

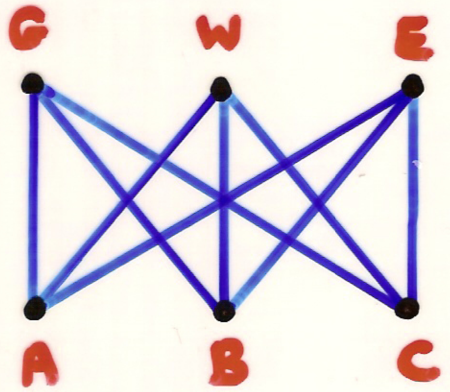


be drawn in the plane without crossings?

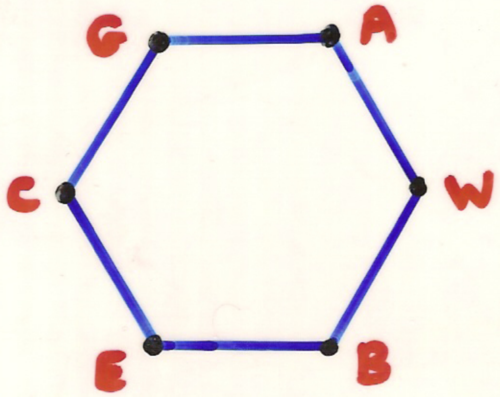
Gas, Water and Electricity

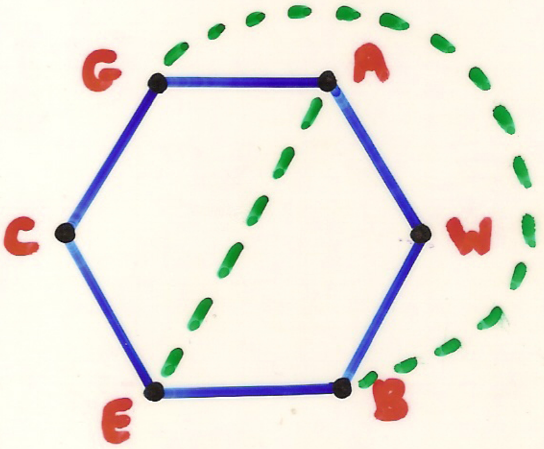


Can



be drawn in the plane without crossings?



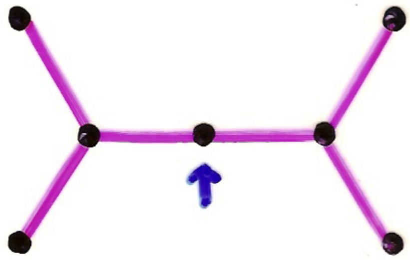


GOOD WILL HUNTING

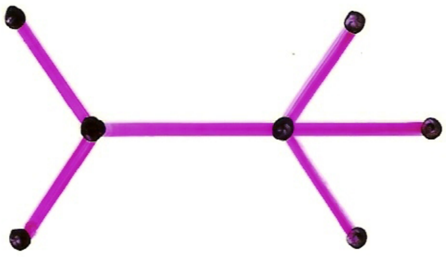
Draw all the homeomorphically irreducible trees with $n = 10$.

Homeomorphically Irreducible Trees

No vertices of degree 2:

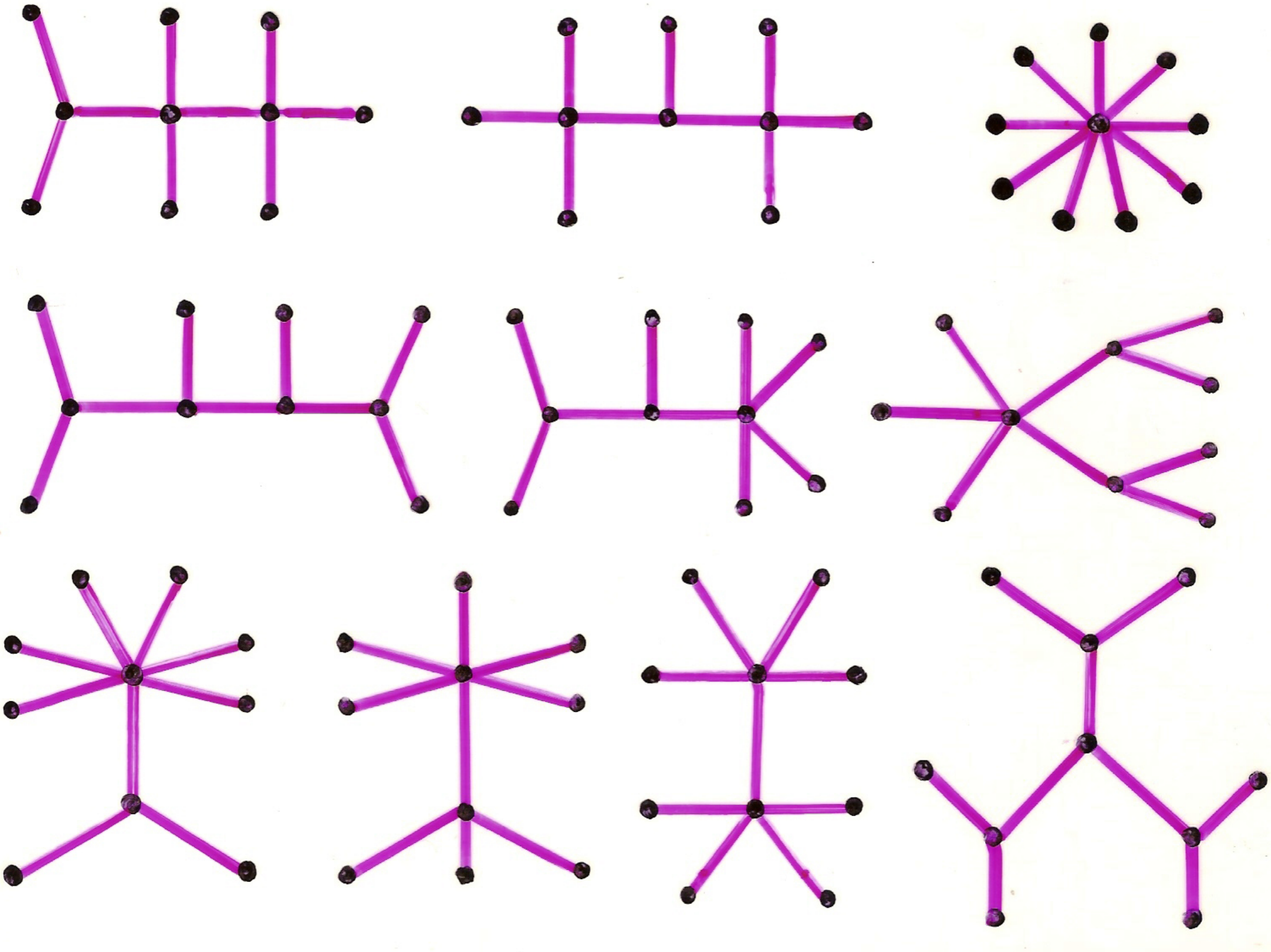


no

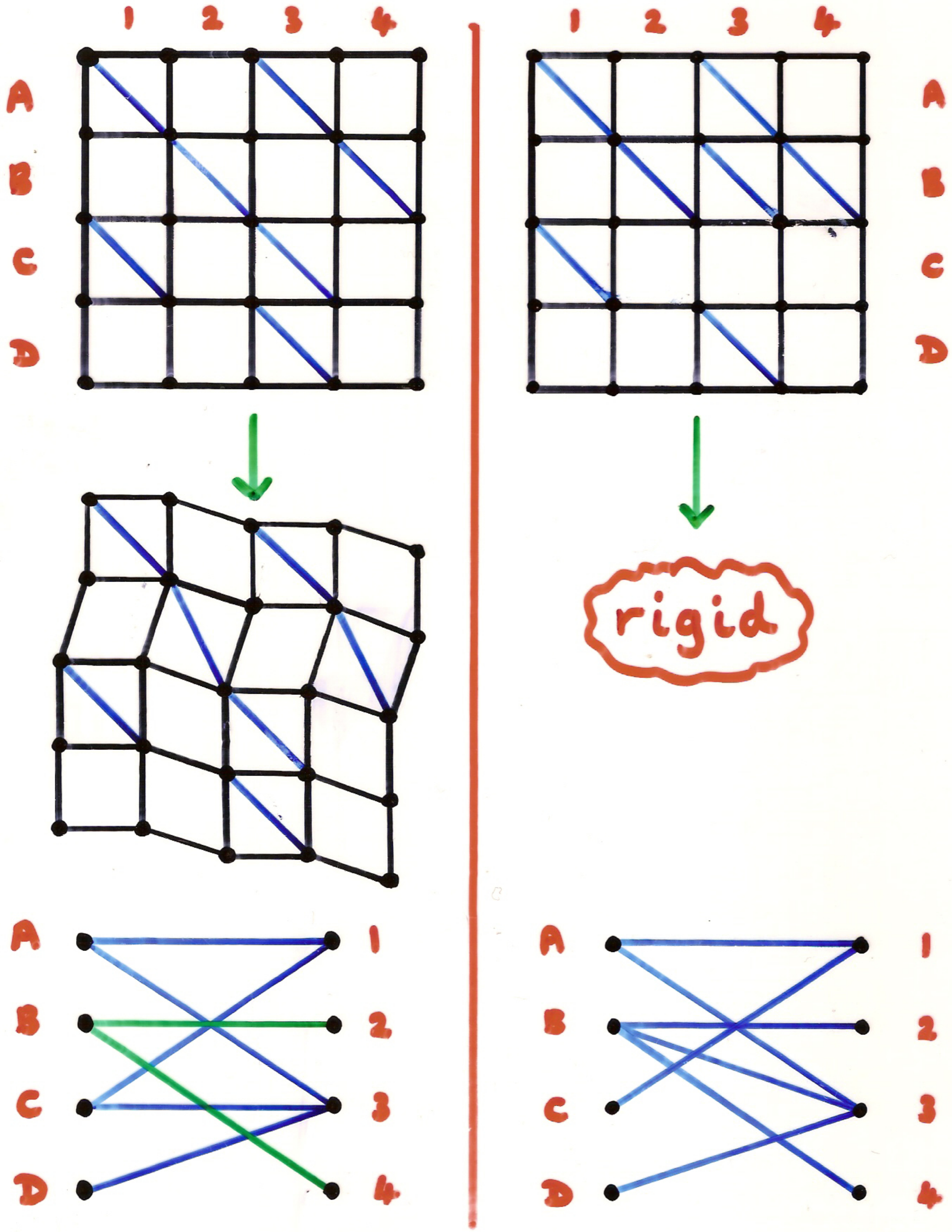


yes

n = 10



BRACING FRAMEWORKS

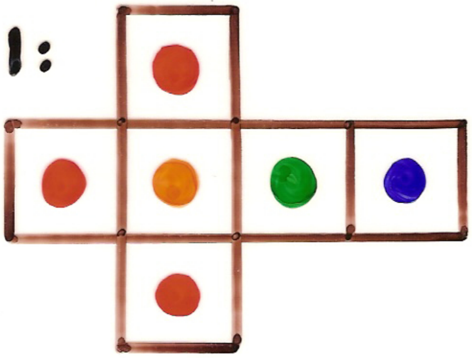


graph connected ≡ rigid bracing

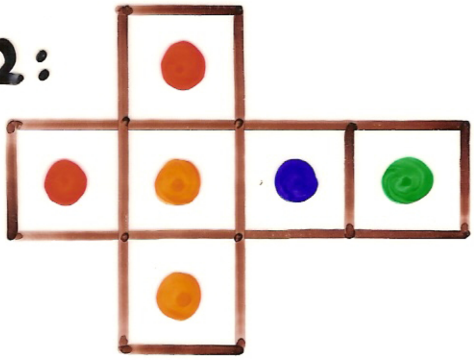
Instant Insanity

[The four cubes problem]

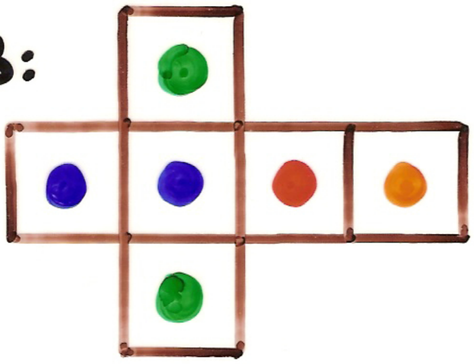
cube 1:



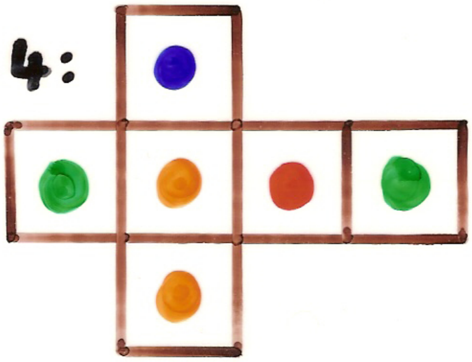
cube 2:



cube 3:



cube 4:



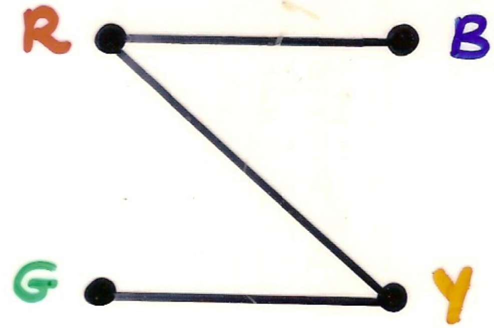
Aim: to pile up these cubes on top of each other so that all four colours appear on each side of the 'stack'.

82944 ways – only one works...

INSTANT INSANITY

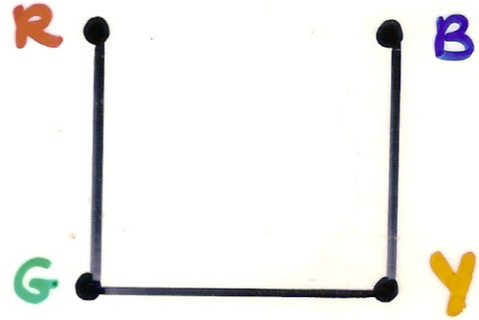
cube 1

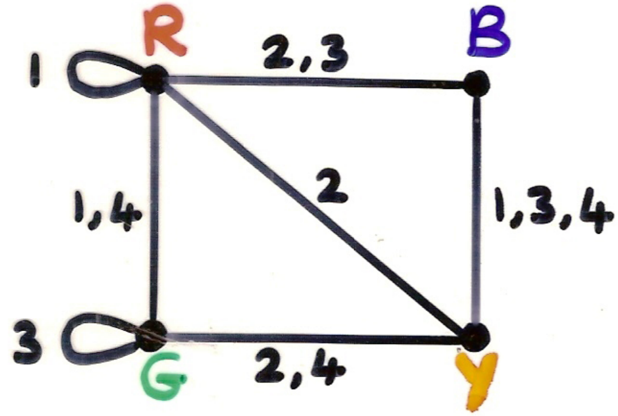
cube 2

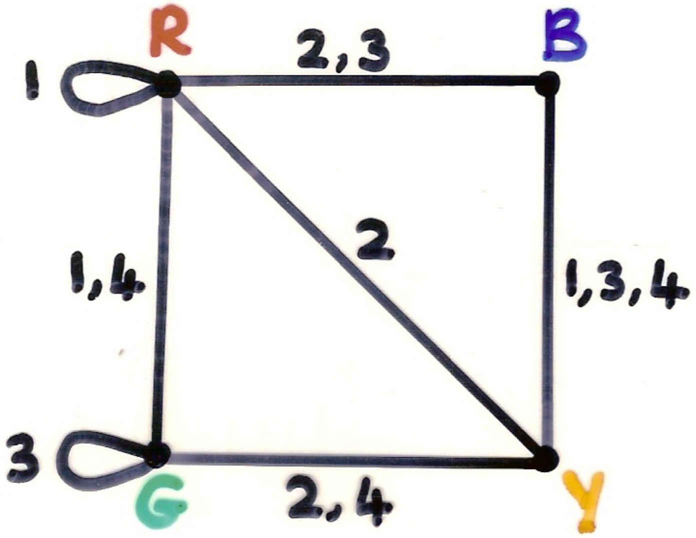


cube 3

cube 4







Find subgraphs H_1 and H_2 such that:

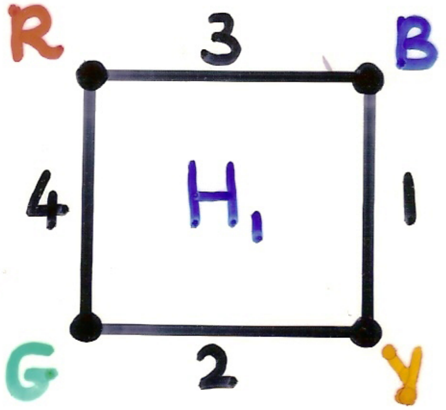
(a) each contains 1 edge from each cube

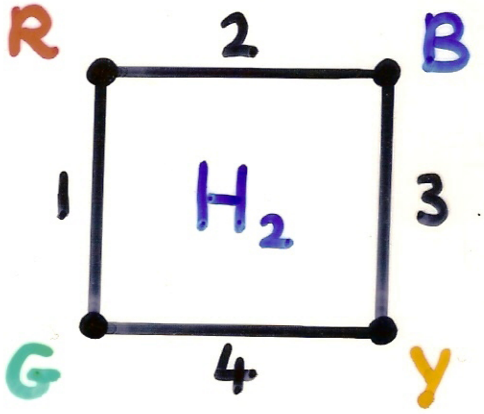
(b) they have no edges in common

(c) all vertices have degree 2

front/back

Left/right





INSTANT INSANITY (2)

Let R = 1, B = 2, G = 3, Y = 5

Each side of the stack multiplies to $1 \cdot 2 \cdot 3 \cdot 5 = 30$.

cube 1 : RR = 1, RG = 3, BY = 10

cube 2 : RB = 2, GY = 15, RY = 5

products : 2, 15, 5, 6, 45, 15, 20, 150, 50

cube 3 : RB = 2, GG = 9, BY = 10

cube 4 : RG = 3, GY = 15, BY = 10

products : 6, 30, 20, 27, 135, 90, 30, 150, 100

✓ × ✓ × × × × ✓ ×

	cube 1	cube 2	cube 3	cube 4	
150×6	BY	GY	RB	RG	← F B
45×20	RG	GY	RB	BY	
6×150	RG	RB	BY	GY	← L R