GRESHAM COLLEGE

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How will we teach maths in the future?

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Introduction

When I was at primary school in 1970 (in the heyday of 'new maths'), my head teacher stopped the teaching of mathematics at her school. The reason she gave was that it was 'not creative'. I was in the ridiculous situation of actively having to disobey her in order to do any maths at school. Ultimately this did me good as it made me want to do maths and gave me the drive to do it. (I was very pleased to see from its website that this particular primary school now takes maths much more seriously and are taking a creative approach to its learning). I was fortunate to have some amazing maths teachers at secondary school, and here we are now. Fortunately, things have changed a very great deal since then. Through a combination of both a lot of time, effort and ideas of mathematics educators, and also a shift in political will, maths is in general well taught at both primary and secondary school levels. However, we must not be complacent and much still needs to be done. Furthermore, new ideas and ways of doing things, especially centred around the use of technology, look likely to transform the way that maths will be taught in the future.

In 2011 I was one of the authors of the 'Vorderman report' [1] which addressed the state of play in mathematics education. In this we stated that not only is mathematics the language by which the sciences, commerce, the internet and the global economics structure all communicate, it is also an essential part of all of our personal and working lives. Mathematics is not only a great subject by itself, it is also critical in fostering the logical and rigorous thinking so essential to the modern world. Unfortunately, the great language of mathematics in general, and of numbers in particular, is one which great swathes of our population cannot speak. This is why mathematics education is so important, and its future is important for the wellbeing of us all. In this lecture I will look at both the great recent achievements in mathematics education, and also some of the problems and challenges that it faces, both now and into the future. I will also talk about the very important role that technology plays in mathematics education, both as a means to learning and communicating maths, and also the way that mathematics linked to technology allows it to have an impact undreamed of by past generations of students.

However, none of this will matter much for the future unless we have a good supply of well trained, skilled, motivated and rewarded, maths teachers at primary and secondary schools, and at FE, HE and in adult education. Teachers are the gate holders of mathematics education, and their role is critical. I will finish this lecture by considering the question of the future of mathematics education as really being that of the future of mathematics educators.

The Importance of Having A Good Mathematical Education

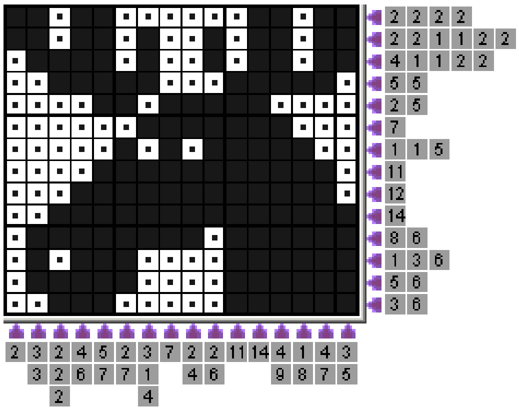
There are many good reasons to teach mathematics, and closely related disciplines such as statistics, data processing, operation research and computer programming) at schools. I will consider three of these.

Firstly, and perhaps the most obvious is that basic numeracy is a necessary requirement for 70% of all jobs. This is a fact well recognised by the CBI as reported in [2]. Indeed you need basic maths for such important lifetime skills such as applying for a mortgage, balancing the family budget, reading a bus time table, planning the family holiday (such as working out the flights and packing your luggage), moving furniture, driving a car and even going shopping and working out the best buys in the shops. Thus, teaching basic mathematical skills will equip the majority of our young people for both the work place and for life in general. This need is only going to increase as (in the manner I have explained in several previous lectures) our technology becomes increasingly reliant on high level mathematical algorithms and ever increased sophistication in the applications of mathematical ideas. A recent report by Deloitte [3] showed that mathematical research contributes £208 Billion and 10% of all jobs in the UK economy, and the 2018 Bond Review showed the critical importance to all aspects of our life, of transferring mathematical knowledge from universities to schools and the workplace. This shows that it is essential to the growth of the UK that we continue to teach maths at the highest levels, and that we inspire young people to do so. As a way of encouraging young people to study maths. Indeed, according to the BBC [4] studying for a degree in maths in 2018 will give you a higher boost to your earnings than almost any other subject.

However, it is not only the immediate utility of mathematics, which make it important to teach it well. A well-known journalist said recently that he had studied all sorts of maths at school including algebra and differential equations, and whilst he had mastered all of it, he had never found any use for it since. This may well be the case; however, he was thinking of mathematics as just a shopping list of methods and techniques. (Similarly, the teaching of English might be incorrectly criticised by saying that none of the books that I read at school ever applied to any real-life situation that I have been in. For example, I love Winnie the Pooh, but have never been in a situation where I have had to escape from a rabbit hole.)

Whilst it is true that the methods and techniques of mathematics (certainly including differential equations) are very important, this alone is a poor description of a great subject. Instead, as the Dame Celia Hoyles, President of the Institute of Mathematics and its Applications (IMA) said in her presidential address [5], mathematics is much better thought of as begin a collection of strongly connected *ideas* and ways of logical thinking and of understanding the world. It is those ideas and modes of thinking, which have led to the invention of Google, the Internet, mobile phones and much, much more besides. And I would make a bet that the journalist concerned has made heavy use of all of these. Similarly, it is precisely the logical way of thinking that maths trains you in, plus the ability to understand and to represent data, which makes it such a powerful training. Mathematics is currently the most popular A-level, and the students who study it make use of the ideas that they learn in many other subjects, including engineering, medicine, economics, music and the social sciences.

Thirdly, mathematics is worth teaching (as are all subjects), because it has the potential for being both fun and hugely creative. This may come as a surprise to many of you, but it is certainly true. One (and perhaps the best) way to demonstrate this is through solving puzzles. Examples of these are published regularly in the newspapers (including those for which my afore mentioned journalist writes). A very good example of this is a Sudoku puzzle. Sudoku was (to all intents and purposes) invented by the great mathematician Leonhard Euler and requires mathematical skills to solve similar to those needed to invert a matrix. As a second example, for a Griddler puzzle you are told the number correlating to an order of blocks both horizontally and vertically and you must find the picture.



The mathematics for doing this is closely related to that of solving inverse problems, such as locating oil under the ground.

It is my great privilege, and honour, to be the current Chair of the United Kingdom Mathematics Trust (UKMT).

The UKMT organises mathematics competitions for secondary school students based on problem solving, both as individuals and in teams. It currently works with over 80% of all schools (??) and the competitions each year are taken by over 800,000 young people. The challenges are carefully graded so that anyone can take them at the lower levels, and you can then progress on to more difficult competitions later. The popularity of doing these contests is testament enough to the enormous popularity of problem solving as a way of learning and appreciating mathematics. Similar problem-solving competitions, or problem-solving based learning sessions are organised (by different bodies) for primary schools and include the *National Young Mathematicians Award* and the *Mathletic*s website. Problems are not only fun, they also play a vital role in stretching the brightest children and in exploring their mathematical horizons.

> *The UKMT were really great for me. They enabled me to study topics exterior to the A level syllabus and treated everyone as very capable mathematicians, imbuing us with confidence to attempt problems we would have else been content to ignore.*
>
> *Luke Betts, medal winner in two IMO competitions*

Of course, the challenges of problem solving is one which can be of great use in other aspects of life. However, I would not in any way want to claim that the UKMT challenge problems are anything other than recreational. As an example, here is a UKMT puzzle [6] first appearing in the 1999 Intermediate Maths Challenge, which you can think about for the rest of this lecture:

It is evening and Meg, who is 1m tall, casts a 3-metre-long shadow. If she stands on her brother's shoulders, which are 1.5m above the ground, how long is the shadow which she and her brother will cast?

The real purpose of problem solving as a way of teaching maths, is that to solve a really good problem you have to think up new ideas and to become truly creative. This in turn develops a spirit of discovery, which means that almost anyone can engage in mathematics by trying to solve recreational problems. Indeed, it is fair to say that much of really good modern mathematics has its roots in purely recreational investigations.

Thus, I would strongly argue that it is well worth teaching the aesthetic and recreational aspects of mathematics and encouraging our students to get engaged with them. This should go along side (and is in every way complimentary to) the more functional aspects of maths that I have already described.

Problems With The Way Maths Is Currently Taught In The UK

Whilst I have painted a rosy picture about what a mathematics education can do for you, the reality is that mathematics is not as thriving a subject as we might wish it to be, and I have serious concerns about the future of mathematics education in the UK. These concerns are both general and specific.

1. *False Perceptions of What Maths Is And How It Should Be Taught*

Perhaps we should start with the obvious. Let's be honest, mathematics has an image problem! Despite what I have said above, it is perceived as being hard, confusing, useless, non-creative, male and only done by mad soulless geeks. If you don't believe me (that it has such an image problem) then watch the dreadful film 'Comment j'ai détesté les Maths'. Better still, don't watch it. This perception is widely held in the media, and beyond, and acts to put off young people from engaging with any level of mathematics. Whilst it is certainly true that mathematics can be difficult, and it takes a level of dedication to do many aspects of it, it is far from unique in this. Many other things are difficult, such as playing the piano, speaking a foreign language and kicking a penalty. Yet young people are encouraged to do these, and see proficiency in them as good, even though they are hard. Furthermore, doing well in these subjects is treated by the media as being great, whereas if a child does well at maths they are often treated as a freak and can be bullied as a result. Another frequently heard comment is that people suffer from *maths anxiety* and that this is a problem with mathematics as a subject. I agree that maths anxiety exists, but so do many other anxieties. For example, at school I had anxiety both about playing rugby and appearing in a drama in front of an audience. In contrast I loved doing maths! In neither case was the subject held to blame for my anxiety. I completely agree that we should understand the anxieties that young people have in learning any subject, and we should teach them accordingly. But please don't say that maths is an especially bad example of this. (If it was then I feel it would not be the most popular A level.) And certainly, do not use it as an excuse for not teaching maths.

I have already commented on the fact that maths is incredibly useful. Thus, I find the accusation that maths is useless very hard to understand, or to make sense of. However, I do find it a problem that when maths is taught at all levels (both in school and in HE) the applications of it to many areas are simply not mentioned. Please do not misunderstand me here, I am not saying that the abstract nature of mathematics should not be taught. Far from it. It is essential that the abstract nature and power of mathematics is explained well. However, this in no way contradicts the importance of also explaining the way that it can be used in real life. The fault here lies in many places. One is in maths text-books which are dry and un-motivating and reduce maths to just a set of dreary calculations. The second is (I'm really sorry to say) in maths teachers themselves, who having not themselves learned about the relevance of mathematics are not able to pass this on themselves. An example of this (and other problems I have discussed) arose a few years ago at the NUT when a speaker said that we should not teach quadratic equations at schools. The reason being that not only were quadratic equations completely useless, but in some way, they were frightening young people. Below you can see an example of just such an evil quadratic equation (drawn by the wonderful artists in Plus Maths Magazine) about to assault an unsuspecting student. Needless to say, this view was strongly supported in the resulting media frenzy. It even led to a remarkable debate in the House of Commons where I am pleased to say that the quadratic equation was stoutly defended and carried the debate. (I wrote about this episode in [7]). There are many, many reasons (in fact 101 if you read my article) for teaching the quadratic equation. Far from being useless it opens the door to much of science (for example the study of area, speed, acceleration, differential equations, vibration, stability, quantum theory, matrix algebra, complex numbers and chaos theory to name a few). It is also the first example of a problem which needs more than the elementary operations of addition and multiplication to solve, and thus gives a first glance into the richness that mathematics has to offer. Furthermore, solving the quadratic equation has a wonderfully rich history, starting with the Babylonians, progressing through the Greeks and only finishing with the great discoveries of the Indian mathematicians. This teaching the quadratic equations in schools shows dramatically how many different cultures have contributed to mathematical discoveries.

This brings me neatly to another problem with the way that mathematics is often taught. It is frequently described in a way which implies that all mathematics simply existed for ever in a text book. No mention is made of the people who created it and the rich history that it has. This is unfortunate for two reasons. Firstly, it dehumanises maths and breaks a natural point of contact between it and people. Mathematics didn't just happen. It was created by (highly creative) people all over the world.

2. Stopping Maths Too Early

If you do maths after the age of 16 in the UK, then there is a good chance that you will receive a good mathematics education. The modern A-level maths has been carefully designed and delivers a focused mathematical training. Indeed, maths is currently the most popular A level amongst boys, and the second most popular amongst girls. (Although this fact is not widely known). Another positive fact which we will return to, is the existence of Further Maths A-level and its support network the FMSP. Students doing both A level maths and further maths will receive a comprehensive maths education which prepares then well not only to study mathematics at university, but also to do other courses such as physics and engineering, which have a high mathematical content. Although it is very worrying that recent changes to the way that A-levels are funded mean that schools may be very reluctant to continue teaching further maths as a fourth A-level.

Unfortunately, whilst as I have said maths is the most popular A level, it is still only taken by a minority of students. Indeed, according to the Royal Society, only 13% of young people in the UK (mainly England) study maths beyond the age of 16. Indeed, according to the same survey, one in four of UK adults is functionally illiterate.

This situation has been exacerbated in England (not Scotland) by the practice, until recently, of dropping the teaching of mathematics to most students post-16. The table below (from [1]) shows how out of line England Wales and Northern Ireland are in this.

Students taking mathematics post-16[7.4]

	Any mathematics	Advanced mathematics
Japan	All	High
Korea	All	High
Taiwan	All	High
Estonia	All	Medium
Finland	All	Medium
Sweden	All	Medium
Russia	All	Low
Czech Republic	All	–
France	Most	Medium
USA (Mass)	Most	Medium
Germany	Most	Low
Ireland	Most	Low
Canada (BC)	Most	–
Hungary	Most	–
New Zealand	Many	High
Singapore	Many	High
Australia (NSW)	Many	Medium
Netherlands	Many	Low
Hong Kong	Some	Medium
Scotland	Some	Medium
Spain	Some	Low
England	**Few**	**Low**
Northern Ireland	Few	Low
Wales	Few	Low

Key

Any mathematics	5–19%	20–50%	51–79%	80–95%	95–100%
Advanced mathematics	0–15%		15–30%		30–100%

Data on participation in advanced mathematics were insufficient in Canada (BC), Czech Republic and Hungary.

This has a number of serious consequences. One is that policy makers in England, such as politicians and civil servants, often have had no serious contact with mathematics after the age of 16. Contrast this with Singapore, who's president had a PhD in maths, and who's prime minister had the top maths degree from Cambridge! Thus, English politicians are ill equipped to appreciate the role of mathematics in modern technology. Secondly (apart from some glowing exceptions) most figures in TV and the media have also dropped mathematics at this early age. They then go on to write articles in the press about how useless maths is, having denied themselves the education which would have taught them the opposite.

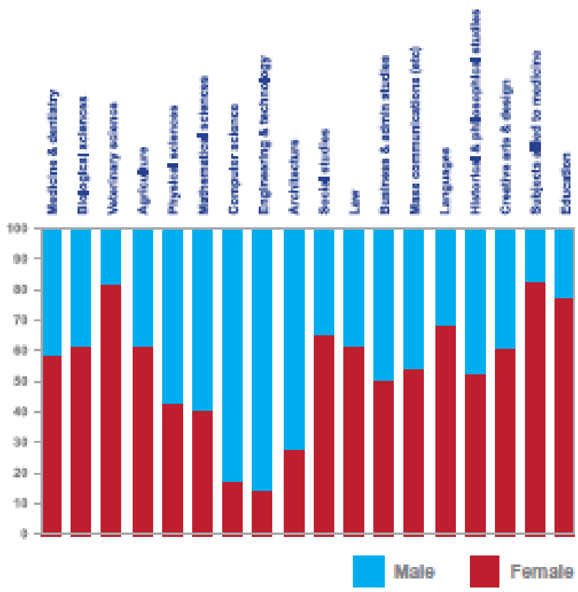
3. Few Mathematically Trained Primary School Teachers

Another serious consequence of the abandonment of maths at the age of 16, is that many, if not most, primary school teachers have had no serious training in maths (or indeed any of the STEM subjects). This means that they struggle to teach the maths curriculum in an informed and creative way and are certainly challenged when having to deal with talented pupils. The mathematics subject knowledge of primary school teachers, and of new trainees, urgently needs to be improved. For example, in a recent survey it was found that only 2% of those graduates studying a PGCE to become primary school teachers had a STEM degree. With no criticism of the teachers themselves, it is terrible that we have a system of education in place that allows this to happen. Unless the mathematical needs of primary school teachers are addressed appropriately, we will be locked into a cycle of poor mathematical performance of their students for generations.

The problem of giving up maths too early has finally been recognised by the government. I am pleased to say that they have now brought in legislation so that all young people staying at school till 18 will learn some mathematics. This will be at one of three levels. One group will continue to do A-level maths. Another will to learn functional and vocational skills at GCSE level. In between these there is a middle level learning Core Maths. I will return to this group presently.

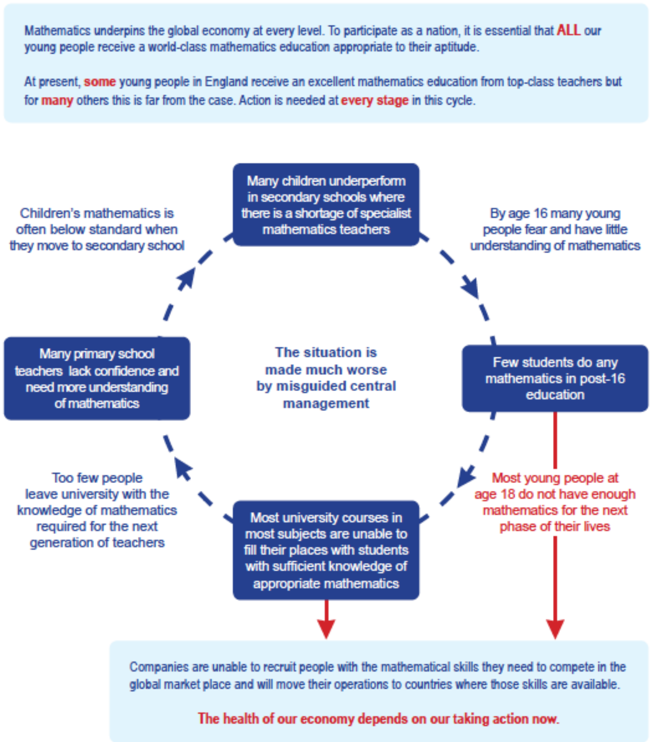
4. Under Representations Of Girls

As I have said already, maths is often portrayed in the media as being male dominated. This is not true, but unfortunately there is a self-fulfilling prophecy that if maths is perceived as being male dominated, then girls are put off doing it, and thus it becomes more male dominated. Thus, it is important not only that we encourage girls to do maths, but that we are seen to be doing so. In fact, at one level the situation is not too bad in this respect. Below is a table of the proportion of female undergraduate doing various subjects. The figure for maths is about 42% and rising. Mathematics is in fact closer to a 50:50 proportion than most other subjects, and is far better balanced than computer science (for men) or veterinary science (for women).



It is important that this fact is made more widely known, as it acts as an incentive for more women to do maths at university. However, it is fair to say that the current situation in the female staffing of universities is not as good, with (according to the LMS Good Practice Guide [8]) 29% of mathematics lecturers and 6% of professors. However, this situation is slowly improving, especially with the Athena Swann programme of awards forcing universities to critically examine the way that they treat, and recruit, female staff.

The rather worrying conclusion of the Vorderman report [1], illustrated below, was that maths education was in a state of crisis and that urgent changes were needed.

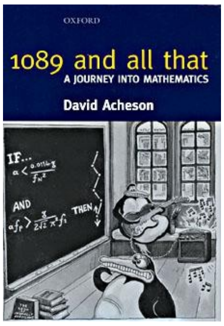


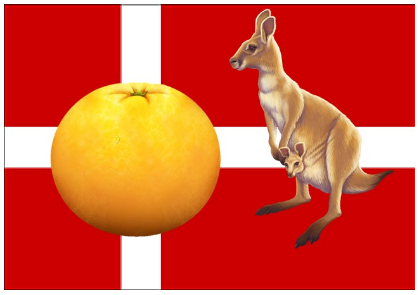
I will now have a look at some of the ways that we can change things, and case studies of real success stories in UK maths education.

How To Encourage Young People To Learn Mathematics

1. Show The Magic Of Maths

One of the reasons that maths has its bad image and people are not engaging with it, is that they simply have never encountered any real mathematics. Once they meet real maths then their attitudes often change. This point is very well made in the lovely book '1089 and All That', shown below [9]. The centre-piece of this book is a famous piece of mathematical magic. You take a 3-digit number such as 729. Reverse it to give 927. Now subtract the smaller number form the larger to give 198. Reverse that to give 891. Add these together and you get 1089. Every time! According to Acheson this trick 'blew his socks off' the first time he met it, and he has loved mathematics ever since. I can confirm that this, and other mathematical magic tricks such as those involving mind reading, card shuffling and the odd orange kangaroo from Denmark (see below), have the great power, not only to amaze young (and old) people but also to encourage them to learn more maths so that they can create their own tricks. I (and many others) have frequently found great success in using mathematical magic to inspire a love of mathematics in an audience. Why does this work so well? Mathematics is perceived as being a dry and logical subject, with no excitement and surprise. However, this is not what maths is at all. It is a deeply mysterious, creative and surprising subject. In many ways it is just like magic, and the magic trick makes this connection clear to all.

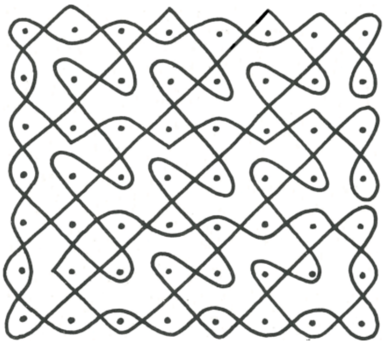




2. Show That Maths Is A Deeply Creative Subject

How can maths be creative shouts the media. What could be less creative than 2+2=4. Leaving aside the enormous creativity in the abstractness of this sum, making a statement that this addition is not creative is like saying that music is not creative because anyone can play a single note on a piano. This misses the fact that it is the interplay of lots of notes which makes the music we so enjoy. In the same way it is the interplay of ideas in maths which gives it its great creative power. It is important in teaching that we bring out this creativity. One way to do this is through examples. We have just considered that a collection of piano notes makes up a tune. The basis of nearly all tunes is the scale. It often comes as a surprise that this foundation of music was invented by the mathematician Pythagoras in the course of his study of musical harmony and the links that this has to simple fractions such as 4/3 and 3/2.

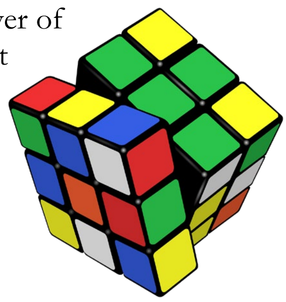
Mathematics also has deep connections to art, with examples in Islamic, Celtic and African Art (which I will explore in depth in a later lecture). The abstractness and creativity of both subjects links them together. Below I show the 'Chased Chicken Sona' design from Sub-Saharan Africa. I challenge anyone not to find this design both artistically striking and mathematically challenging. A workshop based on these not only opens the lid on the creativity of mathematics, but also shows how truly multicultural it is as a subject.



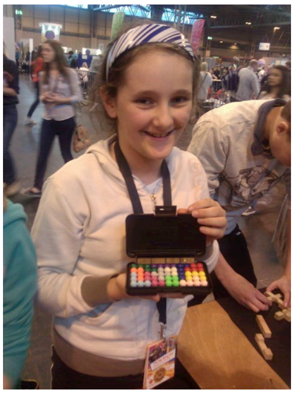
All of this illustrates one of the key aspects of mathematics: the discovery and exploration of patterns. Exploring these (especially in a playful way as I describe in [10]) makes mathematical discovery accessible and enjoyable for all ages.

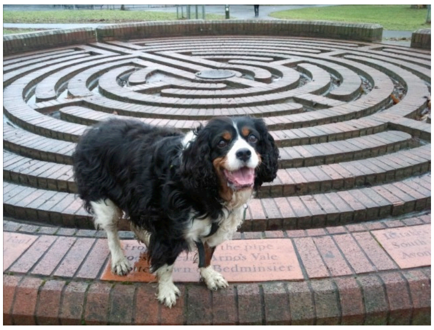
3. Play With Maths

Like all of us I love to play, and to play with toys. We should never underestimate the power of play in learning, and that certainly includes learning mathematics. I was very fortunate that when I was an undergraduate at Cambridge in 1979, the Rubik's Cube had just become available.



There is simply no better way (as we all found out) to learn the intricacies of group theory than by learning how to solve the cube. I have found the cube to be an invaluable teaching tool ever since. Other toys, big and large, can be very useful in teaching maths such as topological puzzles, origami, packing problems (see below), mazes (see below), sundials, and of course mechanical toys to teach mechanics, chaos theory, vectors and many aspects of calculus. All these work through a process of direct engagement and associated discovery.





I urge all classrooms, at all levels, to have a large supply of mathematical toys to inspire and motivate their students, as well as to have a lot of fun in the process.

4. Talk About Careers

Most of our young people are keen to have a job at the end of school or university. As I have said above, mathematics is needed for around 70% of all jobs and leads to a wide variety of careers. This it makes great sense to talk about careers in mathematics as a natural part of any mathematics course. And I don't mean just careers in finance and teaching, worthy careers though these might be. Maths careers can also include telecommunications, aerospace, power generation, iron and steel, mining, oil, weather forecasting, security, retail, food, zoos, sport, entertainment, graphic design, media, forensic service, hospitals, air-sea-rescue, education, transport, risk, health, biomedical, environmental agencies, art, etc. The list is truly endless and lots of great examples of maths careers can be found on the IMA Maths Careers website [11].

Recent Big Changes In Maths Education: Some Case Studies Of Excellence

I hope that I haven't given either an overly negative view on current mathematics education or indicated that nothing good is being done apart from what I suggested in the above section. In fact, this is not the case at all. The UK is doing amazing and leading work in mathematics education at all levels, which is very encouraging.

1. Changes At The Top

Part of this change is due to a much better institutional level investment in, and commitment to, mathematics education. A number of influential reports (see for example [1], the original Smith Report [12] and the more recent report by the same author [13]) have led to greater awareness of the needs of mathematics education and new recommendations to the way it is taught. The Royal Society has the Advisory Committee on Mathematics Education (ACME), which acts to act as a single voice for advice to government from the mathematics community. ACME in turn works closely with the education committees of the learned mathematical societies such as the IMA, LMS, RSS, and the OR society, together with mathematical teaching bodies such as the MA, ATM and the Joint Council of Mathematics. Such a unified voice is vital if the mathematics community is to influence government and sadly this community has been quite divided in the past.

2. Core Maths

I have already talked at length about the problems with students giving up mathematics at the age of 16. One of the groups that this was hitting hardest were those who were competent at mathematics (having done well at GCSE) and who would be using mathematics in their future careers but had interests which meant that they did not feel that mathematics A-level was right for them. This group included future doctors, psychologists, social scientists, and economists as well as journalists, plumbers, carpenters, and those working in the fashion, tourism, leisure and entertainment industries. According to ACME, 330,000 students at university started courses which needed post GCSE level mathematics, but of these 200,000 would have given up maths at 16. This set them at a clear disadvantage when they arrived at university and put the UK well behind our international rivals.

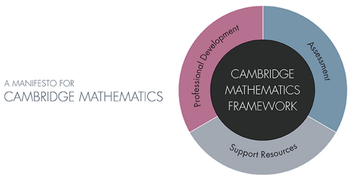
I am really pleased to say that this is now changing. In July 2011, Michael Gove MP, who was the then Minister for Education, announced that '...we should set a new goal for the education system so that within a decade the vast majority of pupils are studying maths right through to the age of 18'. Since then there has been a flurry of activity, which has led to the creation of the Core Maths Curriculum. Core Maths is aimed at those students described. This curriculum takes a much more problem-solving approach to mathematics, and involves mathematical modelling, data handling and the use of appropriate digital technologies. It is still in a relatively early stage of development, but text-books have been written for it, and it is now being taught in schools as an addition to the more traditional A-Level. I strongly welcome this development and very much hope that universities will fully recognise it as an appropriate entry qualification for those subjects listed above. I very much see Core Maths as part of the future of mathematics teaching.

3. Great Educational Initiatives

A number of bodies are now taking the lead not only in developing a creative approach to the way that mathematics is taught and delivered, but also making extensive use of digital and web-based learning methods in the creation of high-quality resources, which are easy to access. Theses bodies also realise that to learn mathematics you need to do mathematics, preferably with a mentor alongside you. Accordingly, the web-based materials are reinforced with summer schools and other people-based activities including continuing professional development for teachers. It is impossible to list all these bodies, but here are some examples.

The Advanced Mathematics Support Programme (AMSP) is a government-funded initiative, which aims to increase participation in Core Maths, AS/A level Mathematics and Further Mathematics, and improve the teaching of these level 3 maths qualifications. The programme provides national support for teachers and students in state-funded schools and colleges in England. I am very pleased to say that additional support is offered to students in areas of low social mobility. The AMSP evolved from the earlier Further maths Support Programme (FMSP) which aimed to give access to further mathematics A-level to students from all schools. This was a spectacular success, meaning that the first-year mathematicians that I teach at Bath nearly all have done Further Maths A-level and thus have a strong mathematical background. (We had to make our courses more challenging as a result!)

The Mathematics for Education and Industry (MEI) is the parent body of the AMSP and itself is a strong innovator in mathematics education and in teacher CPD. One thing I particularly like about MEI is the way that they have challenged the traditional way of assessing A-level maths by introducing a 'comprehension paper'. In this, students are asked to read a piece of maths and are then asked questions about it. Too often young people get the impression that mathematics is simply solving set piece problems which come out of a text-book. The comprehension paper gives a much better view of the way that maths is actually done in real life, and also gives a chance to show some really new maths to the students doing the exam.



Cambridge Mathematics is an organisation rethinking support for curriculum design in mathematics. According to their own web-site, they are developing a flexible and interconnected digital framework to help reimagine mathematics education 3-19. The design of their courses is carefully based on research and evidence and is done in collaboration with the whole mathematics community.

Cornerstone mathematics is a similar initiative based at the Institute of Education which has developed a set of targeted curriculum units of work for key stage 3 maths that 'help pupils achieve deep learning of the most difficult mathematics concepts through guided explorations and activities that use dynamic technology to foster reasoning and collaboration'. Particular areas covered are in algebra, functions and geometry. I apologise to all those other excellent bodies I have missed out. I salute all of you!

4. Changes in higher education teaching of maths

Higher education mathematics teaching (which may be for maths students but is also vital for all of the STEM subjects) should rightly challenge undergraduates and introduce them to the latest ideas and techniques in mathematical thinking. However, the problem with doing this well is that it can easily lead to students getting lost and giving up with mathematics all together. A particular concern in this is the mathematical education of the engineers who are vital to the future of the UK economy. In the past, this was often treated by a 'sink-or-swim' attitude. But this is both unfair and hugely inefficient. The problem has been exacerbated in recent years due to the issues with the school teaching of maths and in 2004 the influential Smith Report [12] made the recommendation that Higher Education has little option but to accommodate the to the students emerging from current GCE process. One very effective solution to this problem has been the establishment of a network of mathematics support centres in universities across the UK. In my own institution in Bath we have the Maths and Stats Help (MASH) centre. This provides an informal opportunity for students to come and receive help from staff, post-graduates and other students, to their various mathematical problems. Coffee, cake and comfortable chairs are provided to make the centre more approachable. Another great success has been the Sigma network, which works to assist both students (especially those with disabilities and learning differences) and university staff in maths teaching. In a recent survey it was reported that 81% of all universities now have some form of mathematical support. The higher education teaching of maths has also been improved by much greater attention to the training of lecturers (through organisations such the Higher Education Academy (HEA), of which I am a National Teaching Fellow) and the use of advanced technology alongside more traditional teaching methods.

I very much hope that the current worsening of the funding of Higher Education teaching will not lead to the restriction, or worse the end, these important support centres.

5. An explosion in mathematical outreach.

This is a subject very close to my heart, and a major reason why I am the Gresham Professor of Geometry. A few years ago, there was little in the way of mathematical outreach and of people prepared to popularise mathematics. Chief amongst these were such stalwarts as Christopher Zeeman and Ian Stewart (both former Gresham Professors of Geometry). Both excellent but very much in the minority. In 1978 Prof Sir Christopher Zeeman delivered the first Royal Institution Christmas Lectures on mathematics. Despite misgivings from the TV producers who thought that he could not do a mathematical proof during the lectures, they proved to be a huge success. As a follow up the Royal Institution introduced the mathematics master class programme. This programme is still going strong and on any one Saturday there will be at least fifty different maths master classes for 100 or so young people at KS3 going on around the UK. It is very exciting to report that since 1978 the UK has seen an explosion of activity in mathematics outreach. Organisations such as Maths Inspiration, the Millennium Maths Project, the learned societies and Numberphile, are all active in putting on high quality mathematics shows or in producing internet based resources, including YouTube videos. There is also an increasing maths presence (but still not enough) in big science festivals. Particularly encouraging are the healthy numbers of people now engaged in promoting maths, including some excellent (but still not enough) TV programmes on mathematics. A number of networks act to support and sustain this activity such as the MathsJam network and the biennial Talking Maths in Public conferences.

I am very encouraged by this, and although there is some way to go before maths has the prominence in the media that it deserves, and promoting maths is adequately recognised and rewarded as a legitimate activity for professional mathematicians, I am very hopeful for the future of maths outreach.

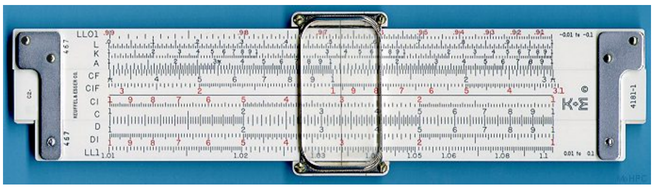
The Role Of Computers And Technology In Maths Teaching.

1. Overview

One of the key areas of growth in mathematics education in recent years has been in the use of ICT and computer-based learning. This is an area in which the UK is world leading thanks in no small part to the pioneering work of Prof Dame Celia Hoyles at the Institute of Education (IoE) [4], The following is a quote by Tony Jones from Australia [1].

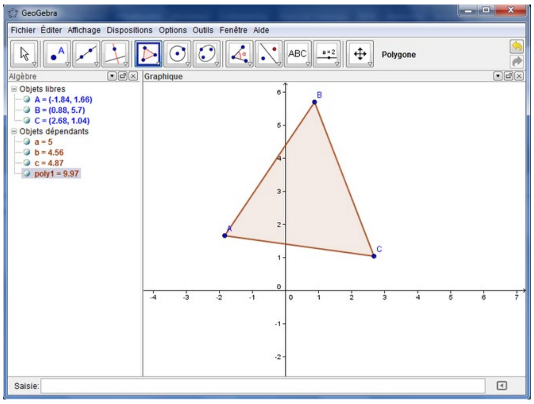
> *'Every country watches carefully to see what other countries are doing in mathematics. The UK has had a reputation for creativity, not least in the use of IT. Many teachers have made large strides in creating exciting and interactive ways to bring mathematics concepts to life using this technology. I see a great opportunity for this key strength to be included in the UK's vision and planning.'*

In my own lifetime I have seen a revolution in the way that mathematics is taught using IT, with both pros and cons in this approach. Computers can be used in many different ways. An obvious example is the use of calculators to do arithmetic and also scientific calculations. I did my O-levels using a slide rule but by the time I did my A-levels I had bought a Hewlett-Packard RPN calculator. This was a great advance, allowing me to be much more creative in my mathematics. Do not believe anyone (including Ministers of State) who regret the passing of doing calculations such as 3123.7834/432.897345 by hand.



However, whilst the use of calculators is (in my opinion) a massively positive advance, it does have the problem that it reduces the need (as we had to do with slide rules) to estimate the answer before the calculation itself. This is still a very useful skill, both for doing routine calculations such as adding up a shopping bill, and for checking the result of the electronic calculation.

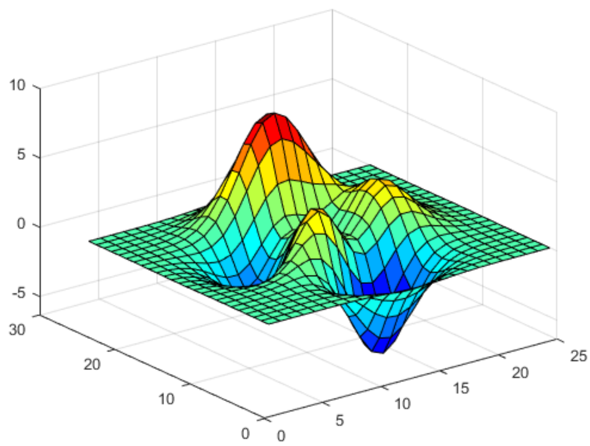
Since the introduction of calculators, we have seen a huge increase in the digital technology available to teach mathematics. This can take various forms. One, is the use of dedicated mathematics software to demonstrate and illuminate a particular area of maths such as trigonometry, or geometry. These allow students to then make creative explorations of these subjects. A good example of this is the GeoGebra package illustrated below.



Similar packages exist to help teach arithmetic, statistics, mechanics and many other areas of mathematics. They are also supplemented with a vast collection of high quality and readily available resources on the Internet on a huge variety of mathematical topics. All of these make it much easier for students to be creative in the way that they explore mathematics and are a very useful addition to traditional teaching methods.

Closely related to this has been the development of 'robots', which can be programmed to do specific tasks and can be used to teach concepts in arithmetic, geometry, trigonometry and mechanics, for example. An early but very influential example of these were the Turtle Robots which were linked to the work of Seymour Papert in the 1970s, and the use of the *Logo* programming language. Now, many school children are learning practical maths through the Lego Robotics challenge.

A third growth area has been in the field of dedicated software languages for general mathematical calculations. When I was a student all that was available to us were languages such as FORTRAN, BASIC, PASCAL and (for the lucky few) APL. Whilst these were very effective as general tools (and FORTRAN is still very much in use today), they were hard to use to do general mathematics and had hard to use graphics. This situation has been transformed with the introduction of programming languages such as Maple, Mathematica, Autograph, Matlab, NetLogo, Scratch, Python, R, and JuPyTr. These languages allow mathematical operations such as inverting matrices, optimisation, solving differential equations, statistical analysis and vector calculus, to be performed with relative ease. They also have powerful and easy to use graphics, which can make movies of three-dimensional surfaces such at that illustrated below (using Matlab). This allows a highly creative opportunity to explore the way that functions behave.



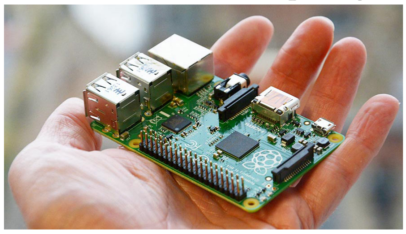
A further feature of most of these languages is their ability to do symbolic calculations, such as the algebraic evaluation of integrals.

The use of these languages has transformed the teaching of mathematics at university level and most mathematics courses make extensive use of them. They are also hugely important in knowledge transfer, allowing mathematicians to work closely with other disciplines to transfer ideas from one field to another and to visualise the results. They can also be used in computer aided assessment of mathematics [14], which allows students to get much quicker feedback on their work. I hope that schools will make much more use of them.

2. *Teaching of programming*

When I was at school in the 1970s, we were taught programming (in BASIC) as part of my mathematics lessons, and (with many thanks to the great support of the then Hatfield Polytechnic) I was able to run computer programs to solve mathematical (and non-mathematical) problems on an IBM PDP 10 mainframe computer. Later in the 1980s the introduction of the BBC Microcomputer allowed all schools to teach programming to their students. Sadly, with the introduction of more sophisticated computers the teaching of programming (as opposed to the teaching of IT and the use of such tools as ExCel and PowerPoint) declined sharply. This was partly because the packages in the computers became more sophisticated and easier to use, and secondly because the computers themselves were getting too complex to program easily. Overall this was a poor state of affairs as it meant that we were not equipping a generation of school children to be able to program computers and to thus write the software of the future. It led in 2011 to Eric Schmidt, the CEO of Google to say in the MacTaggart lecture 'I was flabbergasted to learn that today computer science isn't even taught as standard in UK schools. Your IT curriculum focuses on teaching how to use software, but gives no insight into how it's made'.

I am pleased to say that this has now changed due to three significant developments. Firstly, the Computing At Schools (CAS) initiative founded by Simon Peyton-Jones FRS has led to computer programming now being taught at schools from a primary school level. Secondly, free languages such as Python in particular, allow school students access to powerful programming tools. Thirdly, very cheap computers such as the Raspberry Pi (below) give a perfect platform to run these programs on.



I am now very excited about the future of teaching programming, and the impact that this will have on the future of mathematics education.

3. Is technology good or bad for teaching mathematics?

As a computational mathematician in my 'day job' (both in my research and in my teaching) I would strongly argue that the answer to this question is yes! The development of computers is having the same effect on mathematics as the invention of the printing press had for literature. In both cases they allow for much greater impact, communication and creativity in their respective subjects. And I have never heard anyone say that the printing press has been bad for literature. Computers allow students to do independent creative work in mathematics that would otherwise not be possible. They are not a substitute for mathematical thinking. They only complement it and make it more fun!

The Future Of Maths Education

This is a talk about the future of mathematics education. Whilst we might think that this future is to do mainly with the technology I have described in the above section, we would be wrong. Technology is vital, but the future of mathematics teaching lies primarily in the hands of mathematics teachers.

It is vital that we support existing teachers, and encourage future teachers, if mathematics education is to have to any future at all. However, the current statistics are not very encouraging. It is forecast that pupil numbers are likely to rise by 19% by 2026 and this puts significant pressure on teacher recruitment and retention. However according to the NFER report [15] the proportion of teachers leaving the profession or moving school has increased across all subjects since 2010. Indeed, just 60% of teachers continue to teach in state-funded schools five years after qualifying. For maths and physics, where the sharpest shortages are, this drops to 50%. Due to a shortage of teachers, 24% of all children in secondary schools are not taught by specialist maths teachers, and this increases for the lower years. Therefore, as many as half of Key Stage 3 children in lower sets can be taught by non-specialist maths teachers. According to the Nuffield Report [16] this is particularly bad for schools in disadvantaged areas. A tragedy of this is that mathematics ability has no class, race or gender divides, and many great mathematicians (for example Gauss, the greatest of them all) have come from such a background. It is therefore no wonder that 90% of those children who have failed to reach the target in the SAT at age 11, then fail their GCSE and leave school functionally innumerate. As I have argued above, creative and innovative teaching is especially needed now, and in the future, both for these children and the future of the UK. All subjects are best taught by teachers who know their subject well, are excited by it, and are supported in teaching it with relevant, up-to-date continuous professional development. In the case of both mathematics and science, research by ACME has demonstrated that "specialist teachers play a vital role in inspiring young people in the joy and wonder of science and mathematics, enabling them to achieve their full potential and encouraging further study. Only by prioritising study of these subjects can we ensure that there will be enough suitably qualified individuals to meet future demand for specialist teachers." Whilst I strongly approve of the introduction of Core Maths, we must also realise that if we are going to teach this and to teach A-level maths at the same level as we are doing at present, then we must have many more dedicated and skilled mathematics teachers.

So, how is this to be achieved? I can see two important issues to address. One is in the training of maths teachers and the other is the recruitment in the first place. In the case of initial teacher training, we are seeing significant improvements through the action of the National Centre for Excellence in Teaching Mathematics (NCETM) as well as the bodies I described earlier. However much more needs to be done.

The recruitment problem is possibly even more challenging, but there are some encouraging signs. The Teach First programme encourages undergraduates to go straight into very challenging teaching environments by offering direct on the job training. The Institute for Mathematics and its Applications (IMA) encourages high achieving undergraduates to go into teaching by offering them bursaries.

A further approach is to motivate students to go into teaching by exposing them to teaching related activities whilst they are still undergraduates, and to give them credit for doing so. One very successful way to do this is the Undergraduate Ambassadors Scheme (UAS) launched by Simon Singh. In this, carefully trained students spend time observing and teaching at schools local to their university. This scheme has been taken up by a number of universities. In the alternative scheme which I run at Bath for final year students, called Communicating Maths, a group of about 30 students are again trained to undertake a number of outreach activities including taking an RI mathematics master class, running an exhibition in a science festival (see below) and going into a school. Over the years I have run this course over half of the students who take it have gone on to do PGCEs in mathematics.

Truly I see that encouraging our very brightest undergraduates to go into teaching is the future of mathematics education.

8. Conclusion

I started this lecture on a down beat note, but I hope that I have finished with a much more positive vision about how maths will be taught in the future. Indeed, I see a bright future where a combination of technology and creative teaching will open windows of mathematical opportunity to a new generation who can really appreciate the relevance of maths to their lives. Provided, of course, that we can recruit enough teachers in the future!

Answer to the puzzle: *7.5 m. The ratio of height to shadow length is 1:3. The shadow will therefore be 3*(1+1.5) = 7.5m*

If you want another challenge: *Work out the maximum and minimum number of Friday the 13ths. that there can be in a regular calendar year.*

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