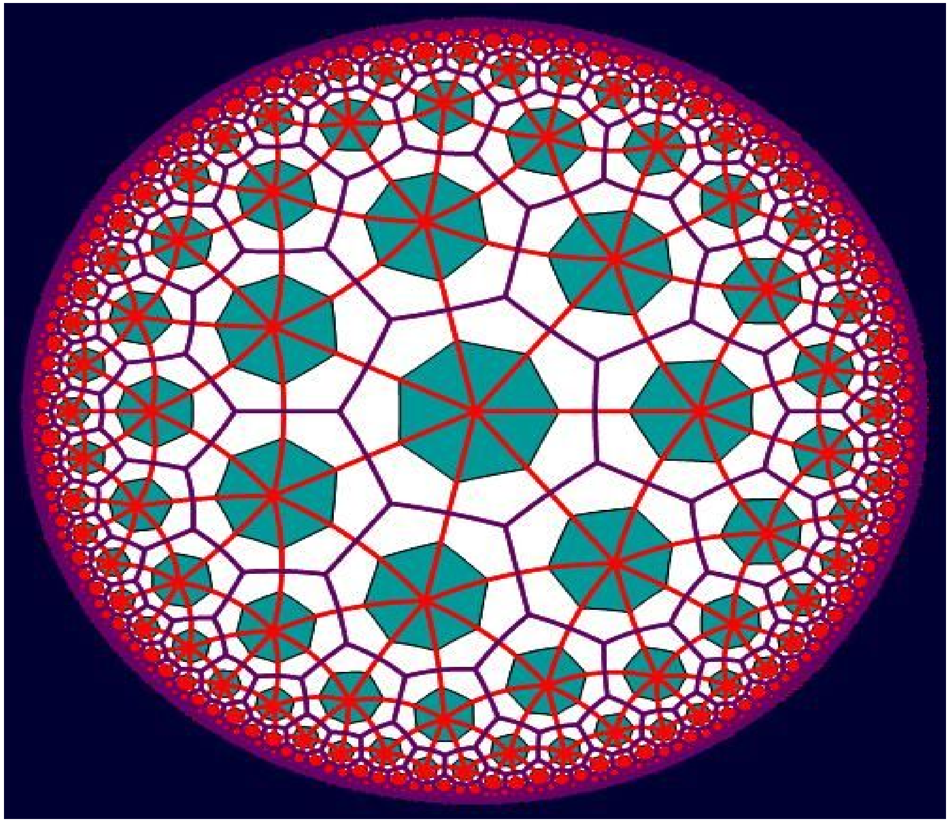
The Art of Maths

Chris Budd



GRESHAM COLLEGE

IMI

UNIVERSITY OF BATH

Are art and maths that different?

BRIDGES AALTO 2020

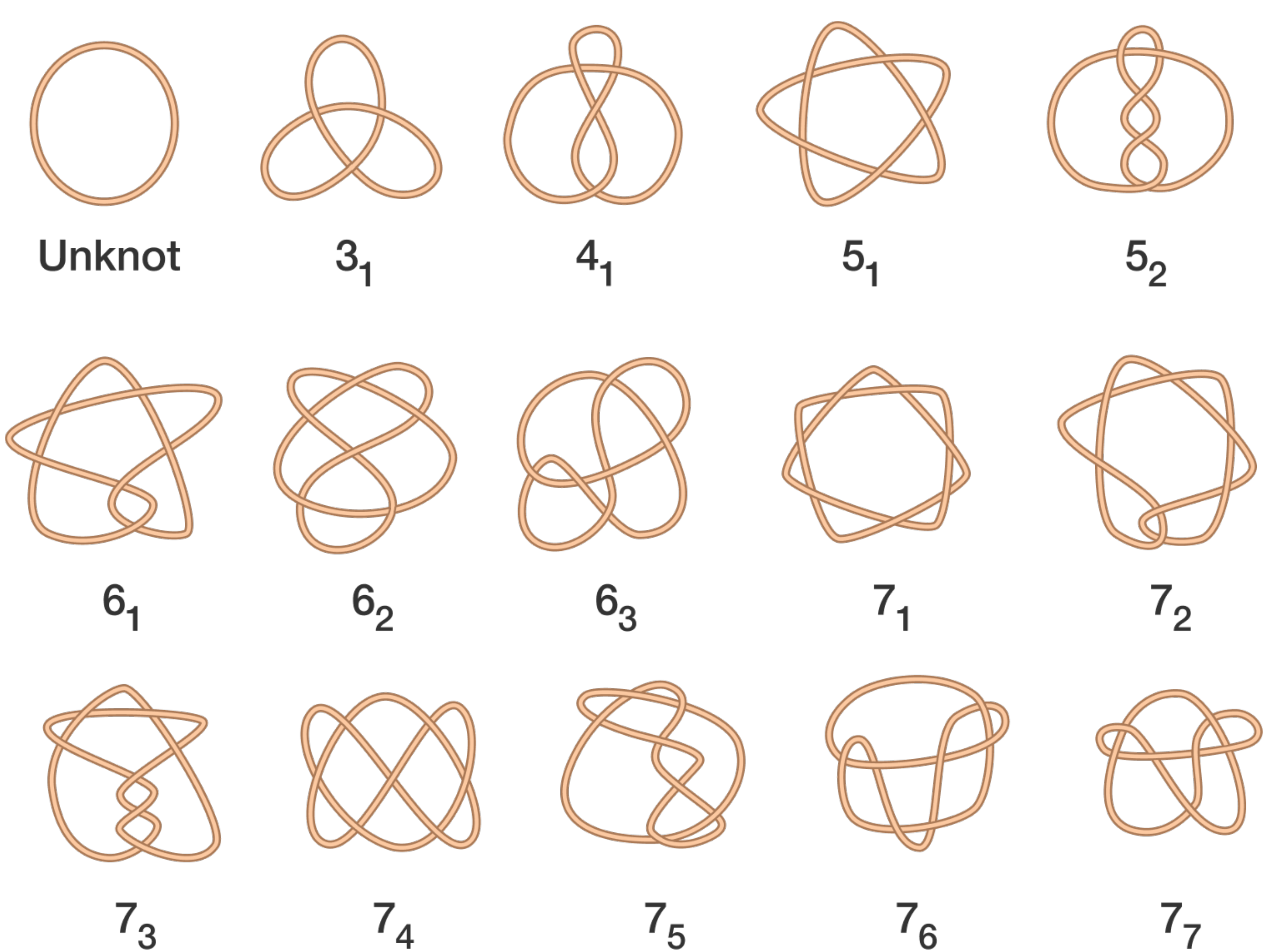
MATHEMATICS / ART / MUSIC / ARCHITECTURE / EDUCATION / CULTURE

Celtic Art

The Book of Kells

Trinity College
Dublin

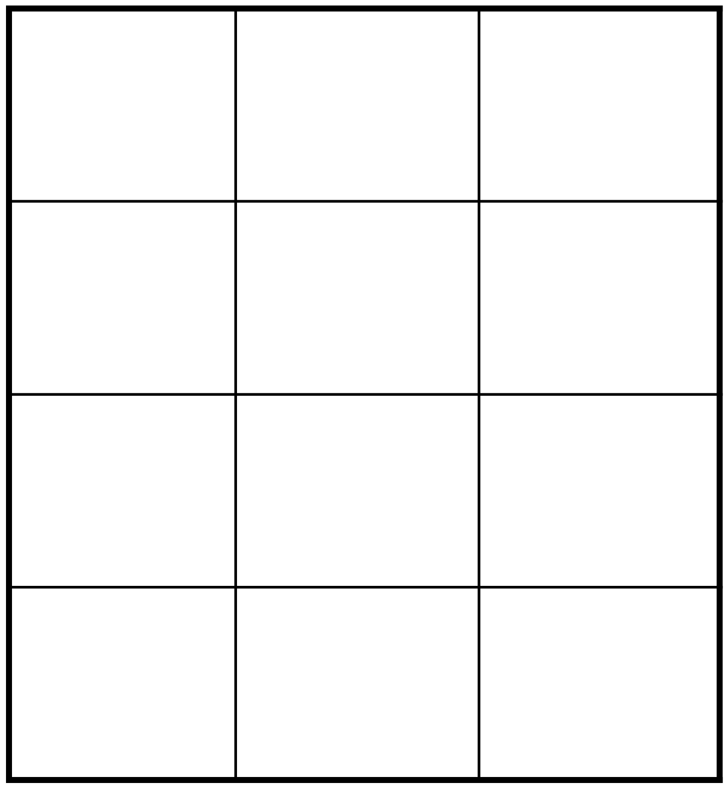
All knots with up to seven crossings



Trefoil Knot

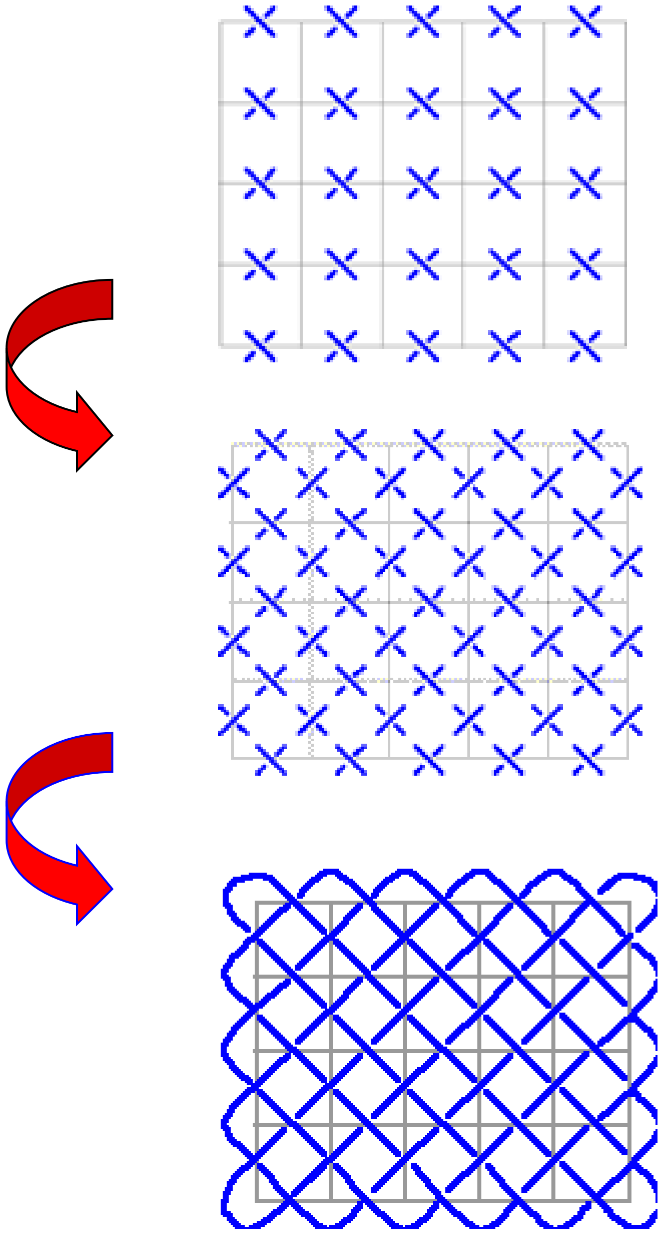
How to draw simple Celtic Knots

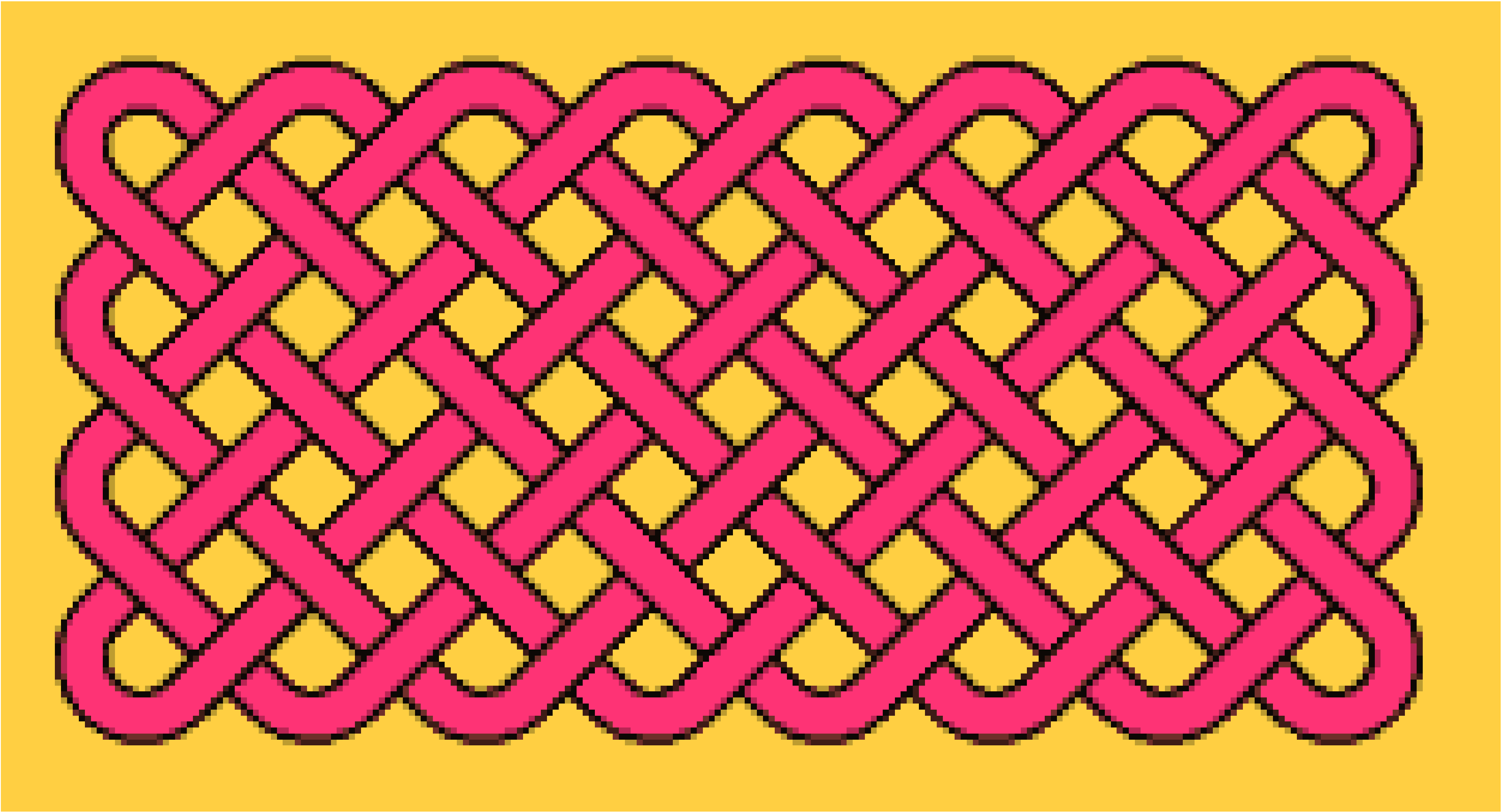
Start with a grid



(5 , 4)   grid

(5,6) Celtic Knot





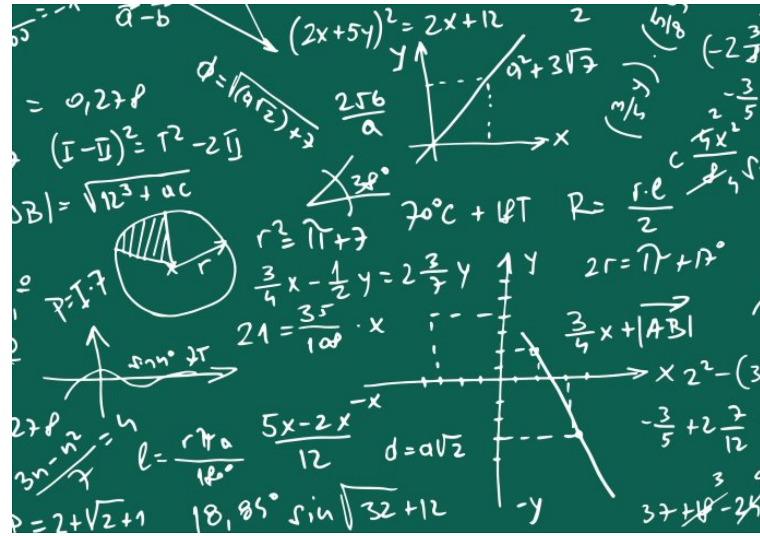
(4,8) Celtic Knot

How many pieces of string are needed?

How many pieces of string are needed?

(2,2)	2
(3,2)	1
(5,3)	1
(4,4)	4

The mathematical method



- Do lots of experiments
- Look for patterns
- Make a hypothesis
- Check your hypothesis with other examples
- Prove your result using mathematical reasoning

Number of strings = gcd(m,n)

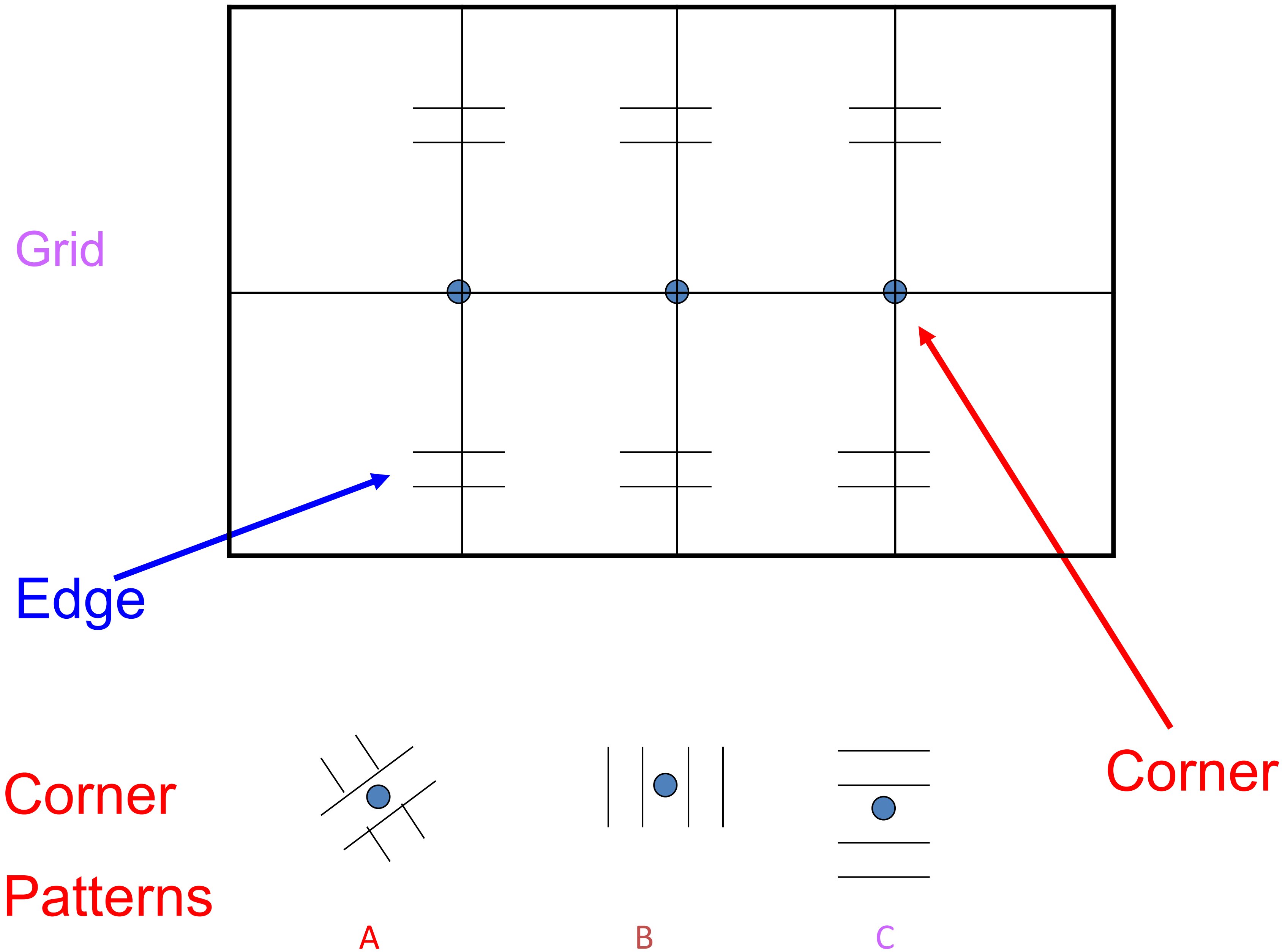
gcd(5,4) = 1

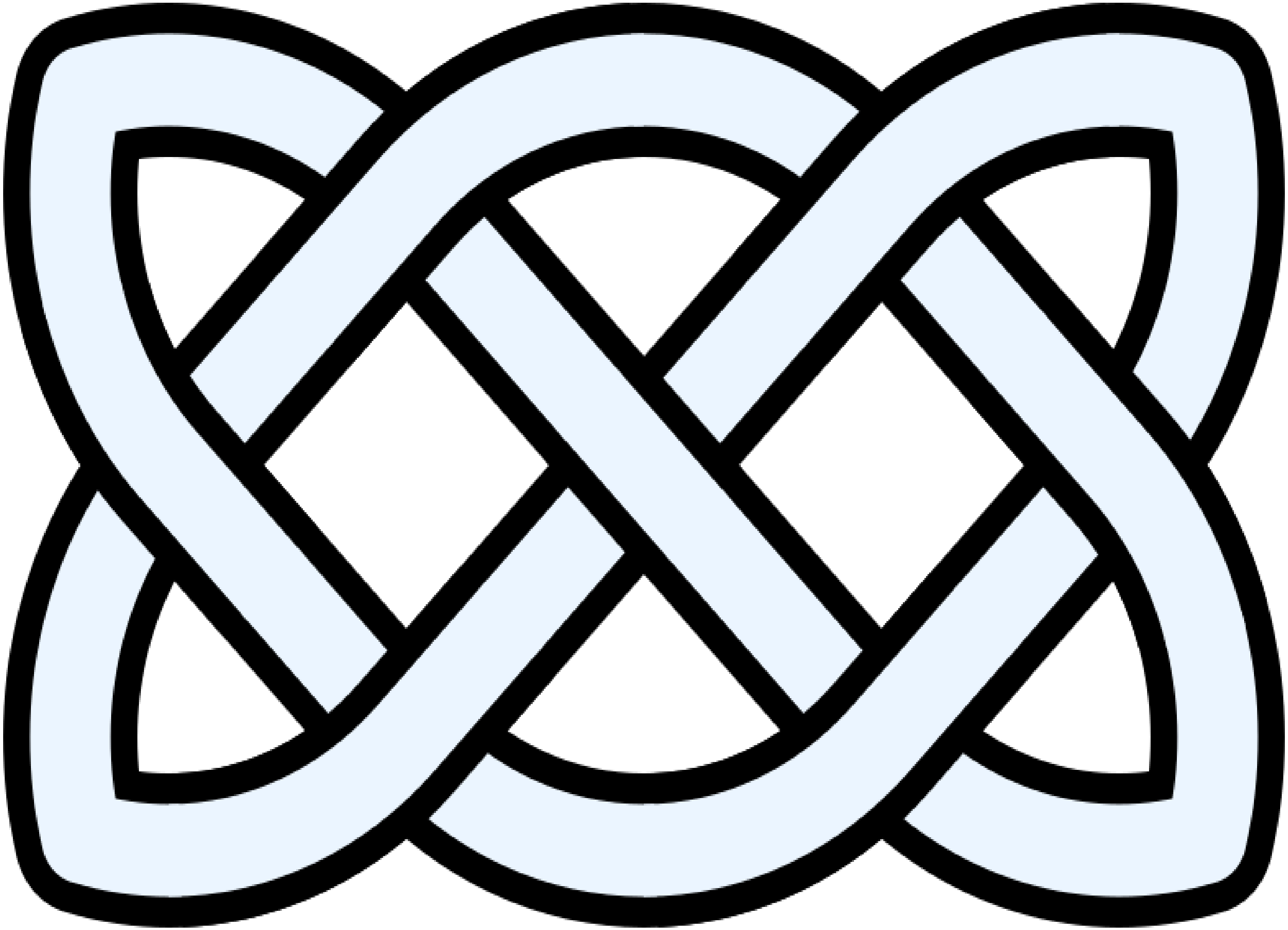
gcd(7,8) = 1

gcd(2,2) = 2

gcd(4,8) = 4

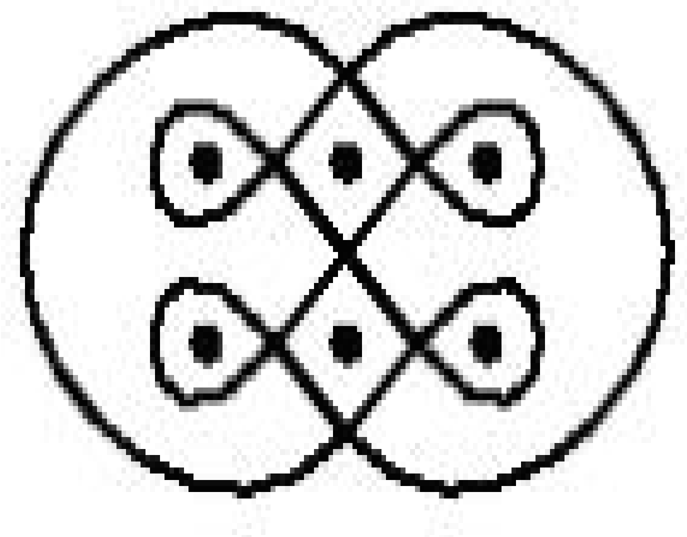
Prove using a geometrical version of Euclid's algorithm

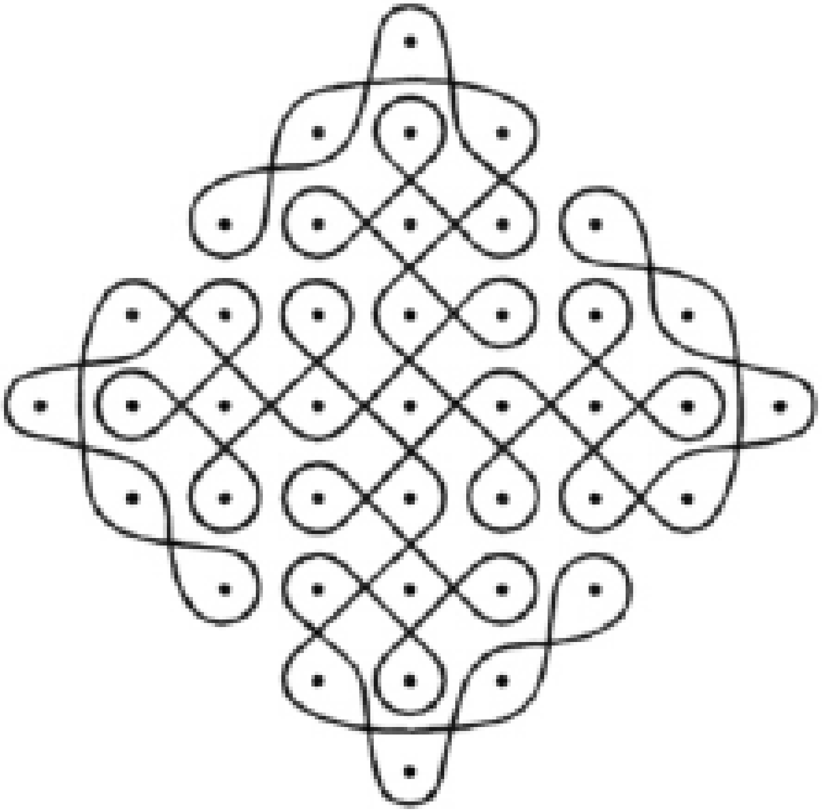

Grid
Edge
Corner
Corner
Patterns
A
B
C

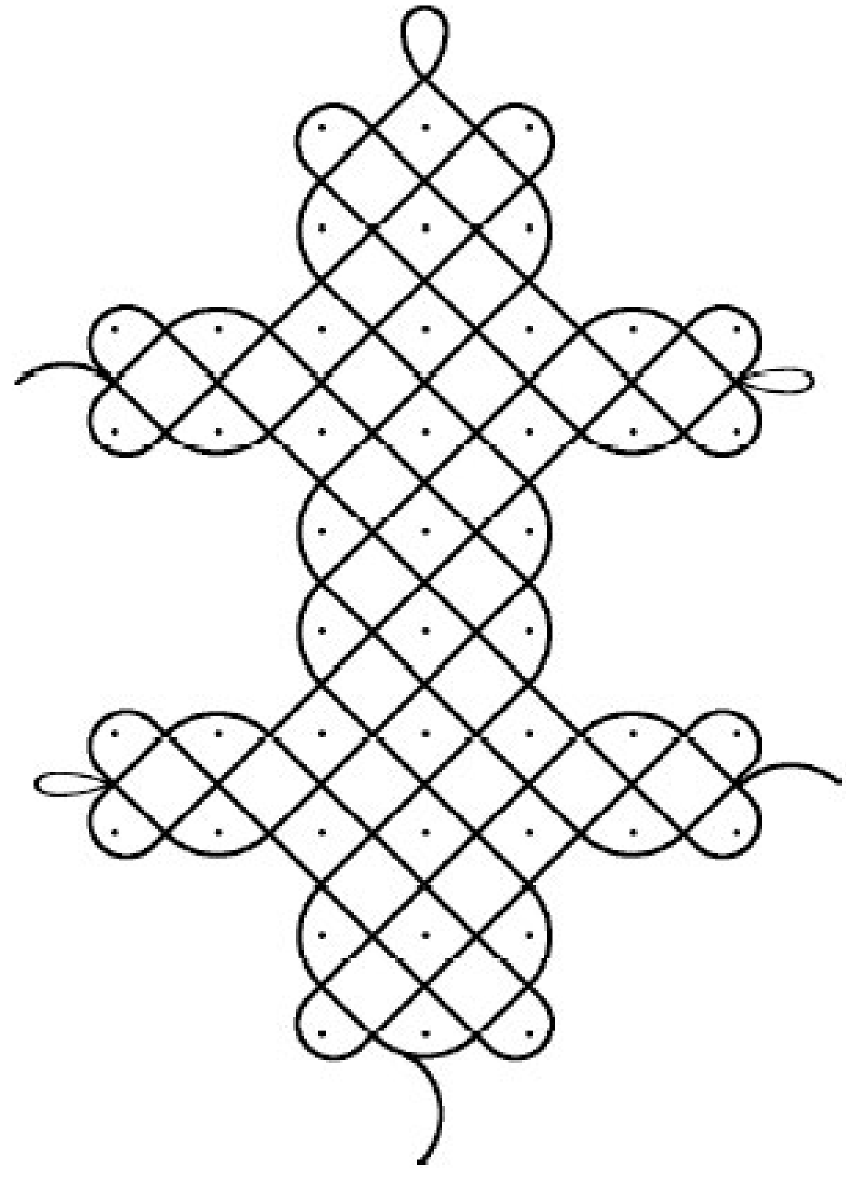


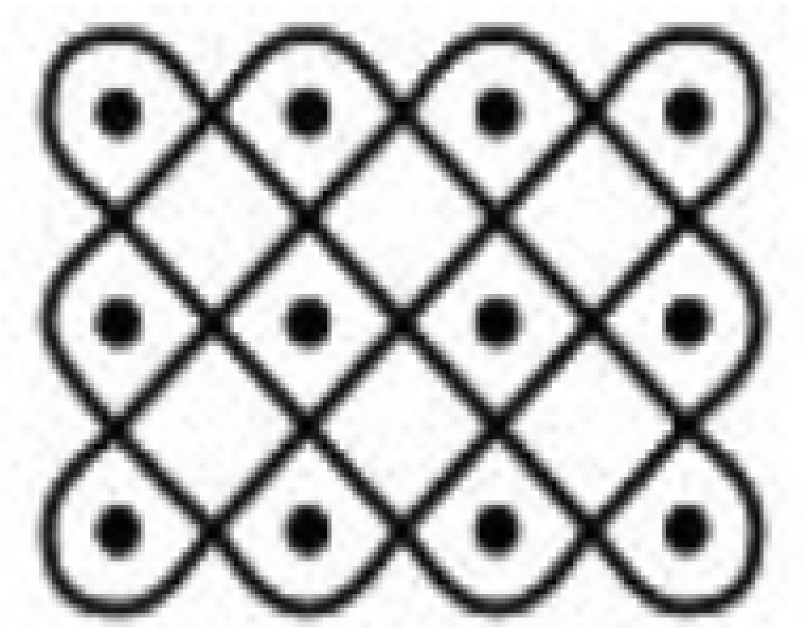
AAA knot

Plaited Mat Sona Art

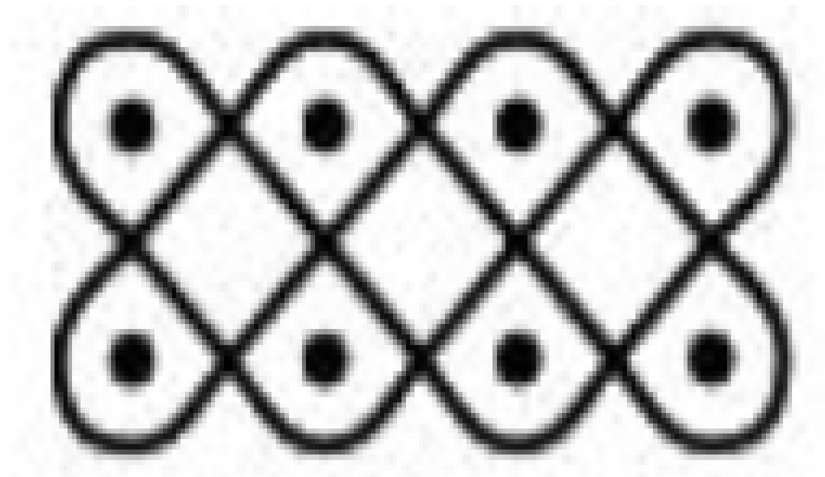








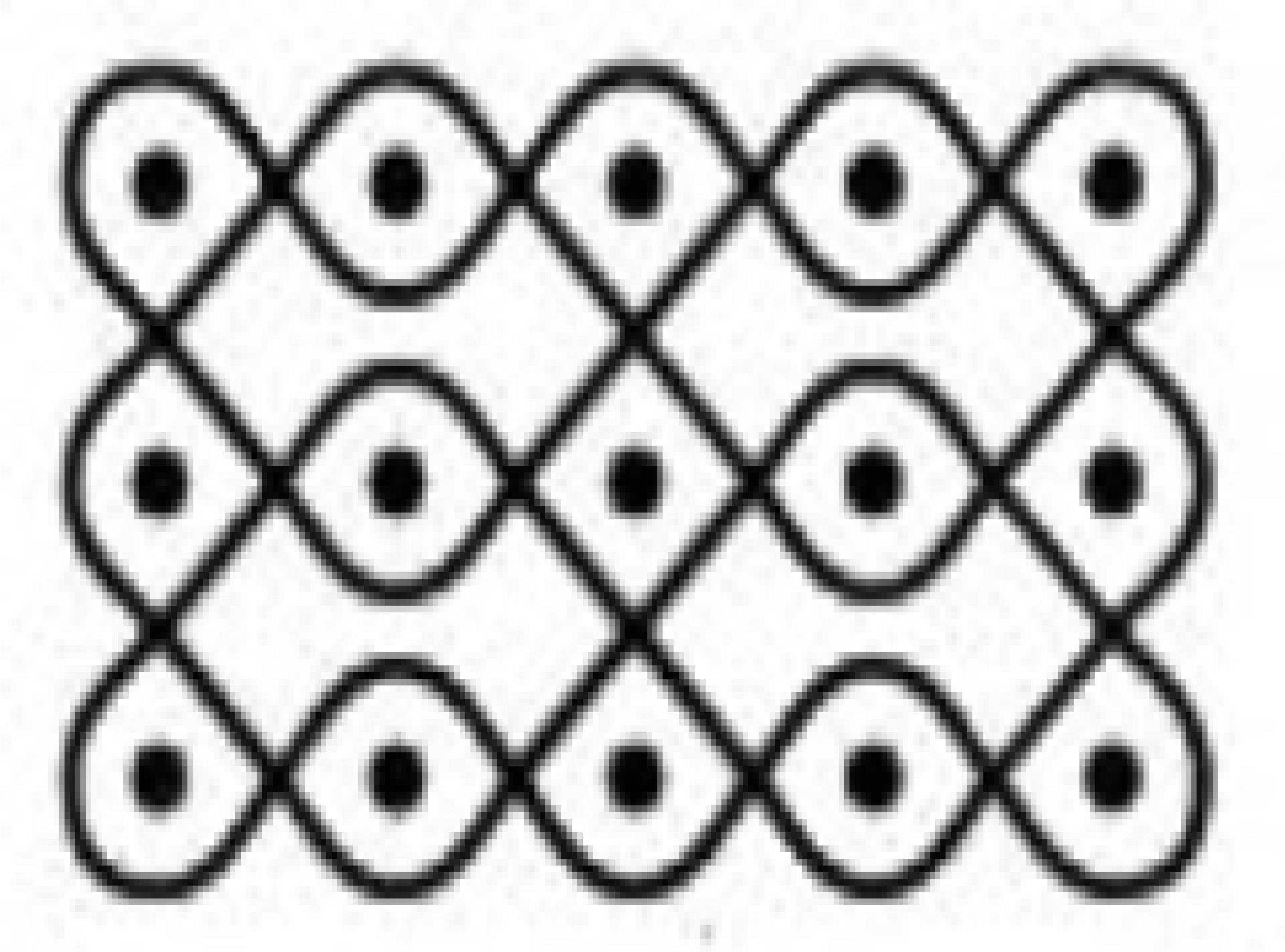
(3,4)



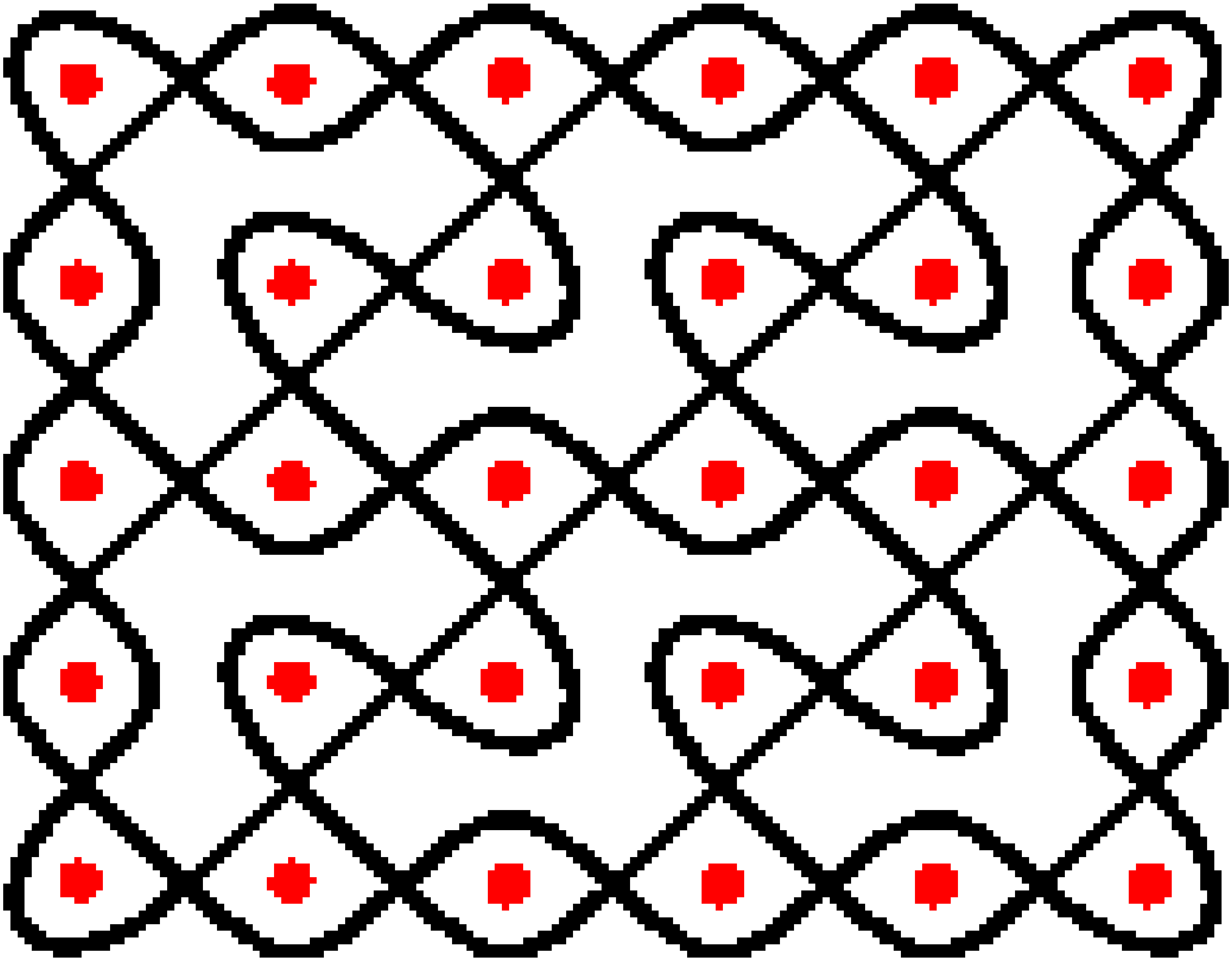
(2,4)

Number of loops is gcd(m,n) as before

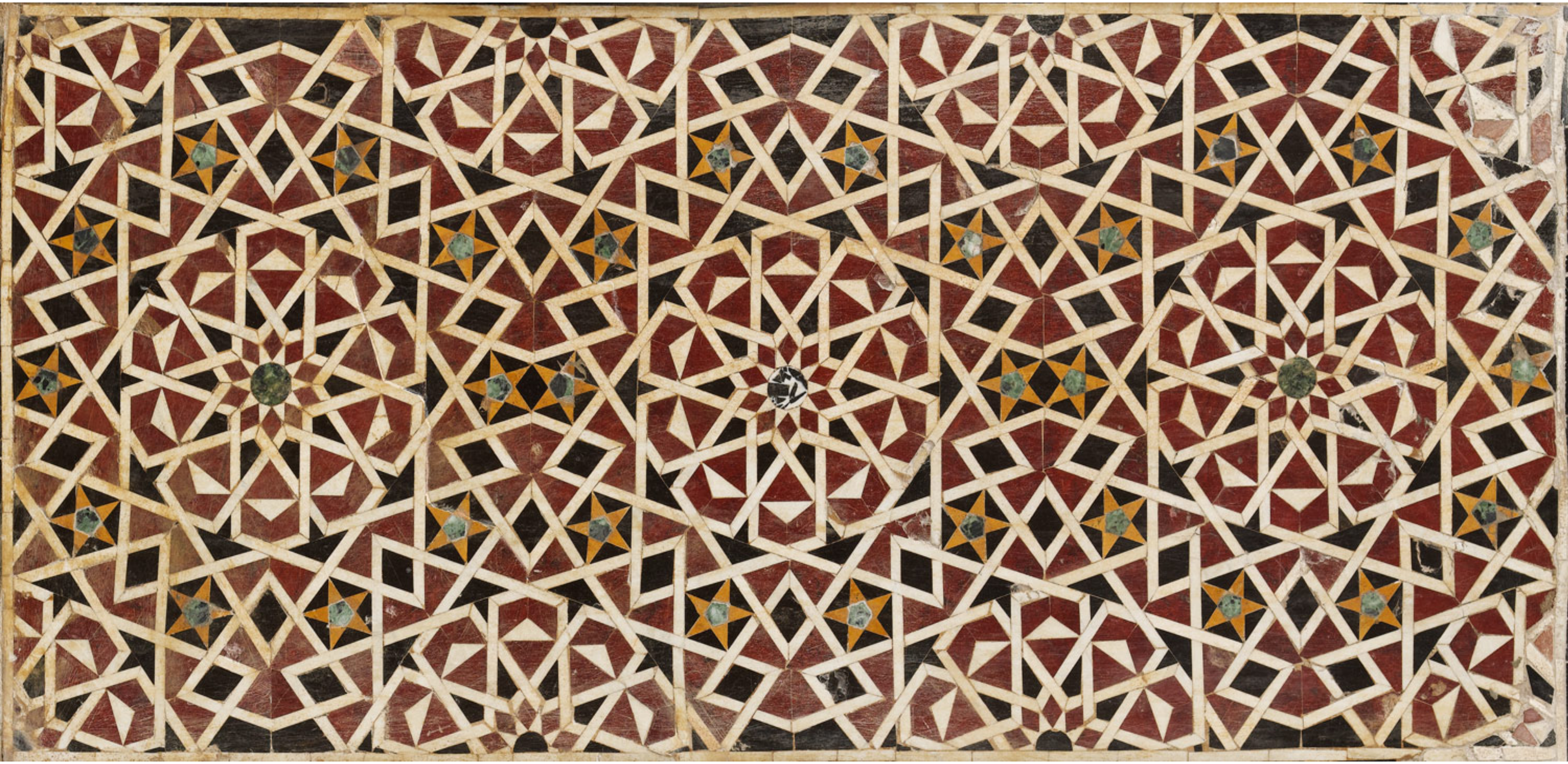
Lion's Stomach

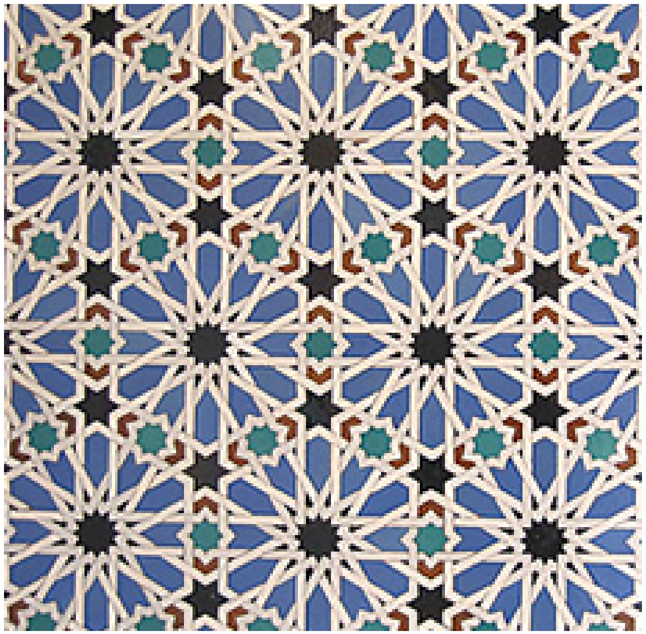


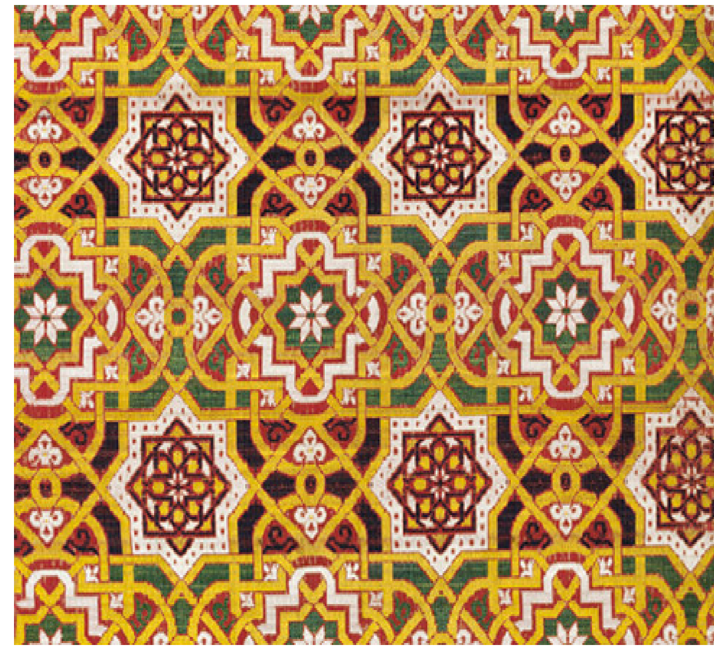
Chased Chicken



Islamic Art

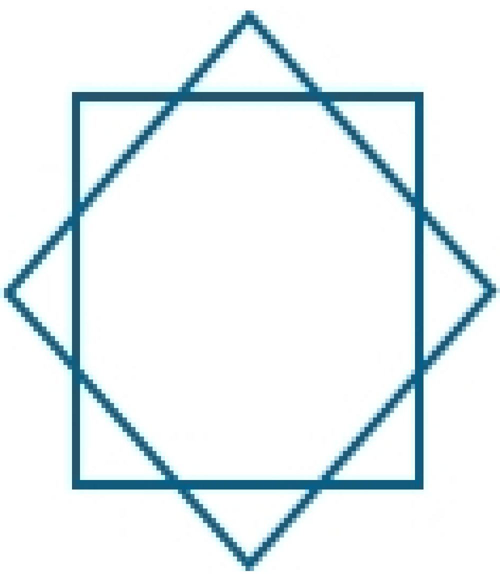


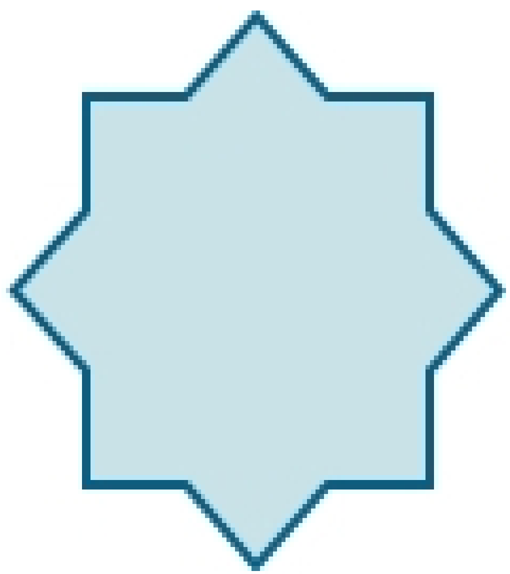


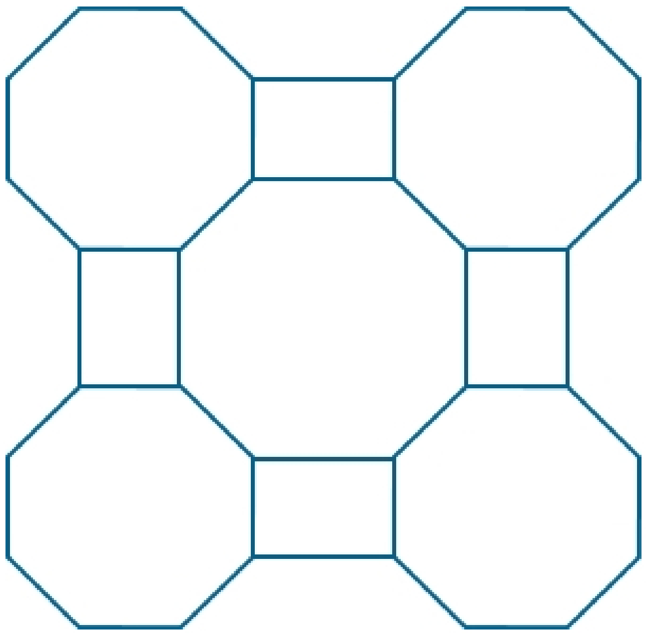


Ruler and compass constructions

Frequently see repeated designs







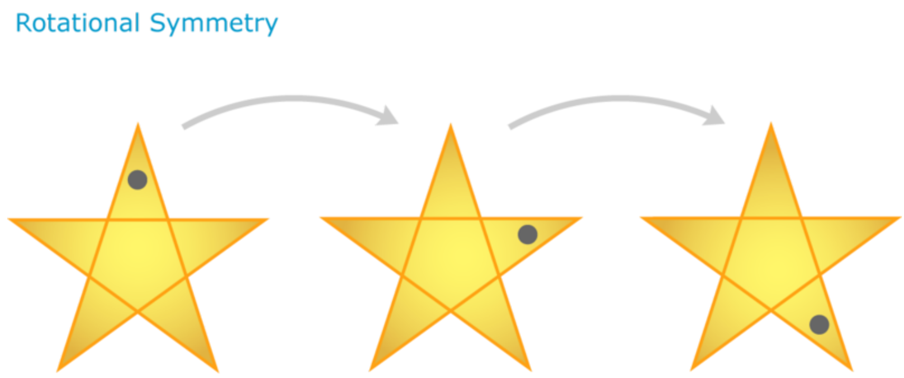
Euclidean symmetry E2 plays a part in most Islamic patterns

Transformations of the plane which preserve distances

Translations: $g(x) = a + x$

Reflexions: $g(x) = -x$

Rotations: $g(x) = Mx$



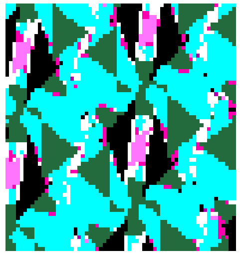
Or combinations of these

$$g(x) = a + M\,x$$

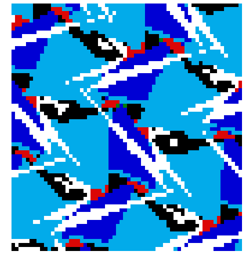
Wallpaper Designs: Invariant under subgroups of E2

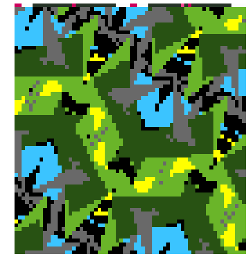
The 17 basic wallpaper designs

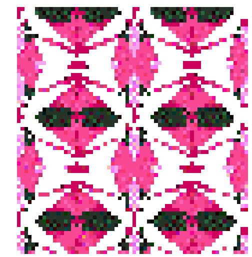

1

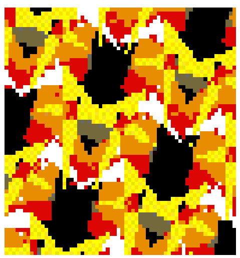

2

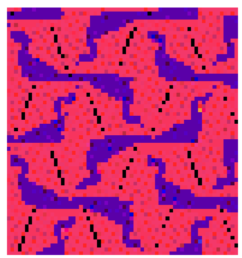

3

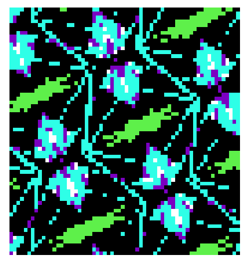

4

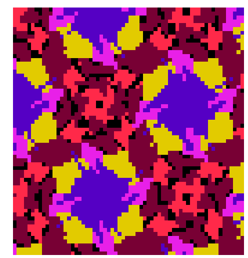

5

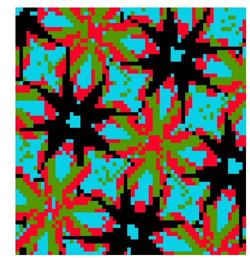

6

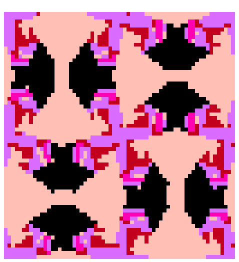

7

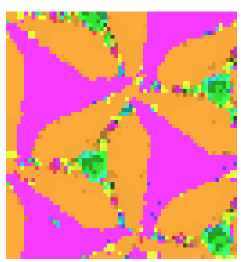

8

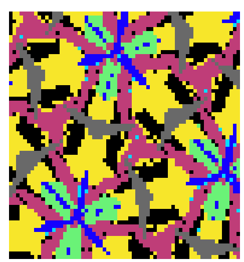

9

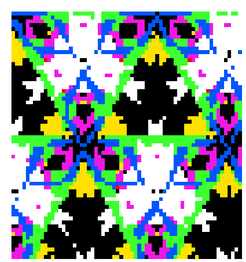

10

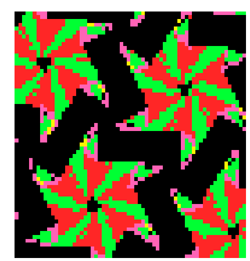

11

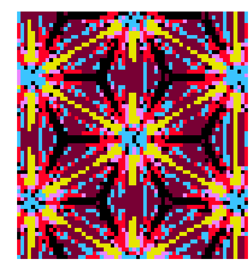

12


13


14


15

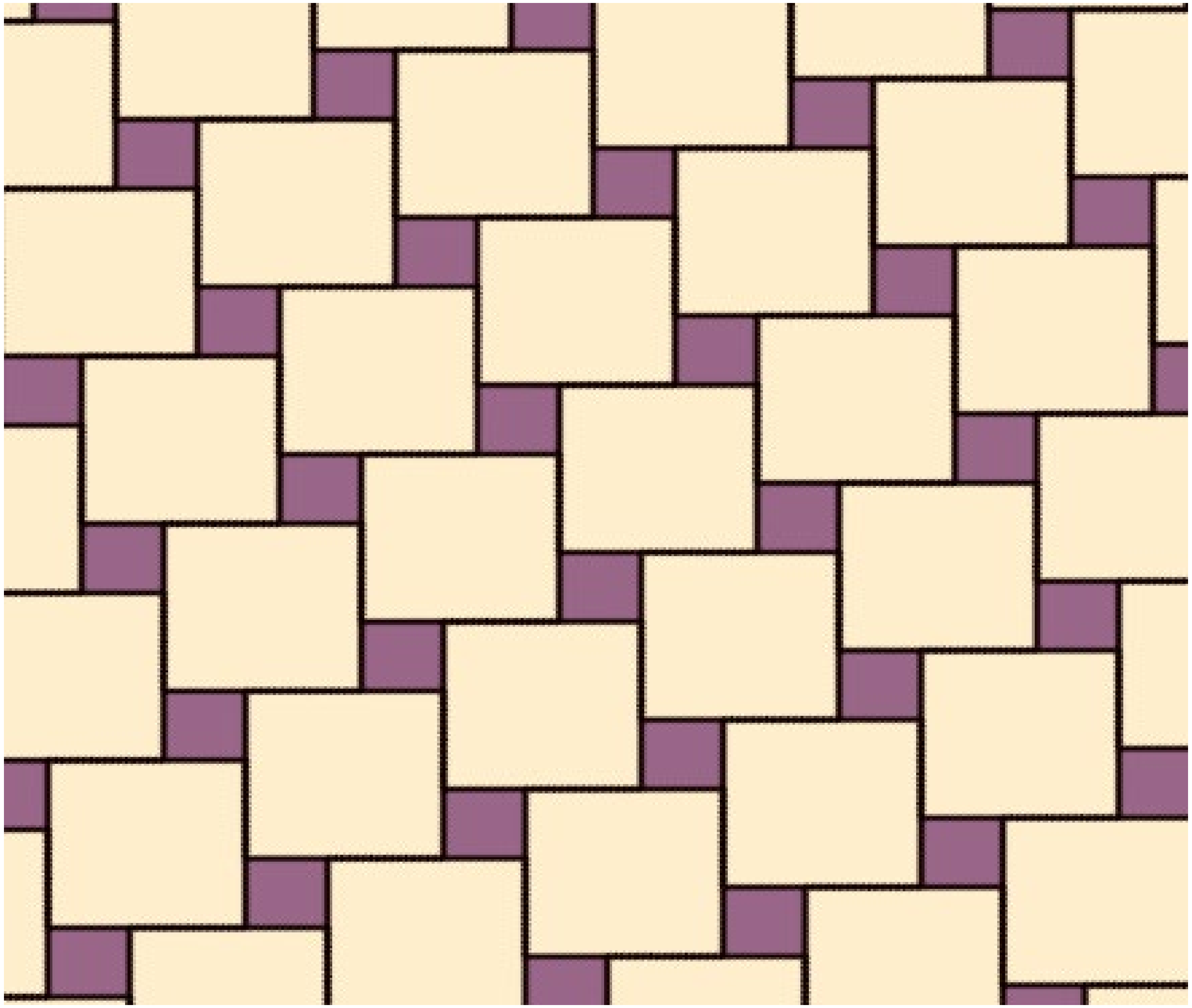

16


17

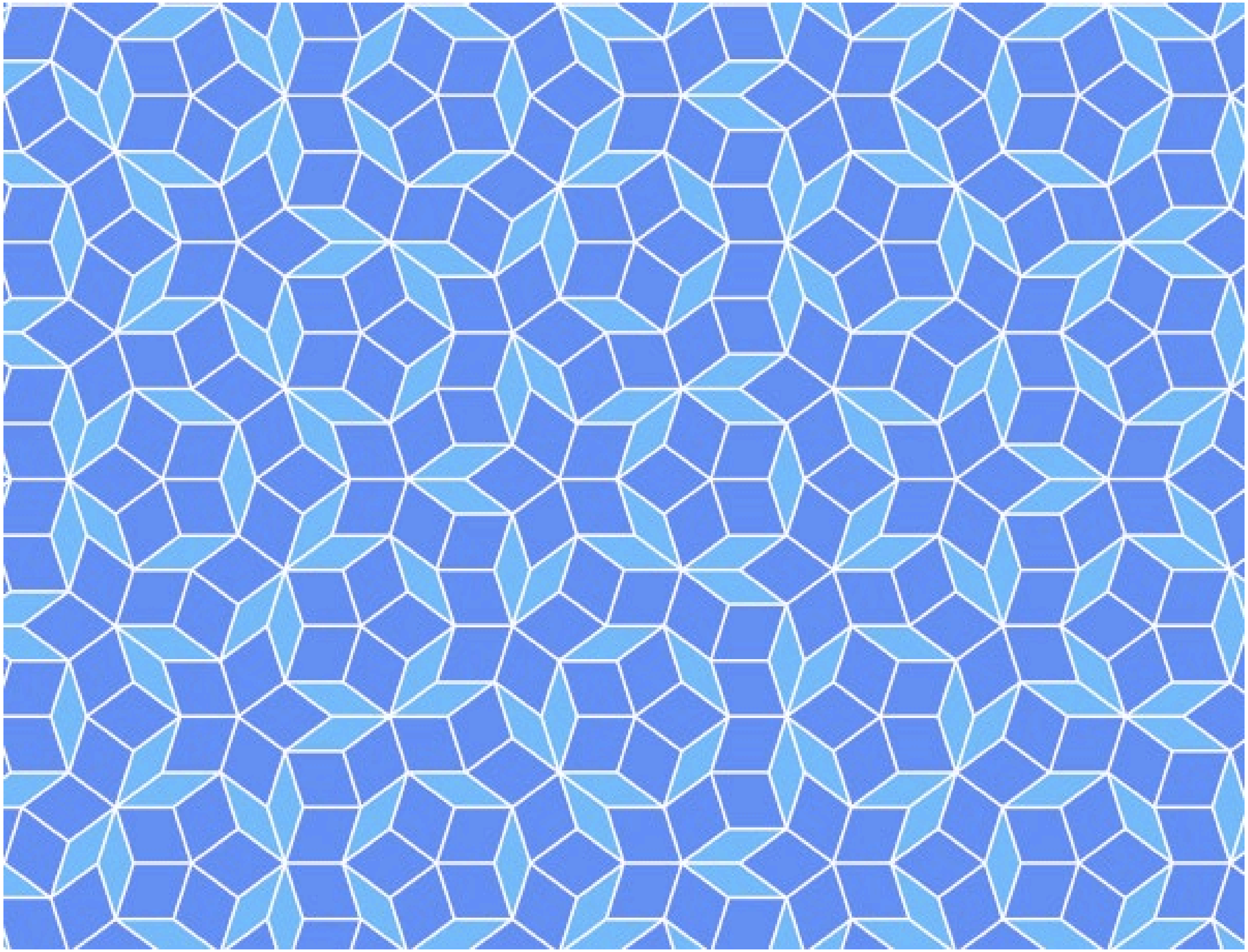
Tilings of the plane

Periodic: Reptiles 1943, Escher

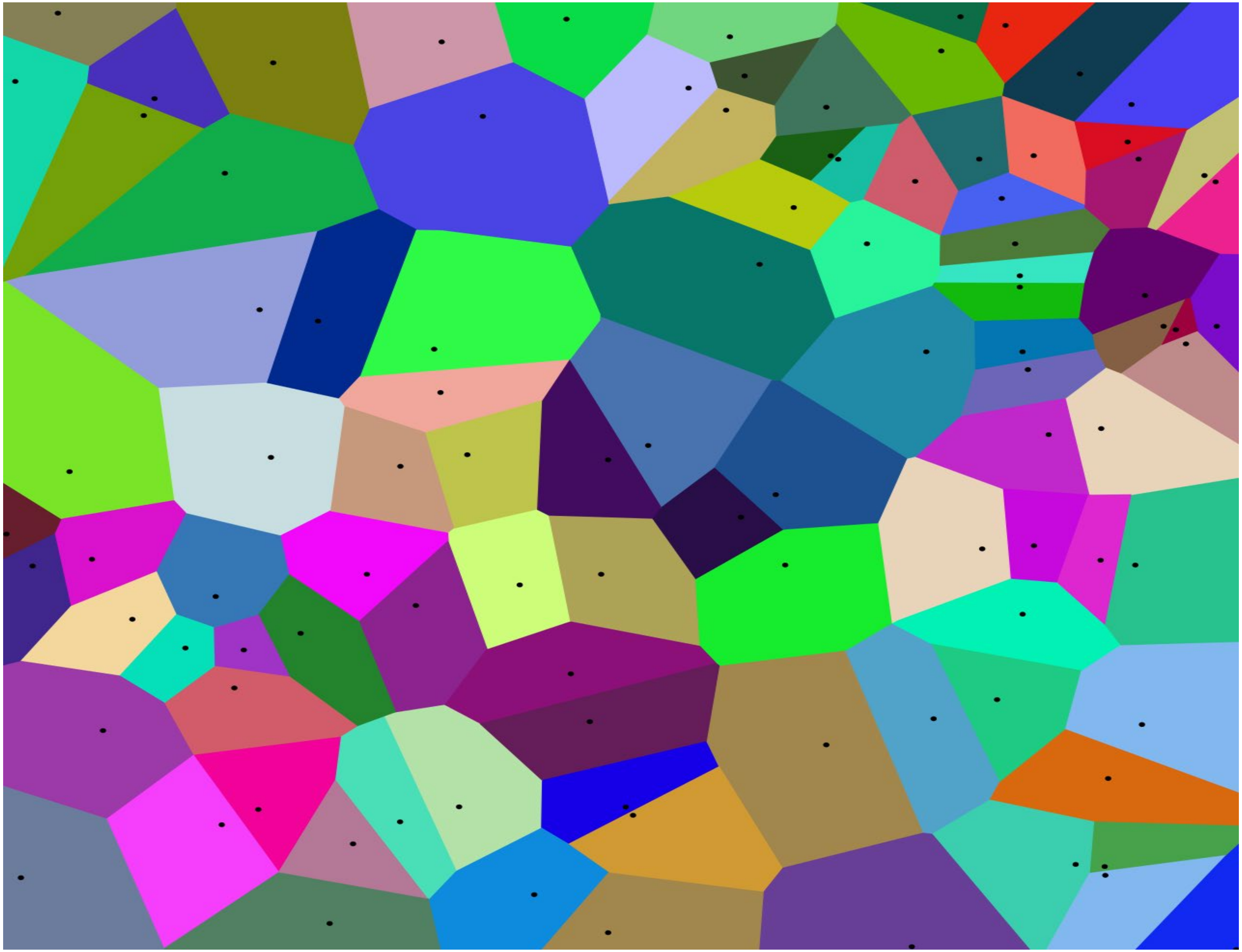
Pythagorean Tiling



A-periodic: Penrose



Random: Voronoi Tesselation



Renaissance art and perspective.

Linear perspective uses mathematical ideas to create the illusion of space and distance on a flat surface

Projection of a three dimensional image onto a two dimensional surface, so that it looks as though it has been seen from a single point.

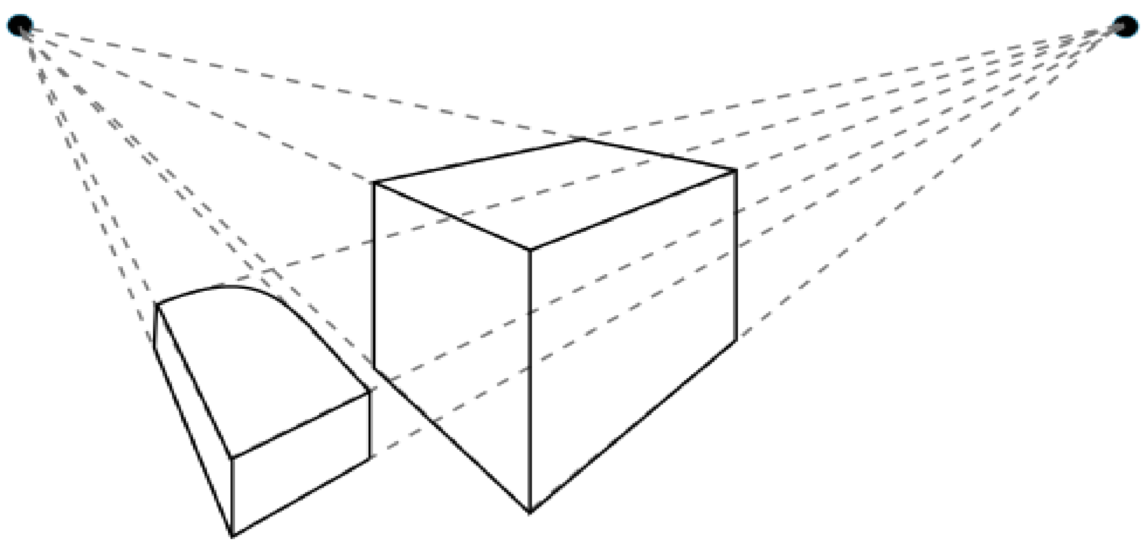
Renaissance pioneers 1400s: Brunelleschi, Alberti, Masaccio, Paolo Uccello , Francesca, Pacioli

Laid the (mathematical) foundation of art as we know it today

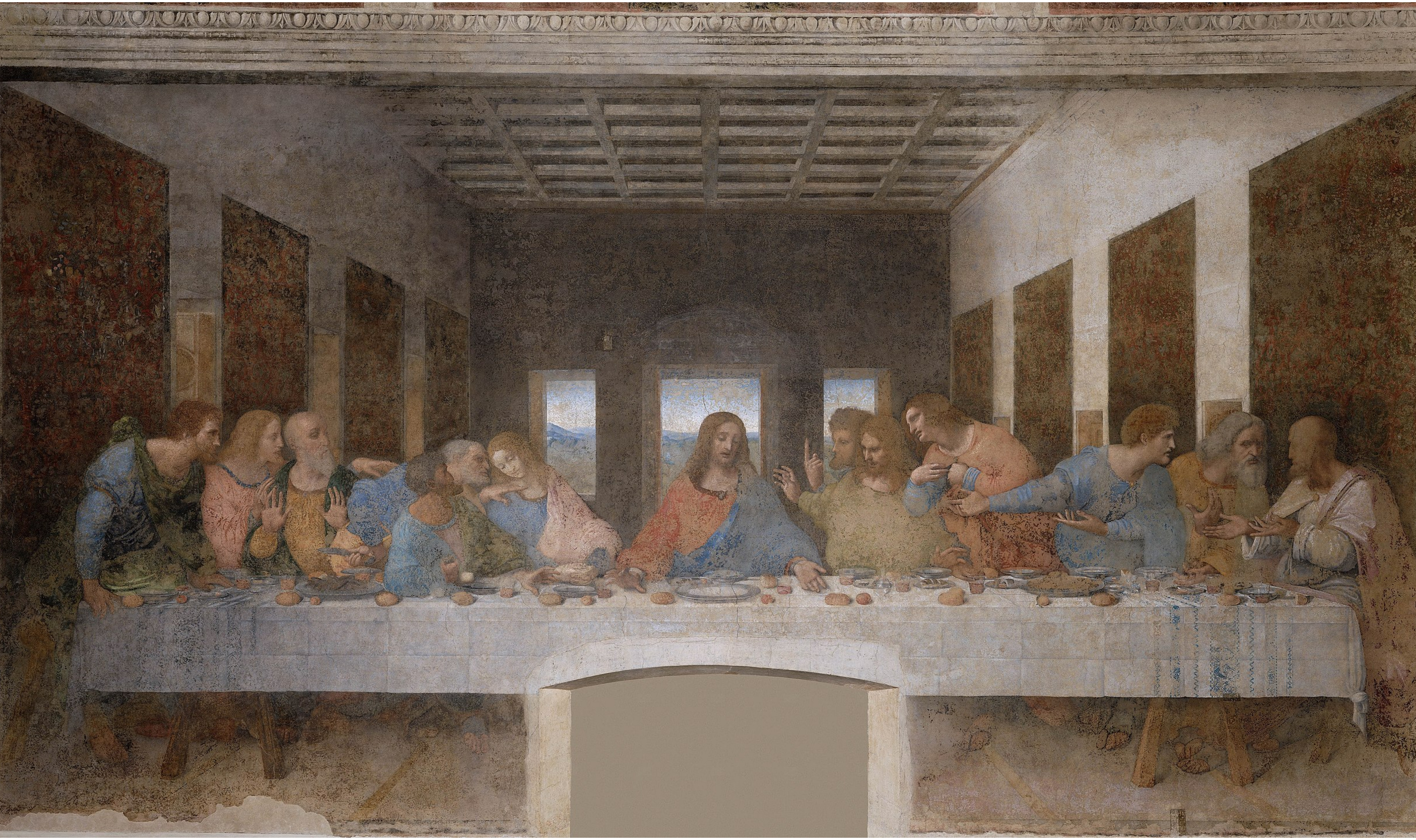
Painter imagines the canvas as a window

In this window they draw straight lines to a vanishing point, to align the edges of walls and floors

Single point perspective has one vanishing point, and two-point perspective has two



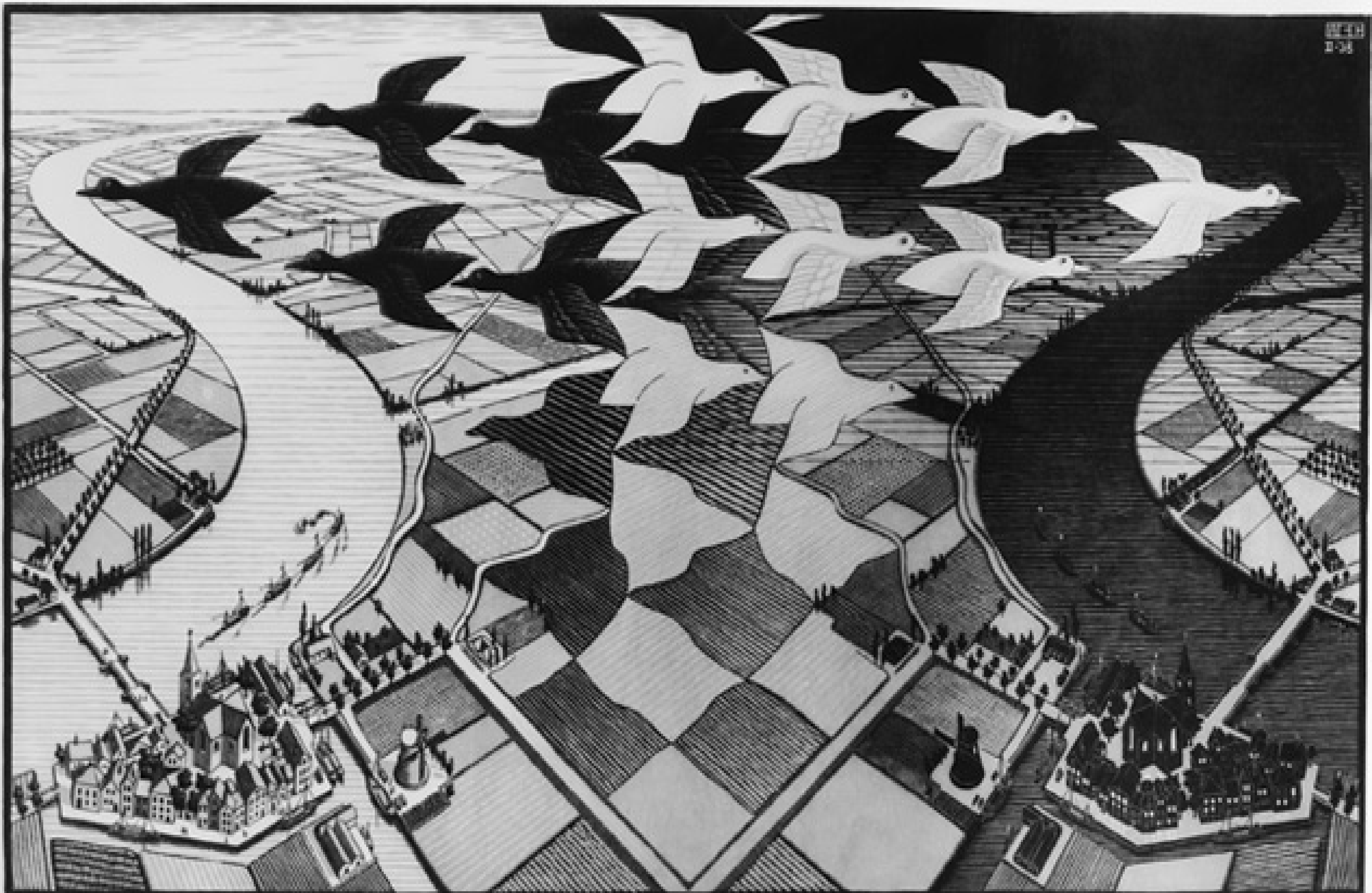
da Vinci Last Supper 1495-1498



Hogarth: False perspective

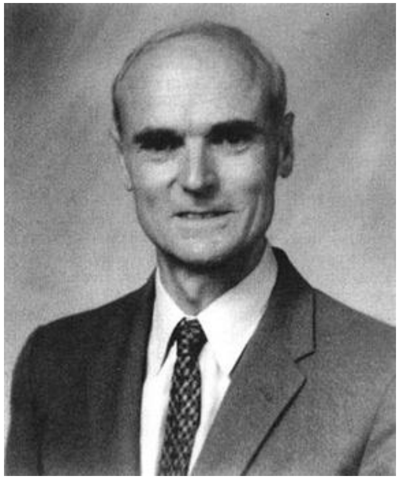
M.C. Escher 1898-1972

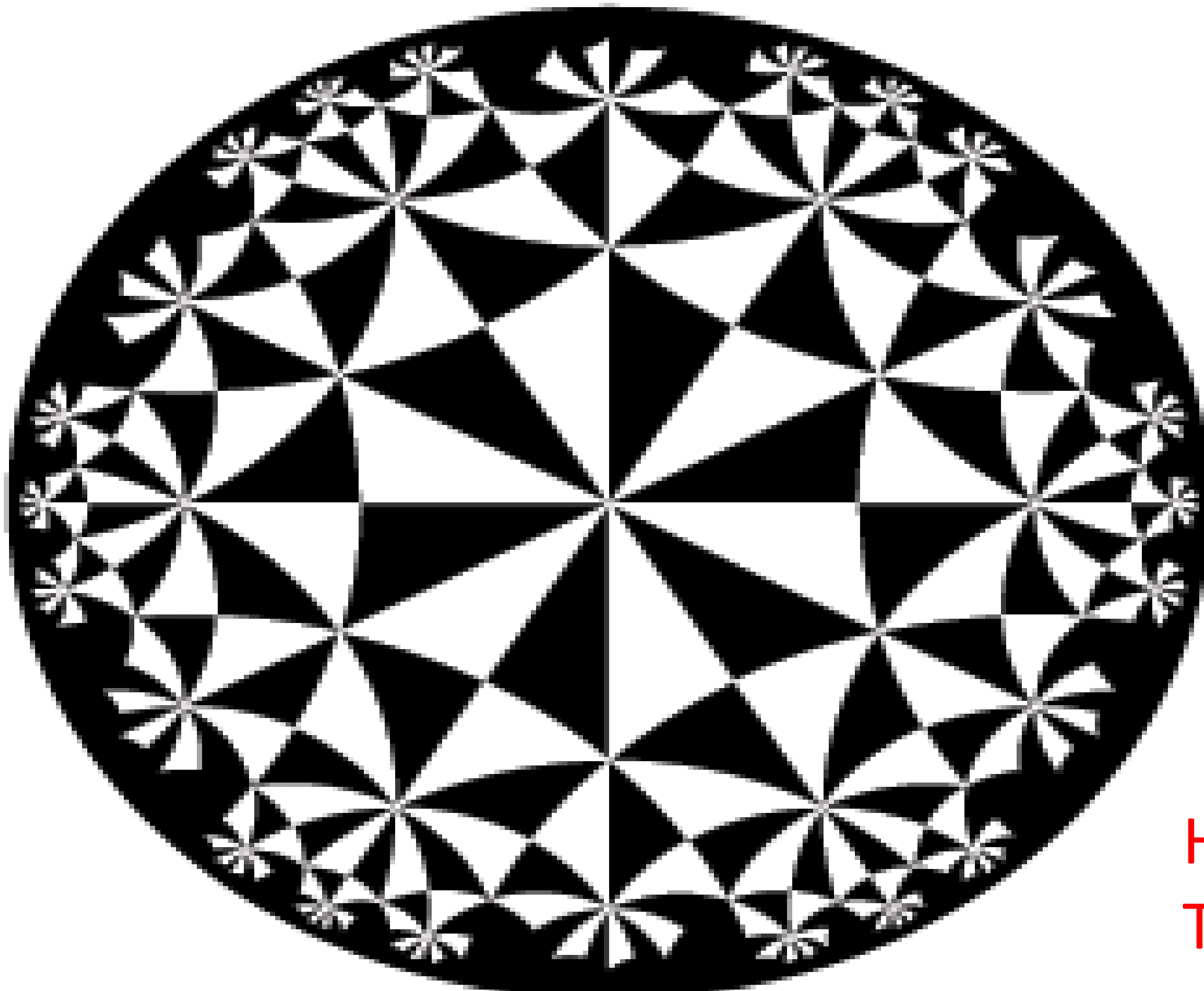
Tesselation of the plane: Day and Night 1938



Tesselation of the Circle: Angels and Devils (1960)

Escher and Coxeter





Hyperbolic Tesselation

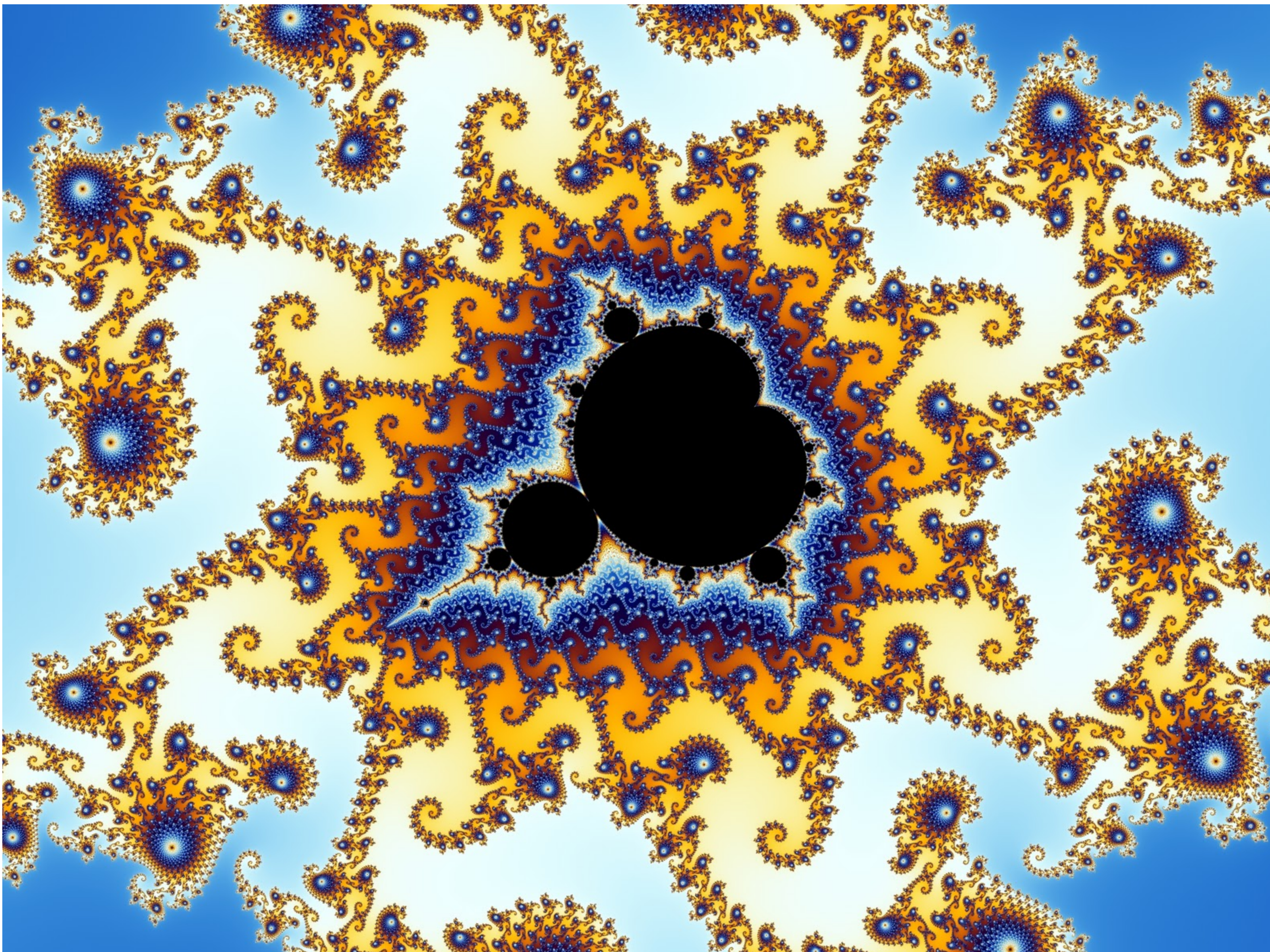
Letter from Escher to Coxeter 1958

Though the text of your article on "Crystal Symmetry and its Generalizations" is much too learned for a simple, self-made plane pattern-man like me, some of the text illustrations and especially Figure 7, gave me quite a shock. Since a long time I am interested in patterns with "motifs" getting smaller and smaller till they reach the limit of infinite smallness. The question is relatively simple if the limit is a point in the centre of a pattern. Also, a line-limit is not new to me, but I was never able to make a pattern in which each "blot" is getting smaller gradually from a centre towards the outside circle-limit, as shows your Figure 7. I tried to find out how this figure was geometrically constructed, but I succeeded only in finding the centres and the radii of the largest inner circles (see enclosure). If you could give me a simple explanation how to construct the following circles, whose centers approach gradually from the outside till they reach the limit, I should be immensely pleased and very thankful to you! Are there other systems besides this one to reach a circle-limit? Nevertheless I used your model for a large woodcut (CLI), of which I executed only a sector of 120 degrees in wood, which I printed three times. I am sending you a copy of it, together with another little one (Regular Division VI), illustrating a line-limit case.

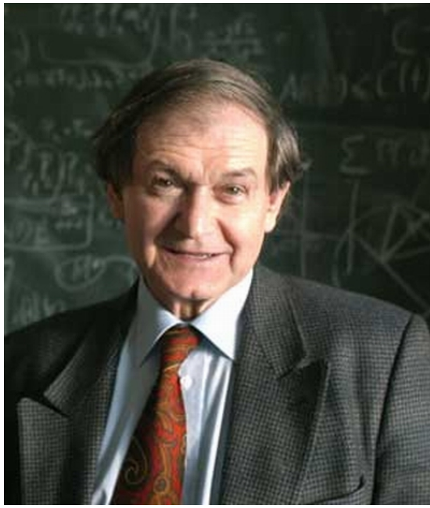
.

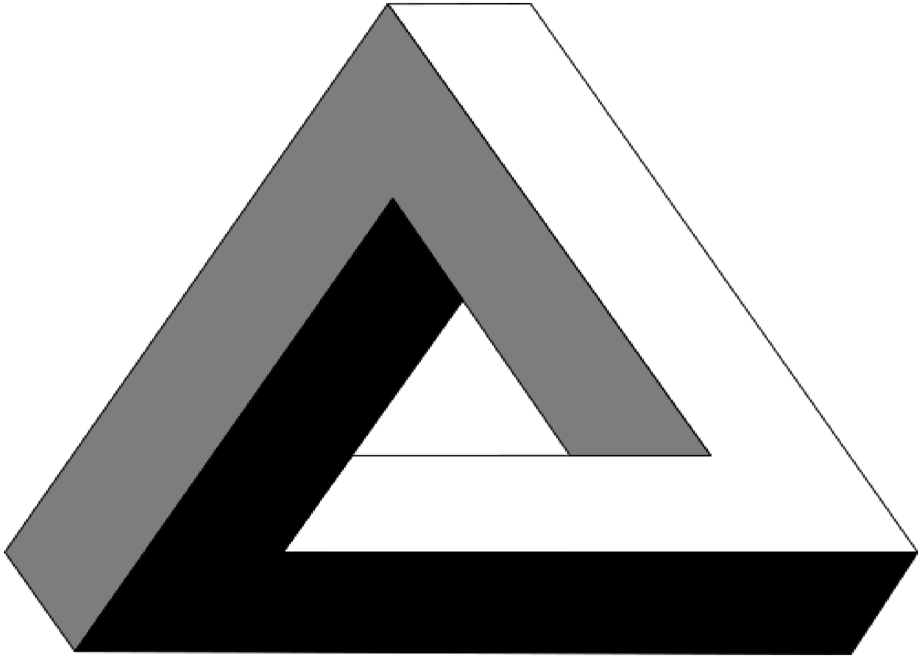
On December 29, 1958, Coxeter replied:

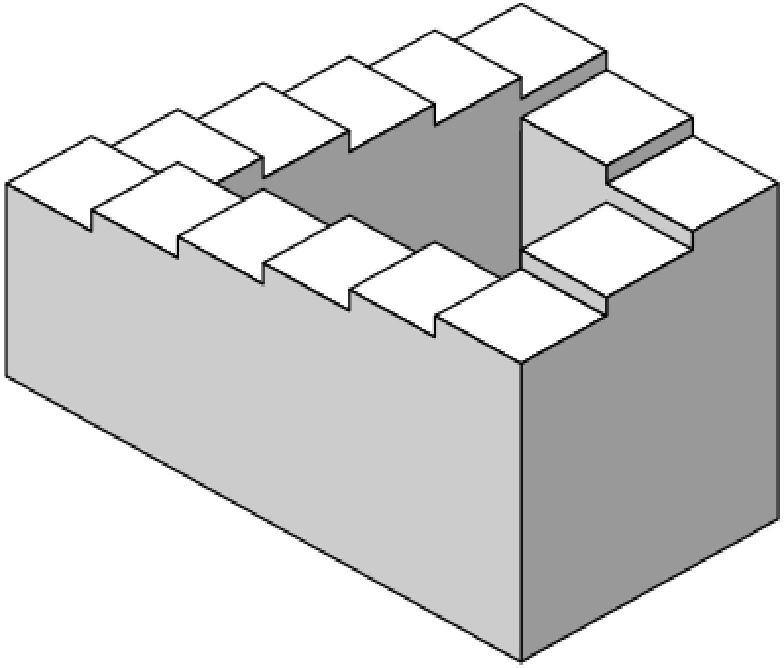
I am glad you like my Figure 7, and interested that you succeeded in reconstructing so much of the surrounding "skeleton" which serves to locate the centres of the circles. This can be continued in the same manner. For instance, the point that I have marked on your drawing (with a red • on the back of the page) lies on three of your circles with centres 1, 4, 5. These centres therefore lie on a straight line (which I have drawn faintly in red) and the fourth circle through the red point must have its centre on this same red line. In answer to your question "Are there other systems besides this one to reach a circle limit?" I say yes, infinitely many! This particular pattern is denoted by {4, 6} because there are 4 white and 4 shaded triangles coming together at some points, 6 and 6 at others. But such patterns {p, q} exist for all greater values of p and q and also for p = 3 and q = 7,8,9,... A different but related pattern, called <<p, q>> is obtained by drawing new circles through the "right angle" points, where just 2 white and 2 shaded triangles come together. I enclose a spare copy of <<3, 7>>… If you like this pattern with its alternate triangles and heptagons, you can easily derive from {4, 6} the analogue <<4, 6>>, which consists of squares and hexagons

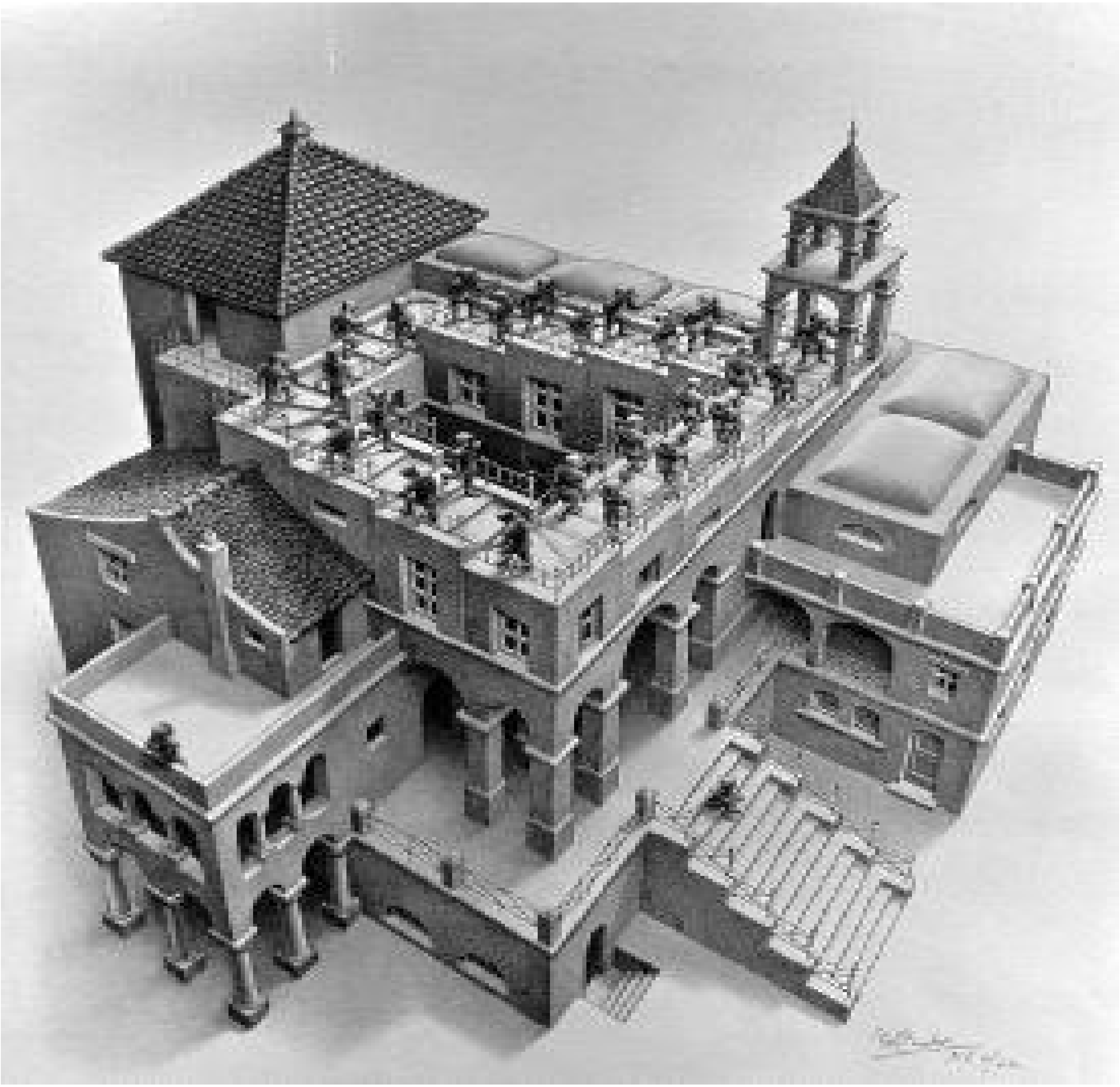


Escher and Penrose









Prentententoonstelling (1956)

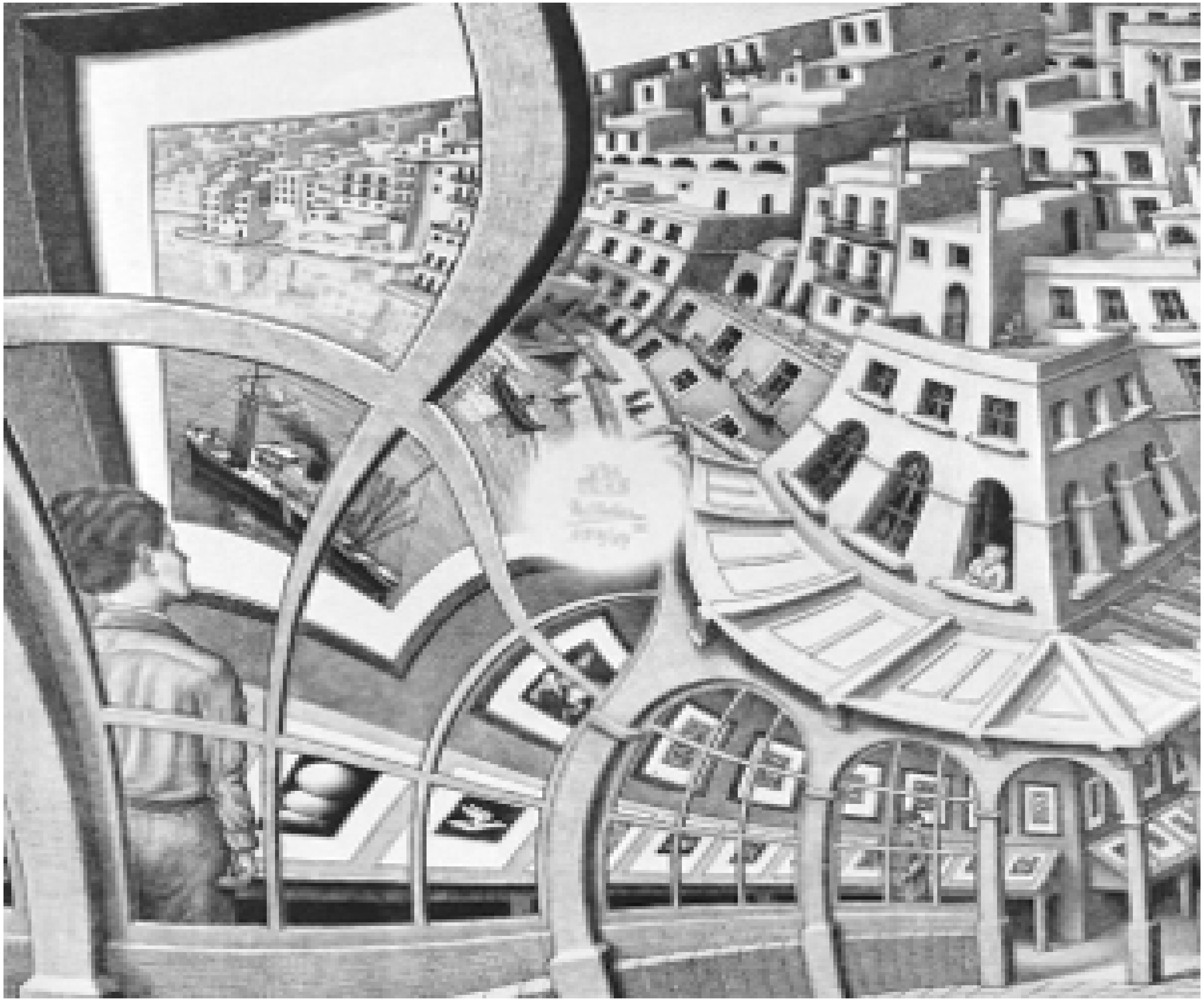
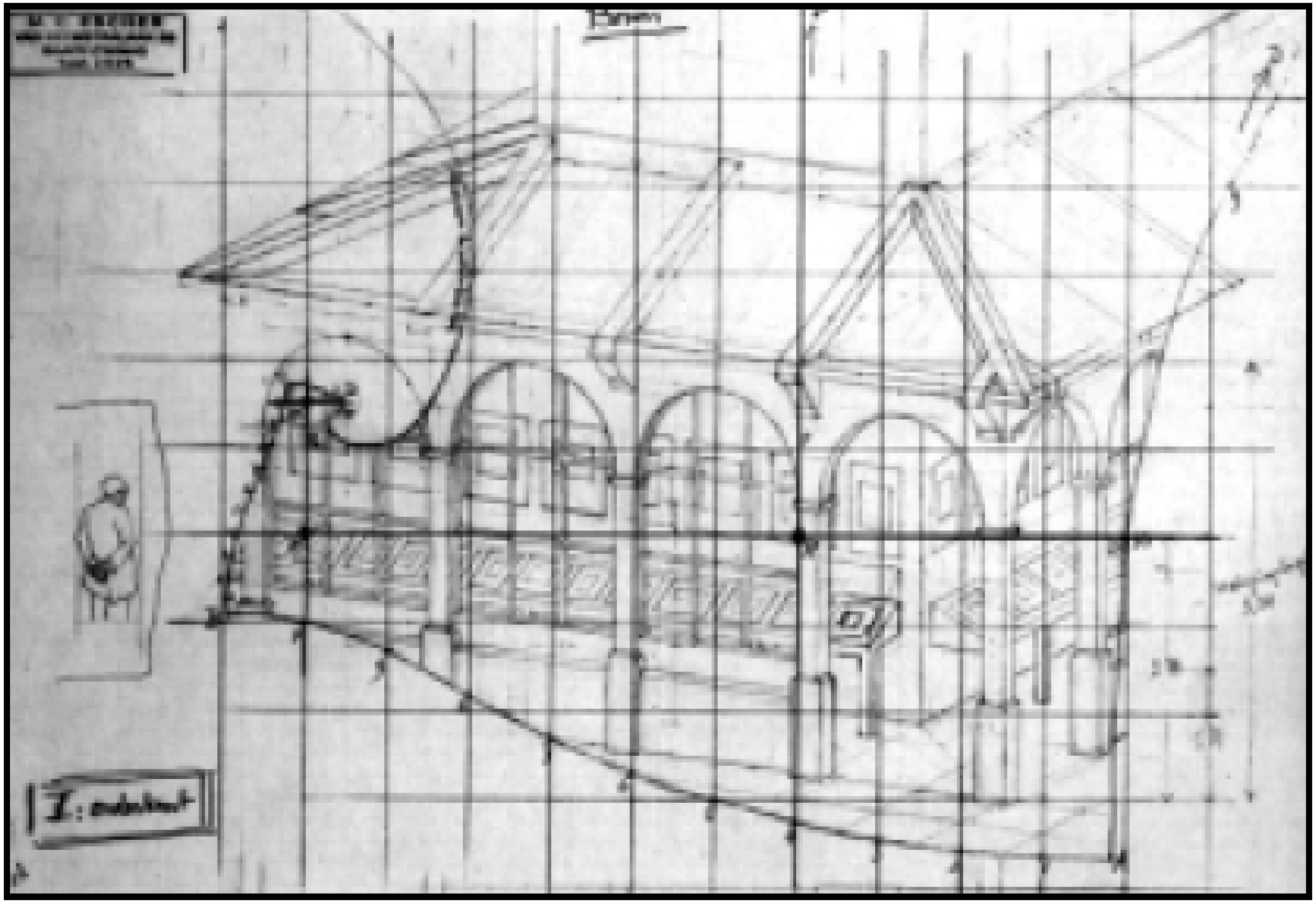
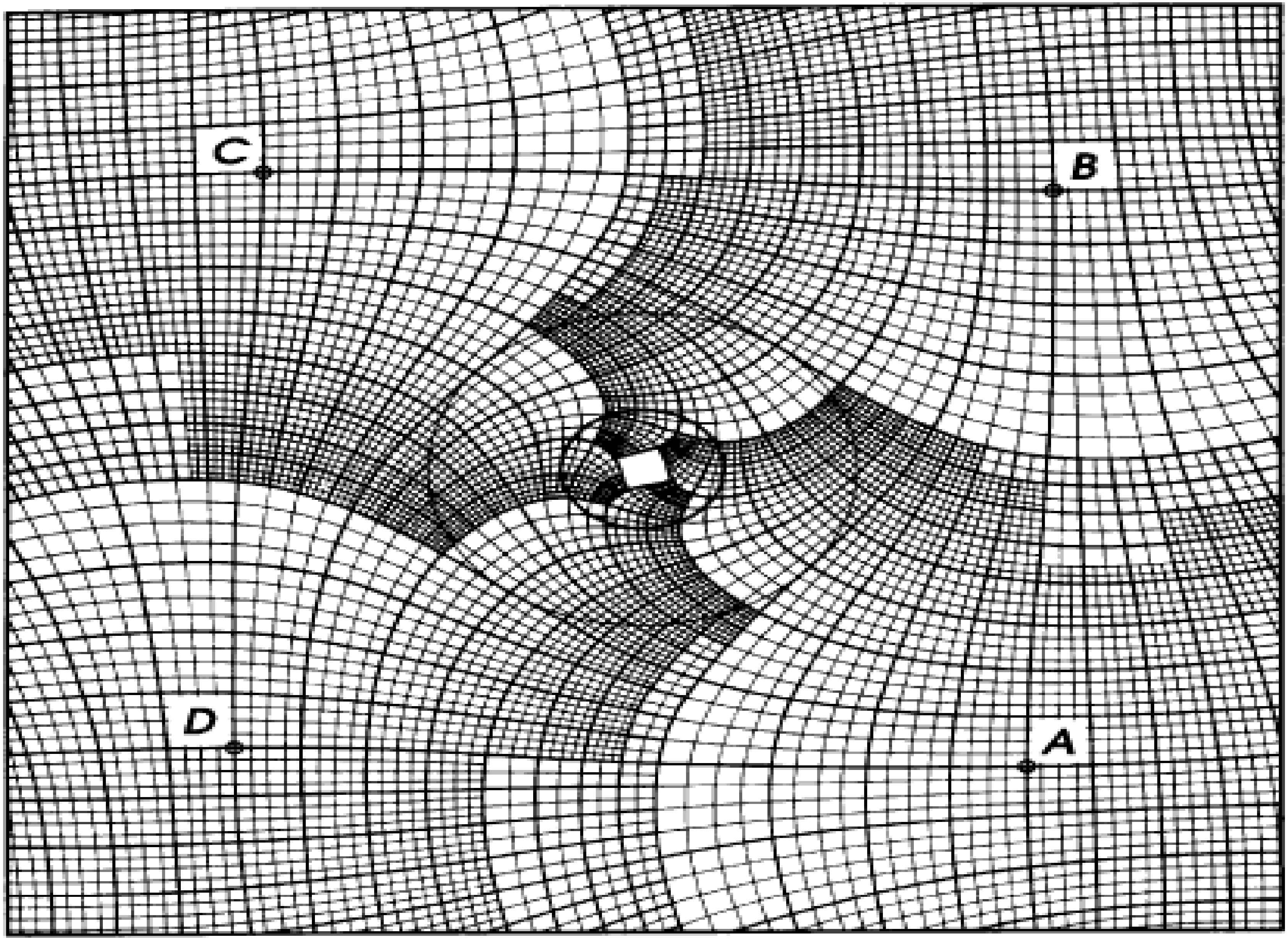


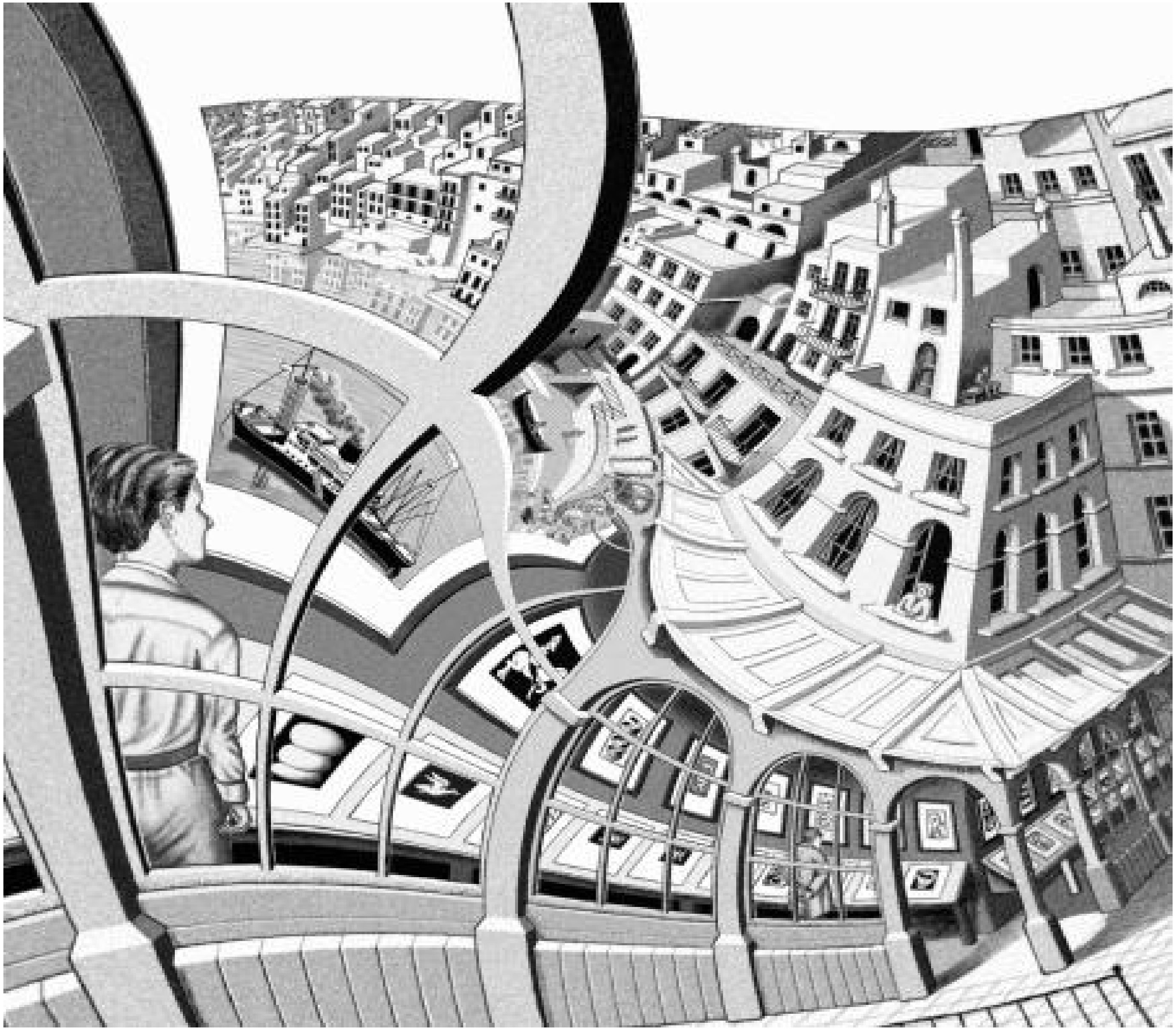
Image starts with a base design:



Which is then mapped onto the grid below

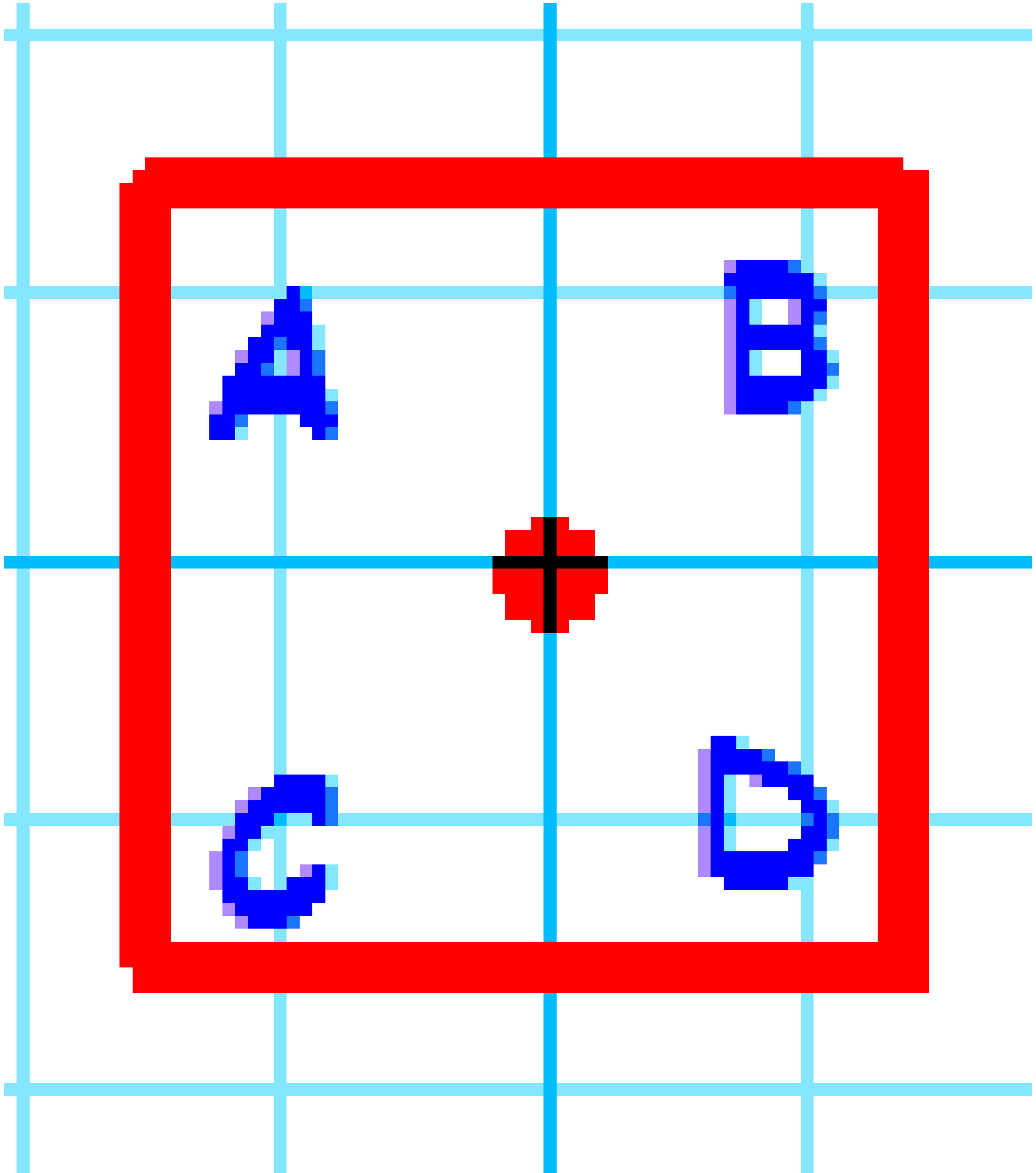


Hendrick Lenstra University of Leiden

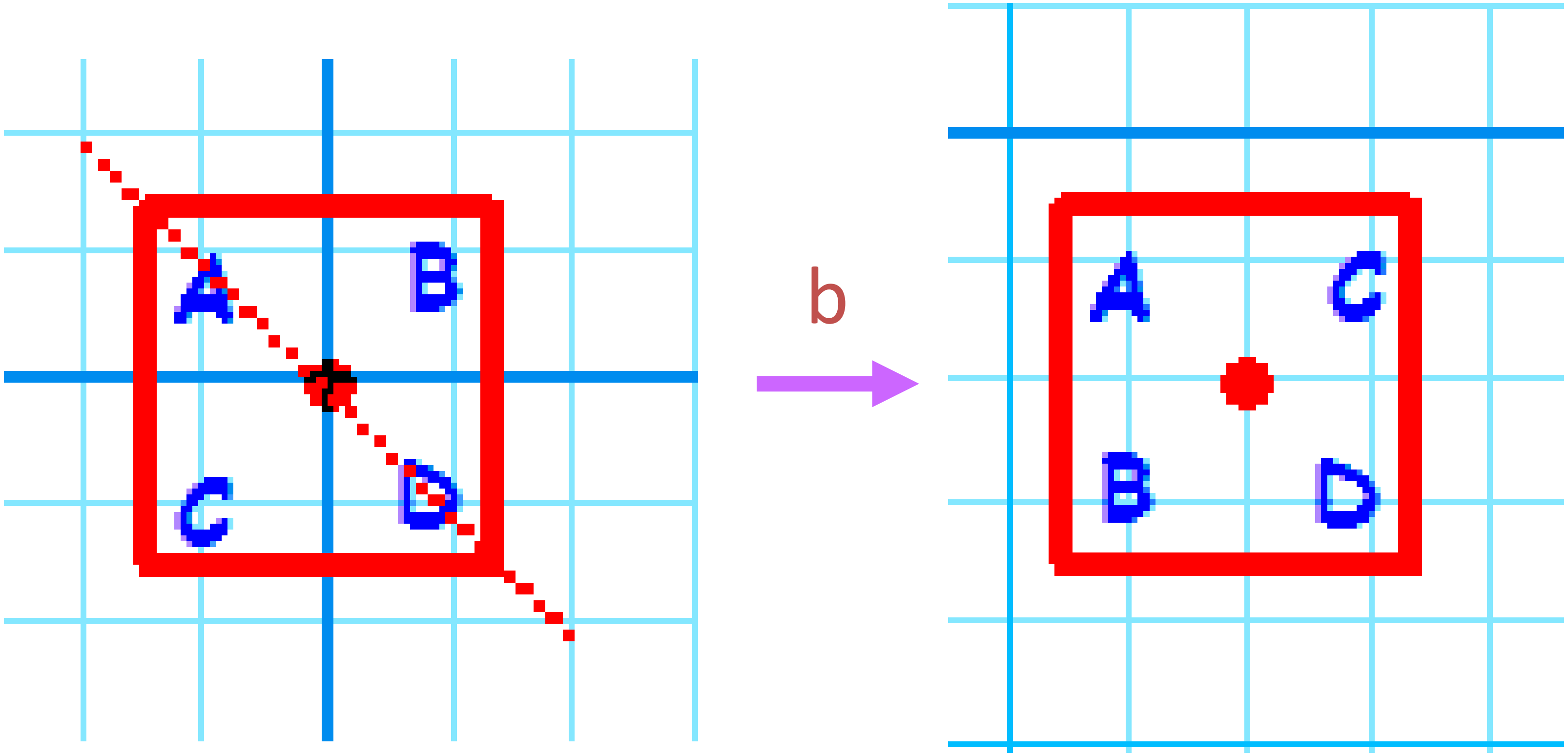


Dancing with Maths

Ceilidh = Square dancing



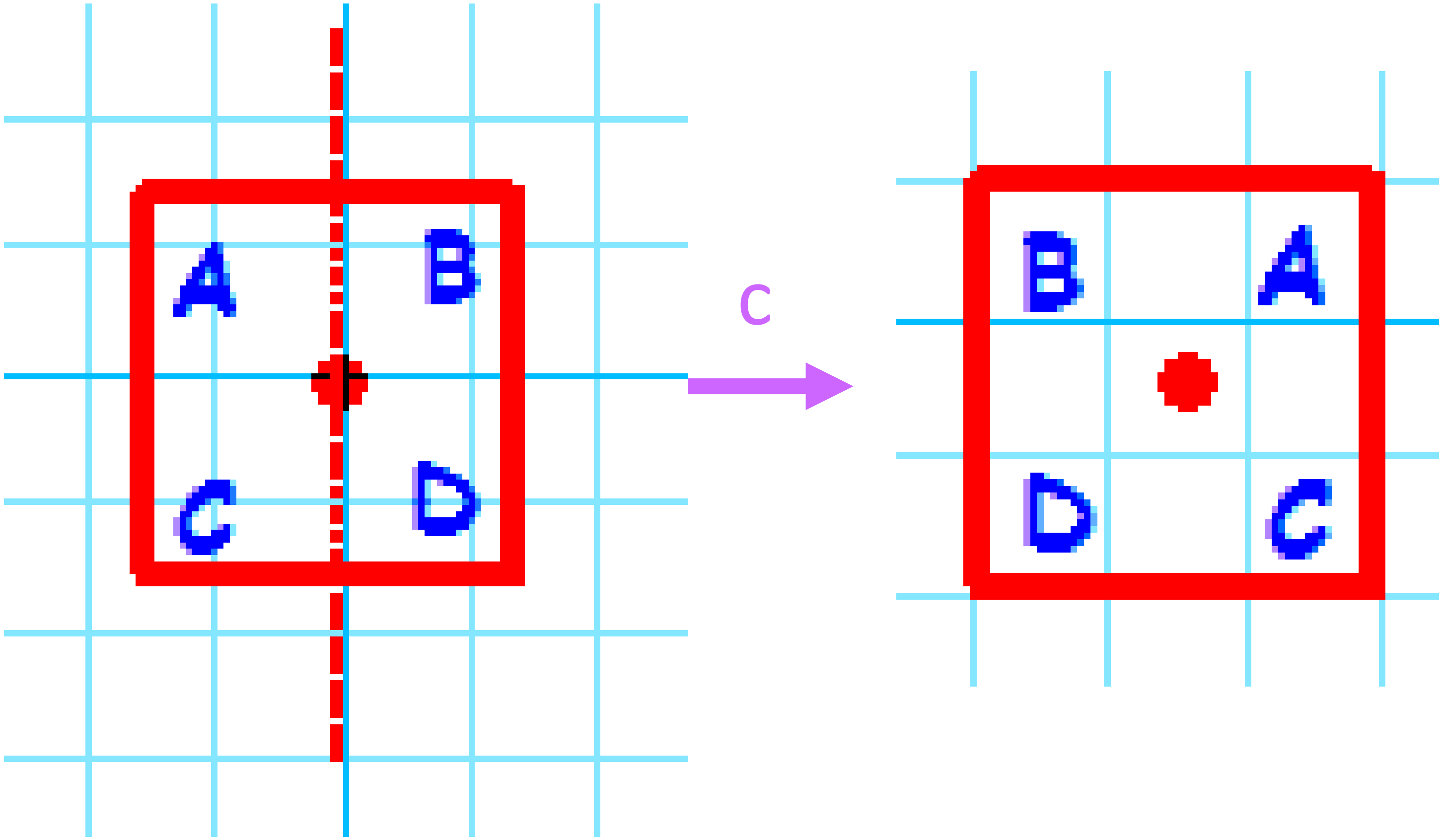
Diagonal Reflexion Symmetry



Dance move: Inner twiddle

A B C D —b→ A C B D

Vertical Reflexion Symmetry



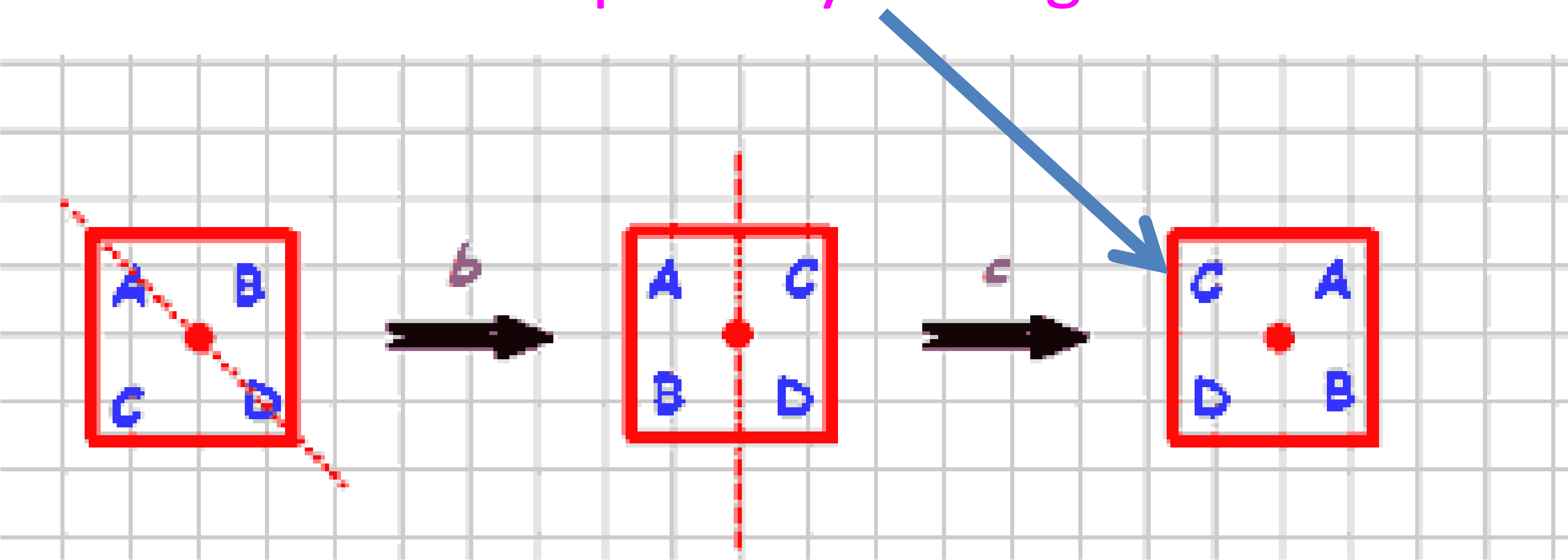
Dance move: Outer Twiddle

A B C D $\xrightarrow{c}$ B A D C

Now for the clever bit!

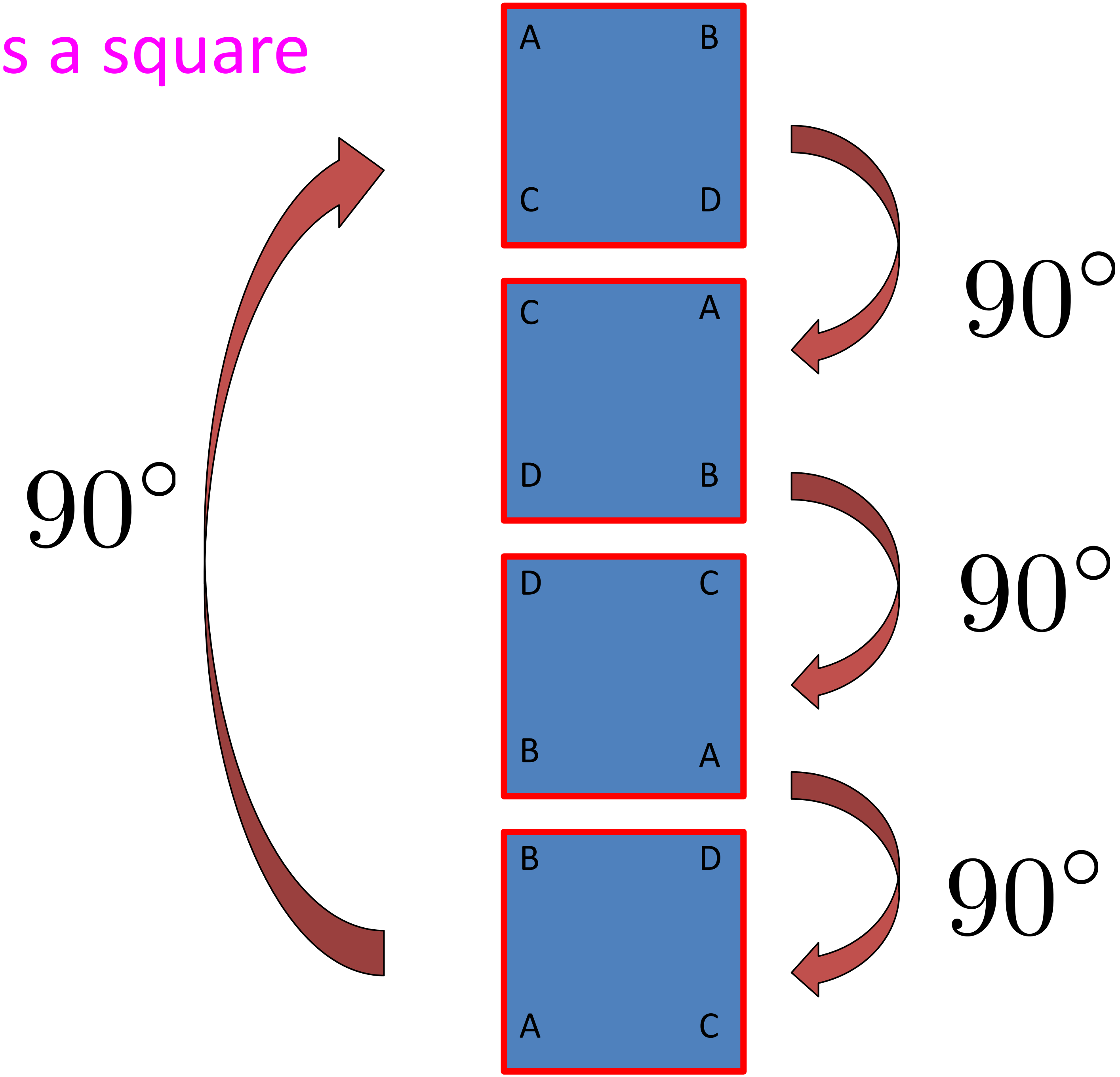
Applying b then c

rotates the square by 90 degrees

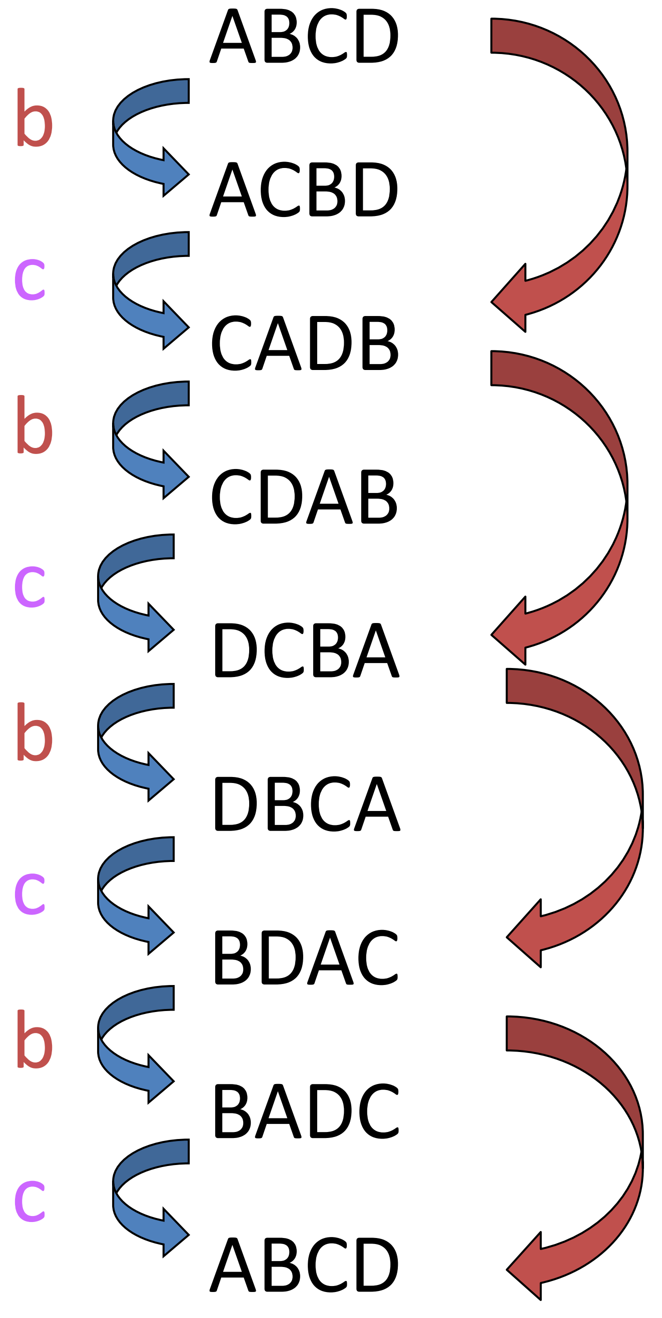


Now do: bc bc bc bc

As a square



As a dance



Now it’s your turn!!