# What have mathematicians done for us?

Google™

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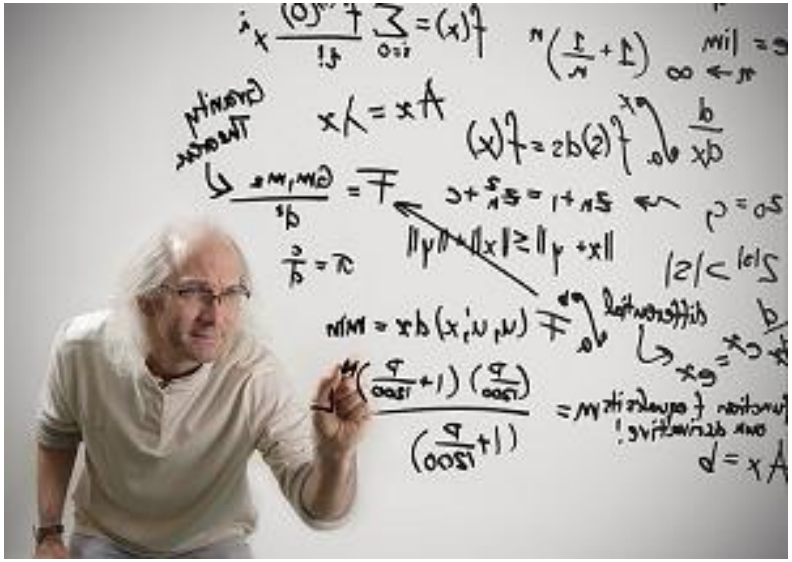
# Some common views on maths and mathematicians

Mathematics is completely useless

Mathematicians are evil souless geeks

All Mathematicians are mad!

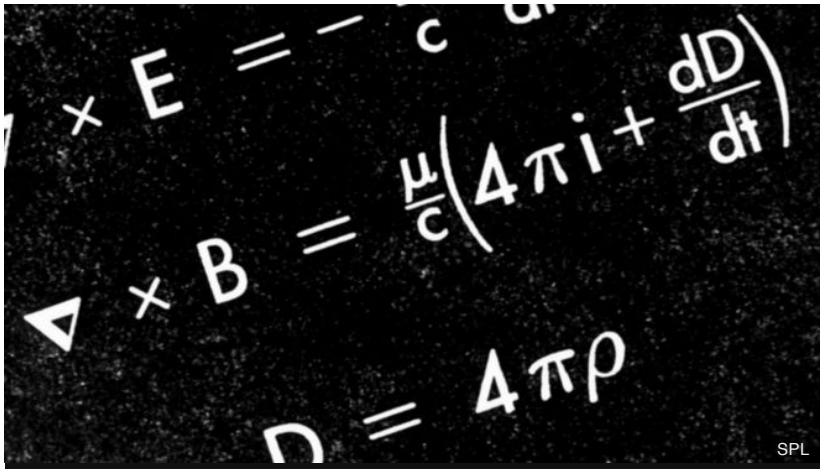


This can cause serious problems



# Flight delayed after passenger becomes suspicious of equation

8 May 2016 | US & Canada Share




We can't fly on together with suspicious maths: the flight was delayed for two hours

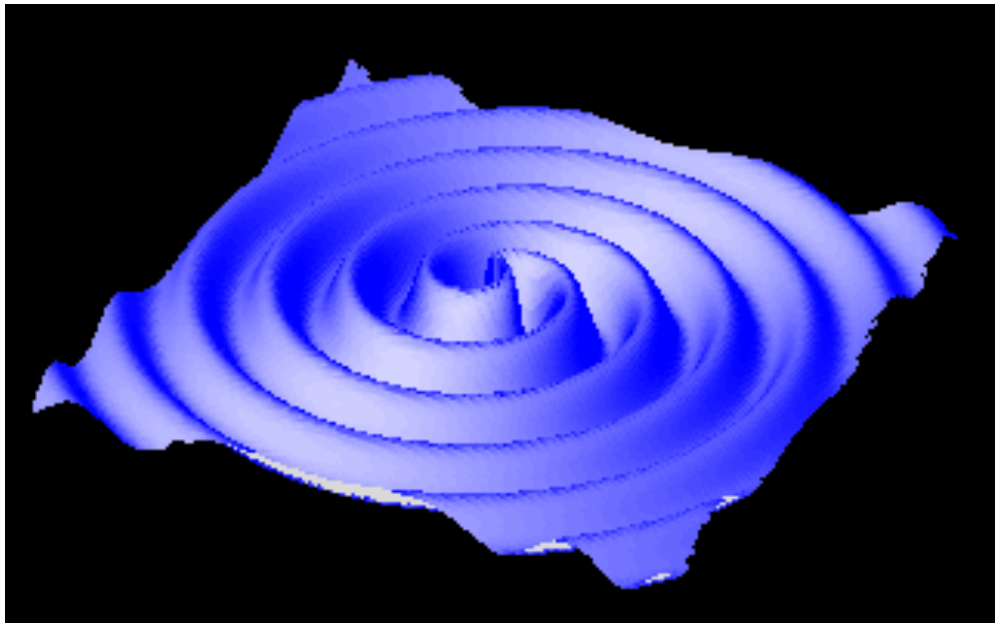
**An Italian economist says his flight was delayed after a fellow passenger saw him working on a differential equation and alerted the cabin crew.**

Guido Menzio was taken off and questioned by agents who did not identify themselves, after the woman next to him said she felt ill.

And it is completely false

The modern world would not exist without maths

Maths lies at the heart of all modern technology



# Indeed ….

Much of industry has problems which can potentially be formulated, and solved using mathematics

Maths connects with all areas and knows no bounds or constraints!

*Too few people recognize that the high technology so celebrated today is essentially a mathematical technology*

Edward David, ex-president of Exxon R&D

## Some possible reasons for this

### 1. Mathematics is quite hard to define

*Today, no consensus on the definition of mathematics prevails, even among professionals. There is not even consensus on whether mathematics is an art or a science. A great many professional mathematicians take no interest in a definition of mathematics, or consider it undefinable. Some just say, "Mathematics is what mathematicians do."* [Wikipedia]

My own 'definition' is that maths is …

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

2. It’s role is as important as the air around us, but like the air it is often invisible

3. Most people don’t seem to realise that mathematicians are real people too

*‘I don’t do maths, as I’m a people person’*

Spot the mathematician, and why are they important?

Maxwell and the discovery of electromagnetic waves

$$\nabla \times E = -\frac{\partial B}{\partial t} - M, \quad \nabla \times H = -\frac{\partial D}{\partial t} + J,$$

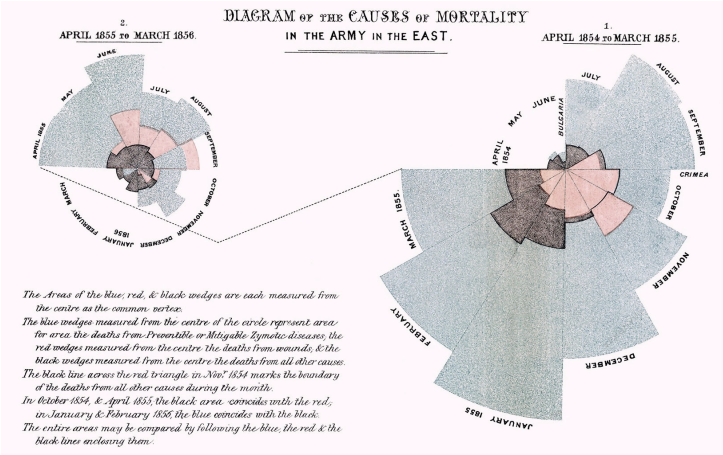
$$\nabla . D = \rho, \qquad \nabla . B = 0.$$

Electromagnetism, radio, WiFi,TV, radar, mobile phones, microwaves all come from the work of Maxwell!

# The most famous ever female mathematician?

Florence Nightingale

Medical statistics




Mathematicians really have made the modern world possible

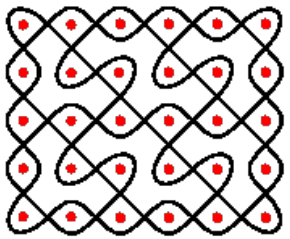
## So … where do good problems come from?

Traditional industrial users of maths are

Telecommunications, aerospace, power generation, iron and steel, mining, oil, weather forecasting, security, finance

But they could equally well be …

Retail, food, zoos, sport, entertainment, graphic design, media, forensic service, hospitals, air-sea-rescue, education, transport, risk, health, biomedical, environmental agencies, art, …



All lead to great and diverse problems. Many/most of which can be tackled or illuminated using math

Good maths (to help solve)


Hard applied problems

(Leads to) More good maths

(Which can help to solve) More hard applied problems

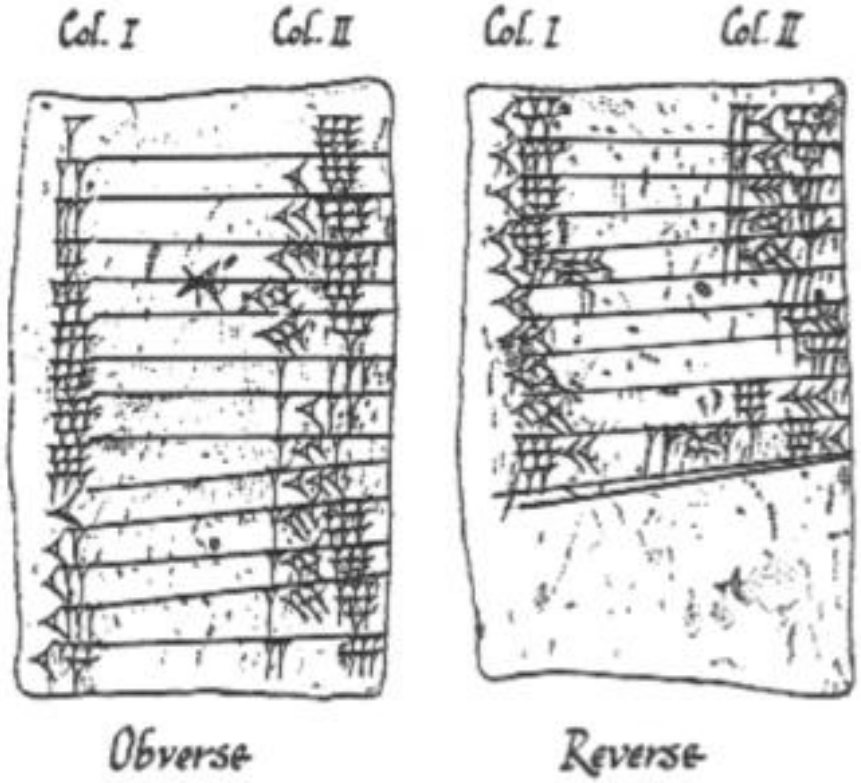
Which can often seem to have no relation at all to the original ones!

So .. Let's see how this process works throughout history

1. Early maths, and the quadratic equation
2. Recreational maths
3. Maths tells us where we are
4. Maths makes waves
5. Maths saves lives
6. Maths helps us communicate
7. Where next?

# 1. Early maths and the tax man

Early people counted on their fingers. This led to the natural numbers 1,2,3,4,5 ….




Numbers recorded on Babylonian cuneiform tablets

Later on numbers were expanded to include zero, negative numbers (debt) and fractions

1, 0 , -1, -2 , 3/5, 7/8, 9/7, …

Problems posed such as: I have 7 cows, the taxman takes 5, how many do I have now.

Called DISCRETE. Still very important in digital electronics

Later on numbers extended to real numbers to allow solutions of other equations such as the quadratic equation


$$x^2 = 2$$

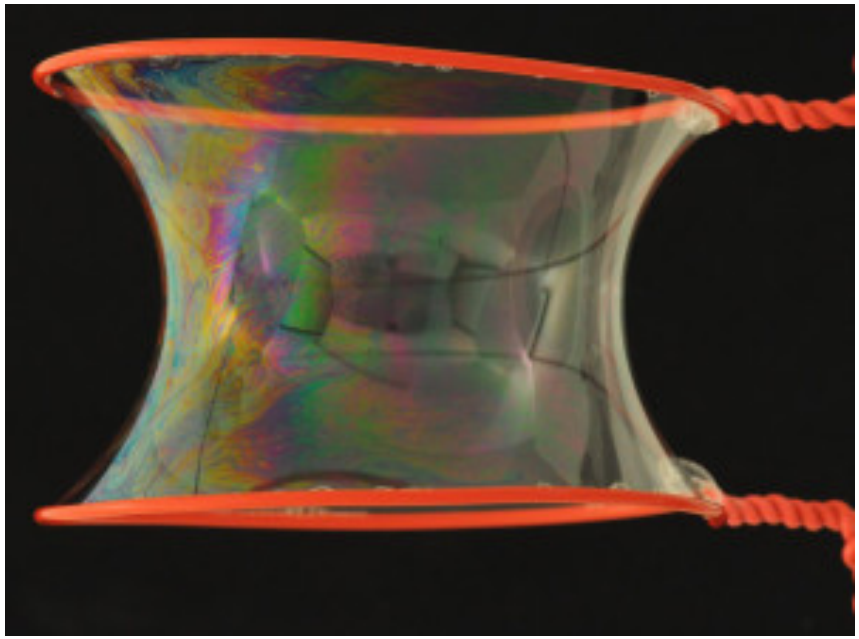
Early calculation of the square root of 2 = 1.41421356….

Important to the taxman to work out the area of fields!

Problems posed in real numbers are called continuous.
Most problems in nature are like this

Tackled using calculus ( around 1690)

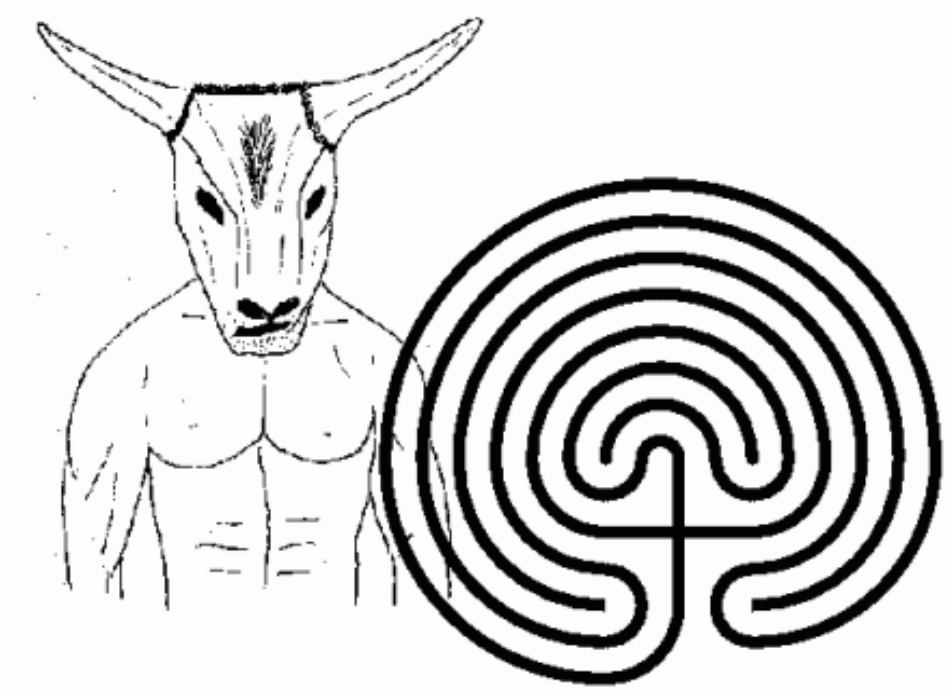
Best tool that we have for understanding the patterns that we see in the real world



## 2. Recreations, music and what they led to

Many people's first meeting with mathematics is through recreational puzzles. Solving puzzles leads to good maths!

An early example of this is the Labyrinth in the story of the minotaur

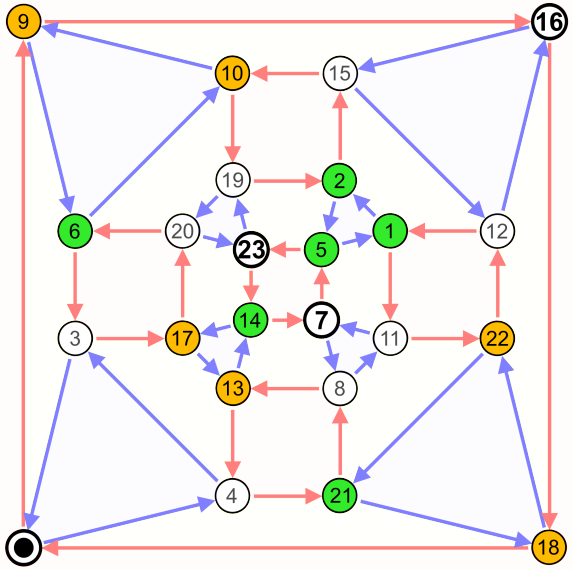


Later became the Puzzle maze eg. Hampton Court

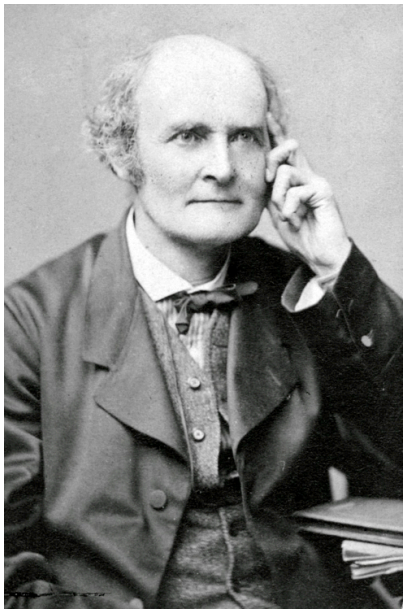
Solved by Euler, who developed the theory of networks to do it

Networks now vital in communications, genetics, medicine, ..

Understanding networks, combined with Matrix Theory (due to Cayley)

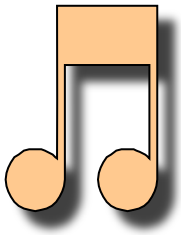



$$\begin{pmatrix} 1 & 4 & 7 \\ 5 & 6 & 9 \\ 4 & 0 & 1 \end{pmatrix}$$



also forms a major part of the algorithms behind

Google™



# Mathematicians are also good for music!

Some musical notes sound better when played together than others

The octave C to C

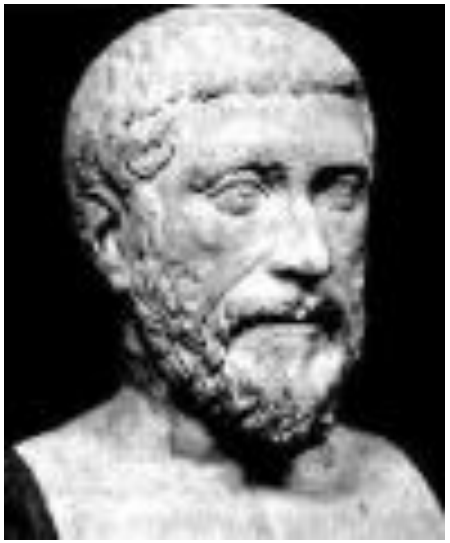
The notes C and G

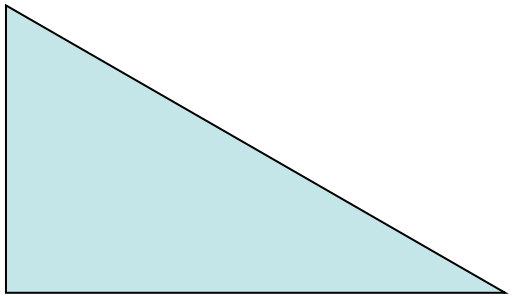
(a perfect 5th)

The notes C and E

(a perfect 3rd)

Reason was discovered by Pythagoras



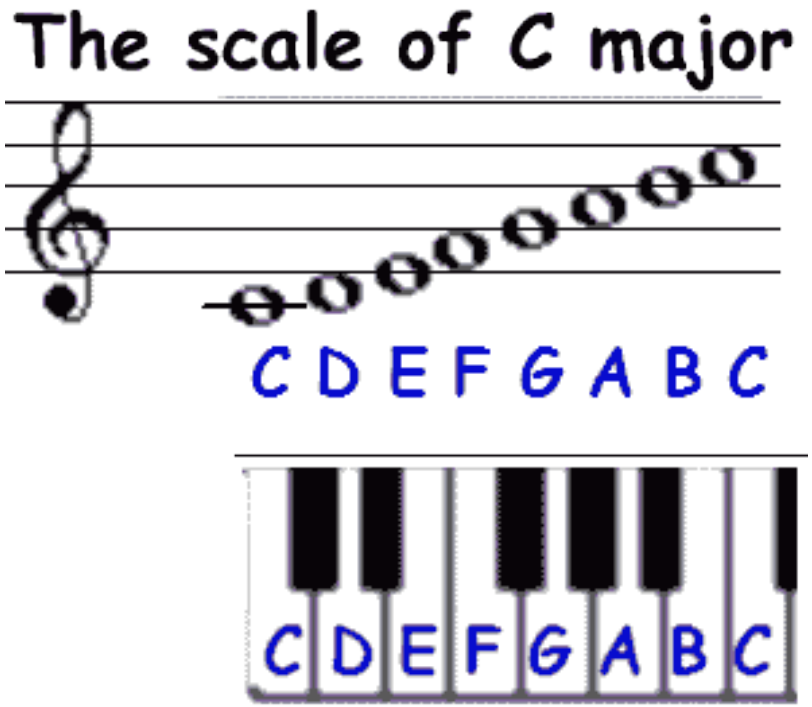


Length of strings giving C and G, and C and E,
were in simple fractional proportions

C:C … 2/1 C:G … 3/2 C:E … 4/3

Pythagoras invented the Just Scale .. Sequence of notes with frequencies in simple fractional proportions

1 : 9/8 : 5/4 : 4/3 : 3/2 : 5/3 : 15/8 : 2



Problem: Keyboard instruments could only be tuned for one key

Mathematicians invented a new Well Tempered scale with all notes in the same proportion

a geometric progression of the semi-tone frequencies, ratio

$$1.059.. = 2^{1/12}$$

which works well in all keys

Mathematics also plays an important role in the study of

Games:

Prisoner's dilemma

Mornington Crescent

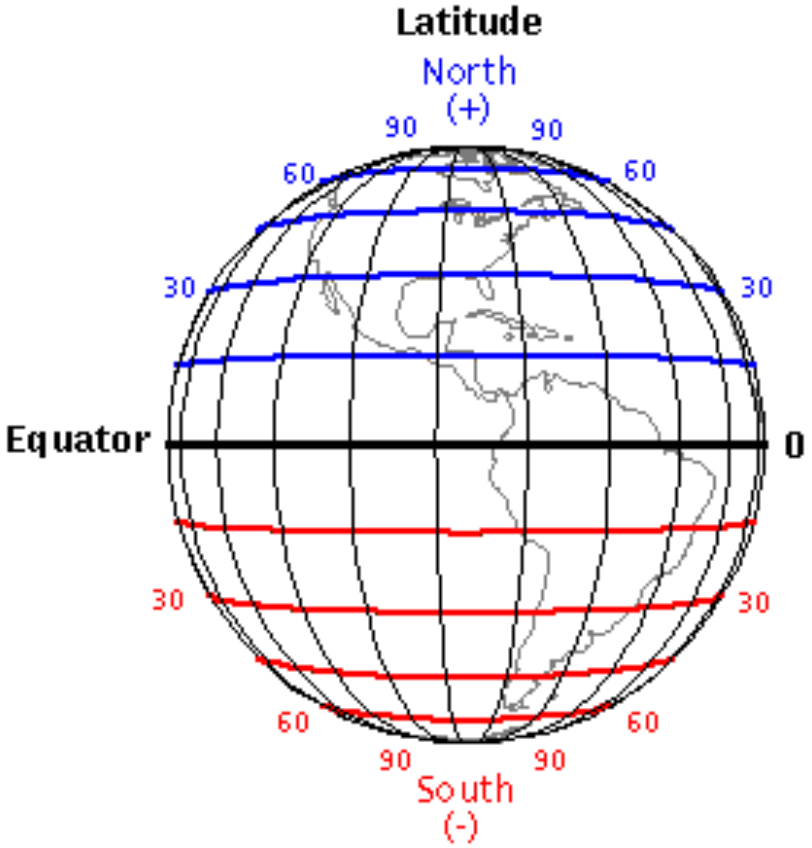
John Nash

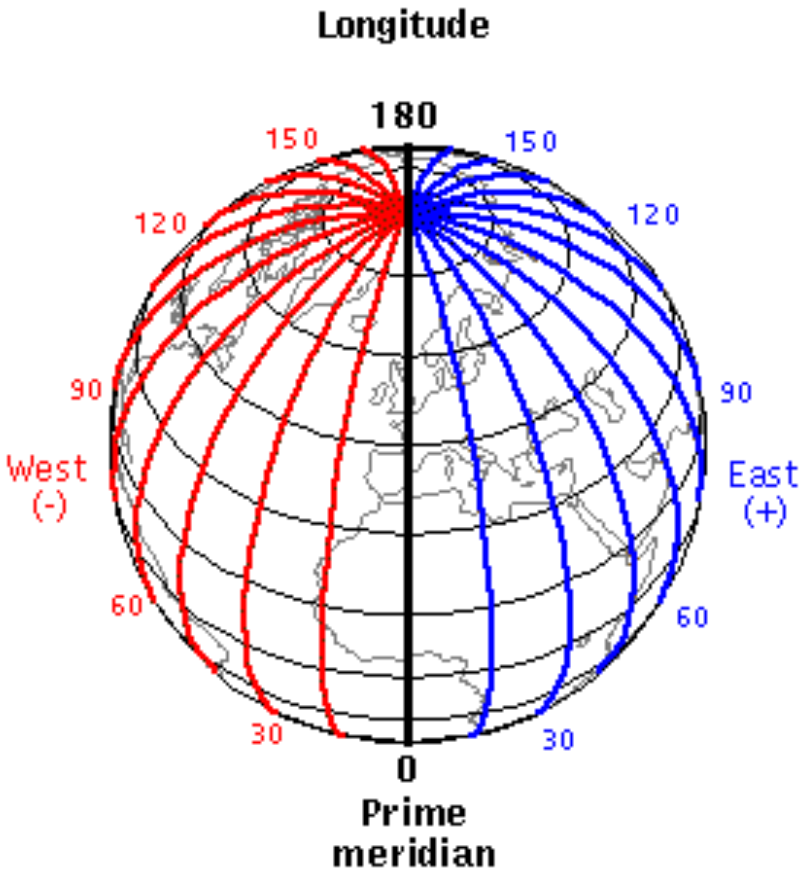
John von Neumann


Game theory developed to solve them is now used widely in auctions and even biology

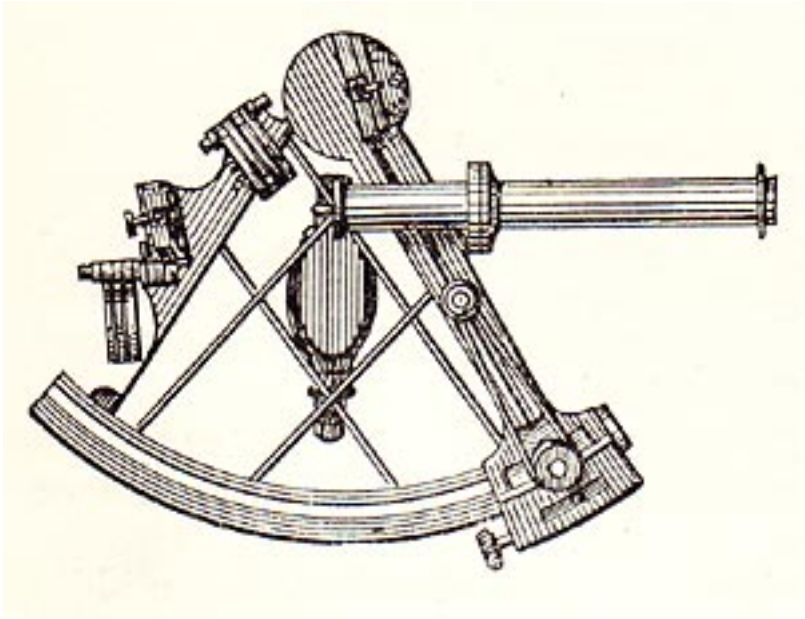
## 3. Maths tells us where we are.

Big problem of the 18th Century was the accurate determination of Longitude.



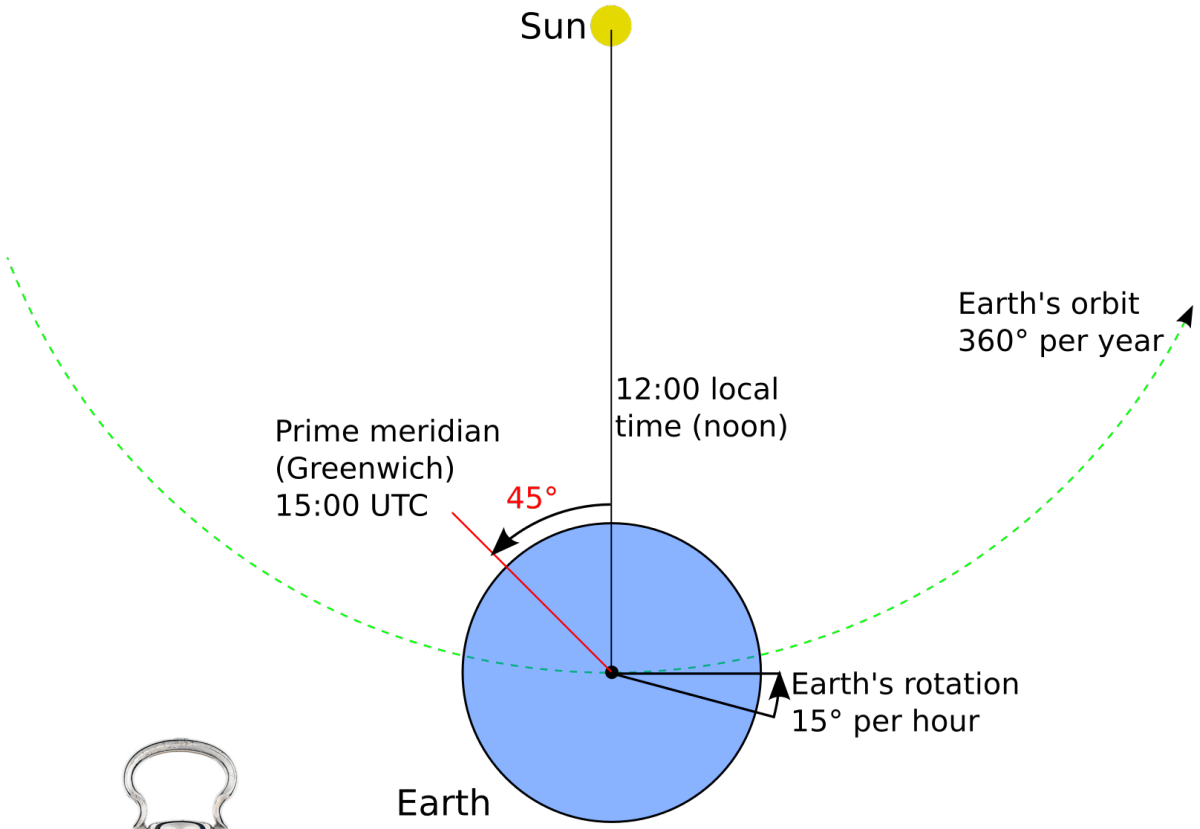



Cloudesly Shovell … many deaths



Nevil Maskeleyne & Newton

lunar method .. Worked but very heavy duty calculations



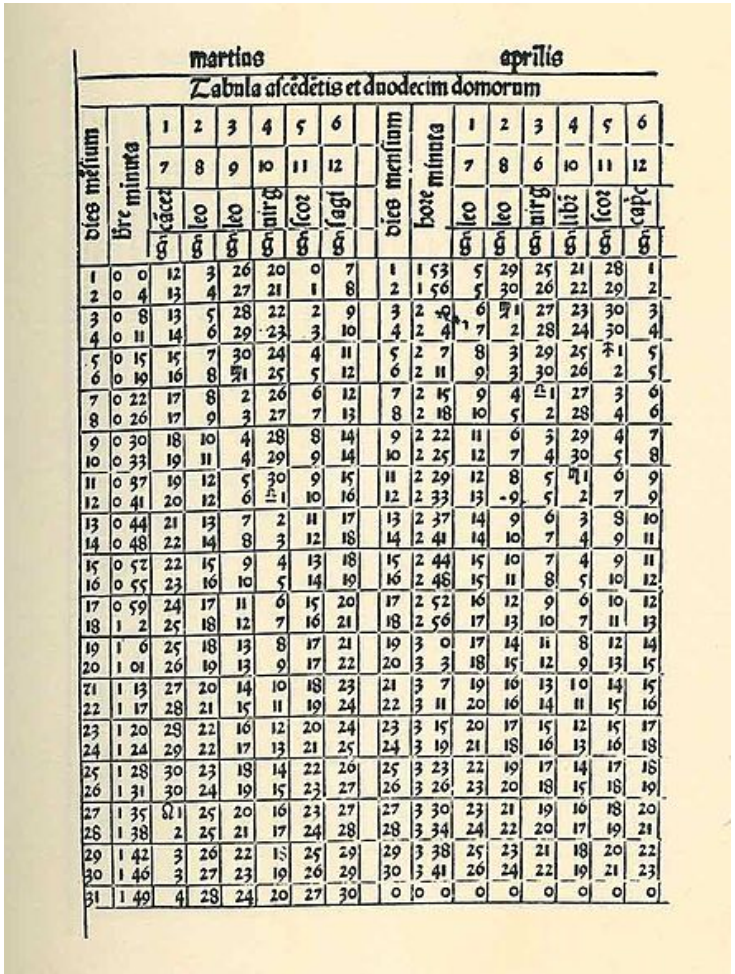

Longitude linked to time

H4 Chronometer

invented by Harrison: Link of mathematics with technology

Ephemerides calculated to give the locations of the celestial bodies.

A major mathematical achievement!



martius — aprilis

Tabula ascēdētis et duodecim domorum

| dies mēsium | hre minuta | 1 | 2 | 3 | 4 | 5 | 6 | dies mensium | hore minuta | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 7 | 8 | 9 | 10 | 11 | 12 | | | 7 | 8 | 6 | 10 | 11 | 12 |
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| 1 | 0 0 | 12 | 3 | 26 | 20 | 0 | 7 | 1 | 1 53 | 5 | 29 | 25 | 21 | 28 | 1 |
| 2 | 0 4 | 13 | 4 | 27 | 21 | 1 | 8 | 2 | 1 56 | 5 | 30 | 26 | 22 | 29 | 2 |
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| 4 | 0 11 | 14 | 6 | 29 | 23 | 3 | 10 | 4 | 2 4 | 7 | 2 | 28 | 24 | 30 | 4 |
| 5 | 0 15 | 15 | 7 | 30 | 24 | 4 | 11 | 5 | 2 7 | 8 | 3 | 29 | 25 | ♑ 1 | 5 |
| 6 | 0 19 | 16 | 8 | ♎ 1 | 25 | 5 | 12 | 6 | 2 11 | 9 | 3 | 30 | 26 | 2 | 5 |
| 7 | 0 22 | 17 | 8 | 2 | 26 | 6 | 12 | 7 | 2 15 | 9 | 4 | ♏ 1 | 27 | 3 | 6 |
| 8 | 0 26 | 17 | 9 | 3 | 27 | 7 | 13 | 8 | 2 18 | 10 | 5 | 2 | 28 | 4 | 6 |
| 9 | 0 30 | 18 | 10 | 4 | 28 | 8 | 14 | 9 | 2 22 | 11 | 6 | 3 | 29 | 4 | 7 |
| 10 | 0 33 | 19 | 11 | 4 | 29 | 9 | 14 | 10 | 2 25 | 12 | 7 | 4 | 30 | 5 | 8 |
| 11 | 0 37 | 19 | 12 | 5 | ♎ 30 | 9 | 15 | 11 | 2 29 | 12 | 8 | 5 | ♏ 1 | 6 | 9 |
| 12 | 0 41 | 20 | 12 | 6 | ♏ 1 | 10 | 16 | 12 | 2 33 | 13 | 9 | 5 | 2 | 7 | 9 |
| 13 | 0 44 | 21 | 13 | 7 | 2 | 11 | 17 | 13 | 2 37 | 14 | 9 | 6 | 3 | 8 | 10 |
| 14 | 0 48 | 22 | 14 | 8 | 3 | 12 | 18 | 14 | 2 41 | 14 | 10 | 7 | 4 | 9 | 11 |
| 15 | 0 52 | 22 | 15 | 9 | 4 | 13 | 18 | 15 | 2 44 | 15 | 10 | 7 | 4 | 9 | 11 |
| 16 | 0 55 | 23 | 16 | 10 | 5 | 14 | 19 | 16 | 2 48 | 15 | 11 | 8 | 5 | 10 | 12 |
| 17 | 0 59 | 24 | 17 | 11 | 6 | 15 | 20 | 17 | 2 52 | 16 | 12 | 9 | 6 | 10 | 12 |
| 18 | 1 2 | 25 | 18 | 12 | 7 | 16 | 21 | 18 | 2 56 | 17 | 13 | 10 | 7 | 11 | 13 |
| 19 | 1 6 | 25 | 18 | 13 | 8 | 17 | 21 | 19 | 3 0 | 17 | 14 | 11 | 8 | 12 | 14 |
| 20 | 1 01 | 26 | 19 | 13 | 9 | 17 | 22 | 20 | 3 3 | 18 | 15 | 12 | 9 | 13 | 15 |
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| 22 | 1 17 | 28 | 21 | 15 | 11 | 19 | 24 | 22 | 3 11 | 20 | 16 | 14 | 11 | 15 | 16 |
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| 24 | 1 24 | 29 | 22 | 17 | 13 | 21 | 25 | 24 | 3 19 | 21 | 18 | 16 | 13 | 16 | 18 |
| 25 | 1 28 | 30 | 23 | 18 | 14 | 22 | 26 | 25 | 3 23 | 22 | 19 | 17 | 14 | 17 | 18 |
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| 29 | 1 42 | 3 | 26 | 22 | 18 | 25 | 29 | 29 | 3 38 | 25 | 23 | 21 | 18 | 20 | 22 |
| 30 | 1 46 | 3 | 27 | 23 | 19 | 26 | 29 | 30 | 3 41 | 26 | 24 | 22 | 19 | 21 | 23 |
| 31 | 1 49 | 4 | 28 | 24 | 20 | 27 | 30 | | | | | | | | |
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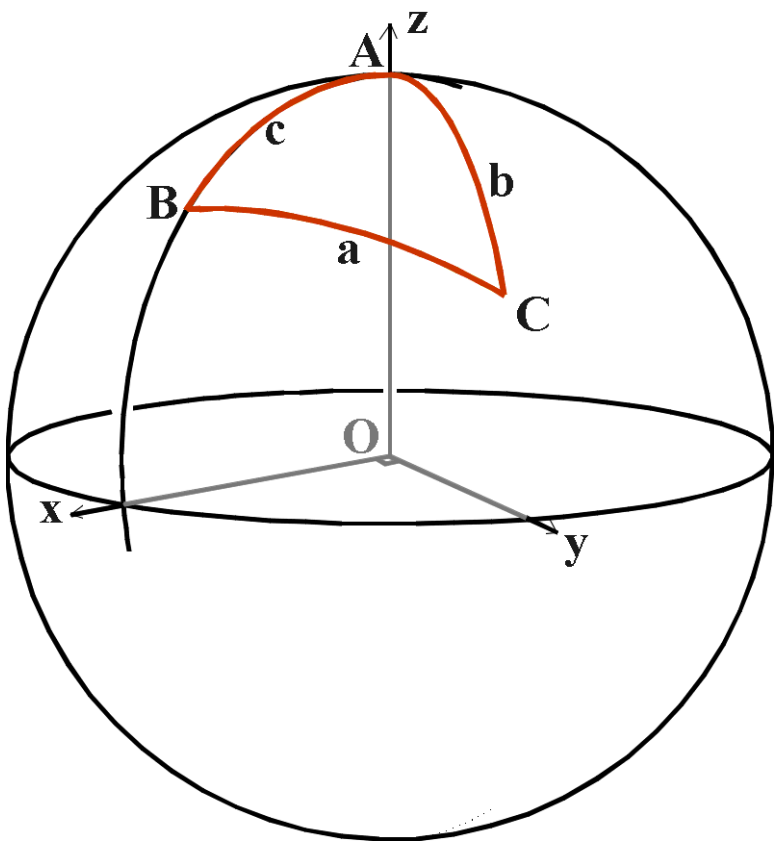
Roots in astrology!

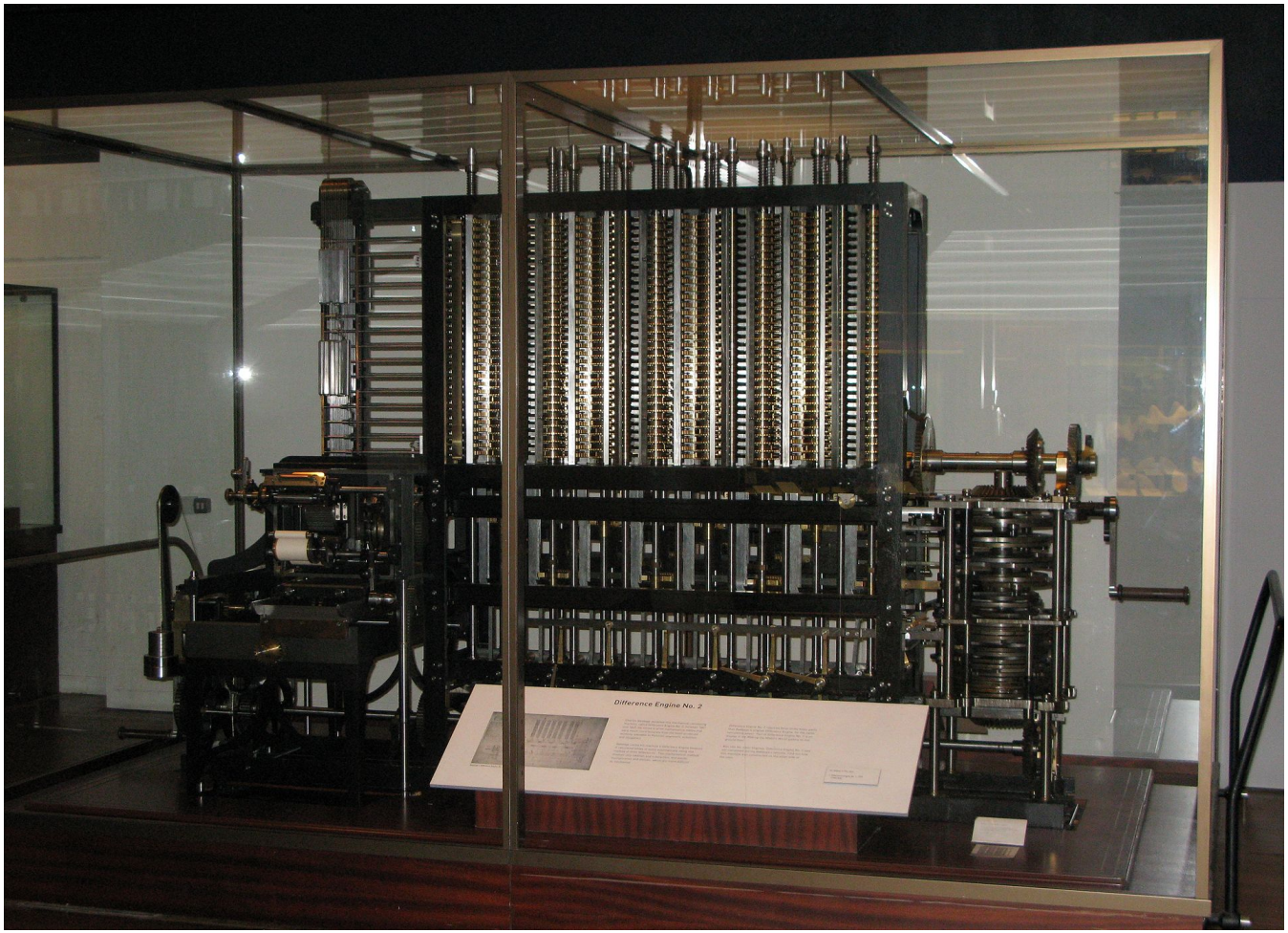
Additional tables were used to solved the spherical triangles encountered in the calculations




*Midshipmen had to 'perform a tedious 22-step mathematical calculation to plot a ship's position'*

But these calculations changed the world

# Babbage's difference engine




Designed to compute the Ephemerides. Ideas behind it led directly to the modern computer

# 4. Maths makes waves

Fourier was studying the
temperature T of a heated bar

Heat
equation

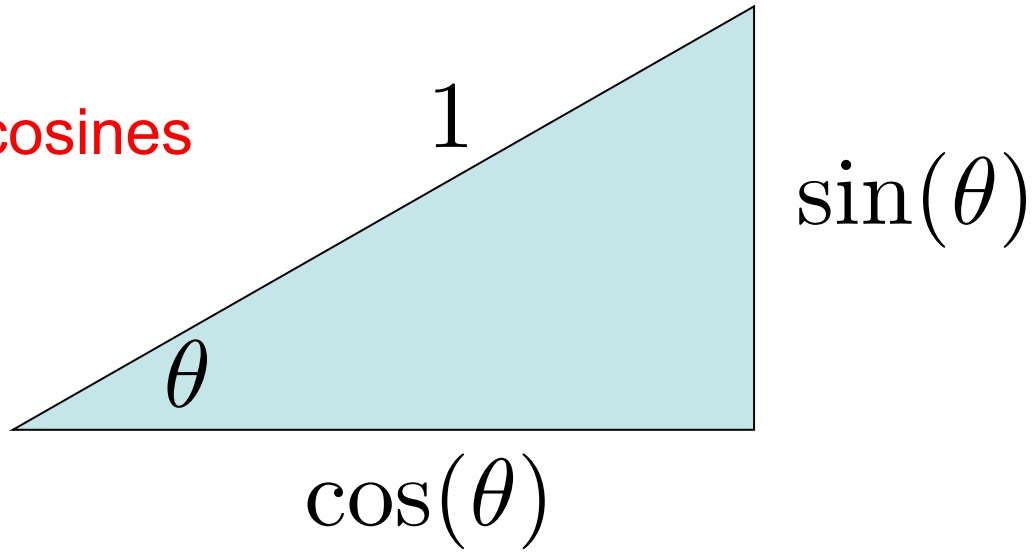
$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

IDEA: Express T in terms of simpler building bricks

Solve a set of simple problems in terms of these

## Astonishing idea

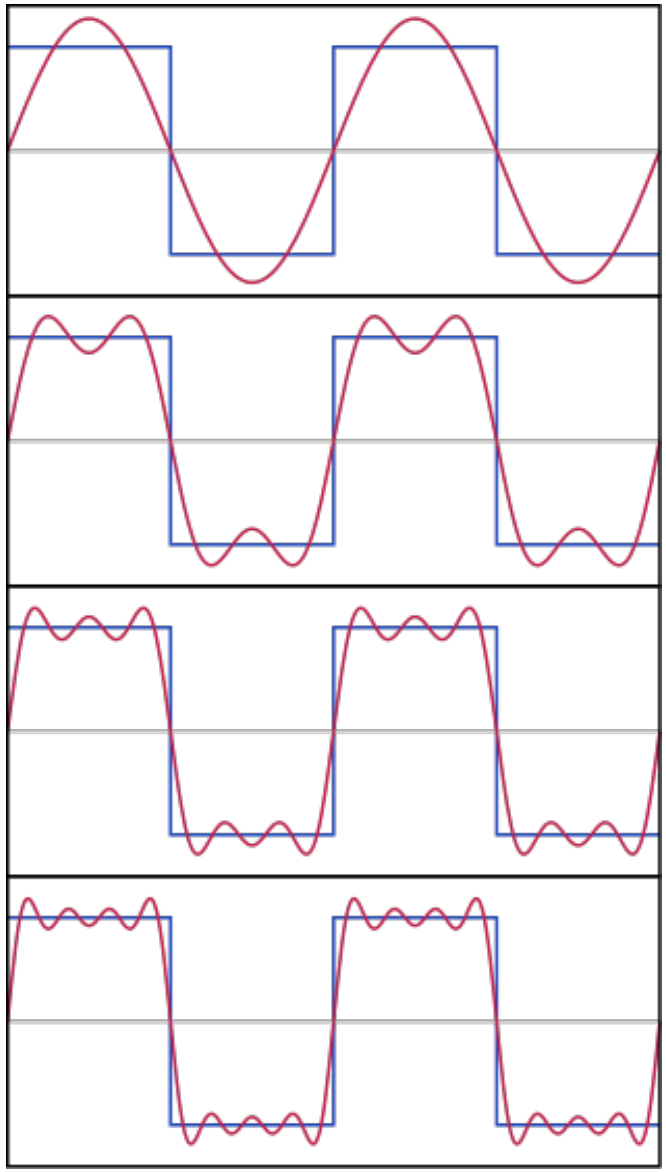
Build T out of sines and cosines




$$T(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n(t)\cos(nx) + b_n(t)\sin(nx)$$

Called a Fourier Series

Now use Fourier series to make up any shape of wave



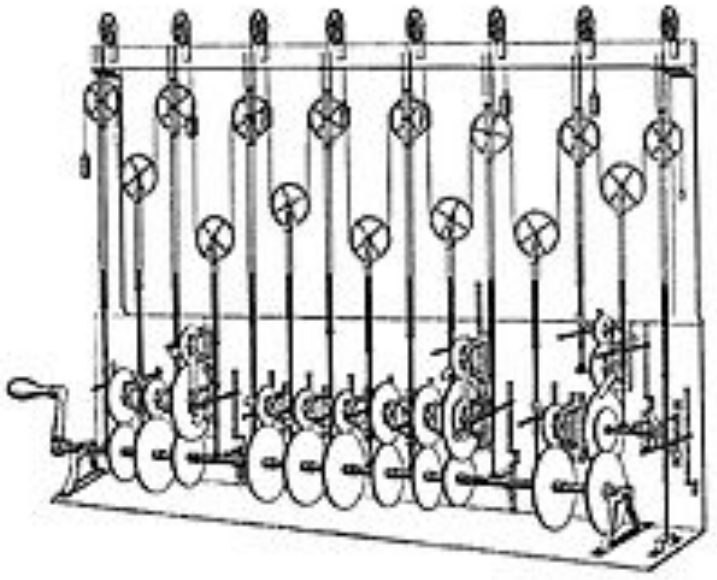
$$\sin(x)$$

$$\sin(x) + \frac{1}{3}\sin(3x)$$

$$\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x)$$

$$\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x)$$

Example: Computing the tides using Kelvin's tidal computer



Used in D-Day

Now the basis of the modern synthesizer and most of modern electronics

# 5. Mathematicians save lives

Radon 1917

Studied shadows cast by objects.

Asked the question of:
whether you can reconstruct the shape just by knowing the shadows?

Shadow

$$R(\rho,\theta) = \int f(\rho\cos(\theta) - s\sin(\theta), \rho\sin(\theta) + s\cos(\theta))\, ds$$

Object

$$f(x,y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{0}^{\pi}\int_{-\infty}^{\infty} e^{ik(x\cos(\theta)+y\sin(\theta)-\rho)} R|k|\, dk d\theta d\rho$$

Modern CAT (Computerised Axial Tomography) scanner implements this and related formulae to look inside you.

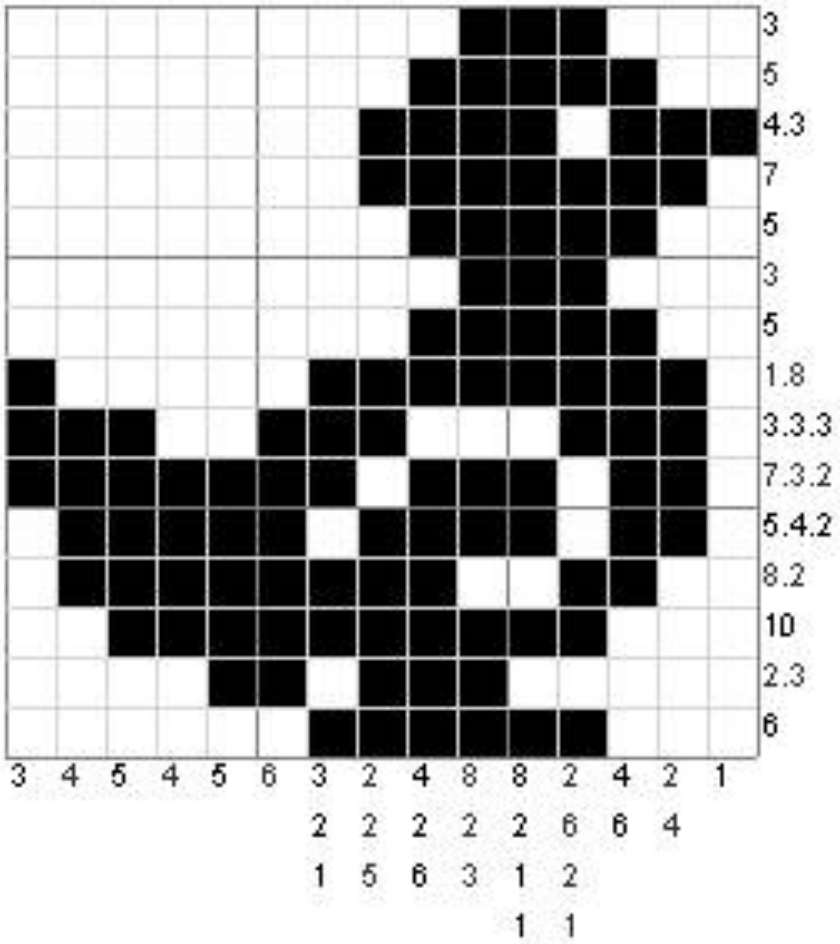
Also used to

X-ray mummies

Detect land mines

Save bees

Solve Griddler




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3 | | 15 | | | 22 | 4 | 16 | 15 |
| 25 | | 17 | | | | | | |
| | | 9 | | | 8 | 20 | | |
| 6 | 14 | | | 17 | | | 17 | |
| | 13 | | 20 | | | | | 12 |
| 27 | | 6 | | | 20 | 6 | | |
| | | | | 10 | | | 14 | |
| | 8 | 16 | | | 15 | | | |
| | | | | 13 | | | 17 | |

If you solve a Griddler puzzle or a Killer Sudoku then you are using the maths of tomography

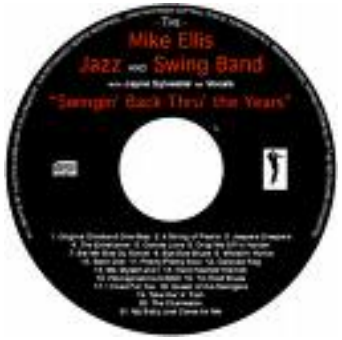
# 6. Maths communicates

# Error correcting codes.

Used to store the numbers 0,1,2,3,4,5,6,7 and other data in such a way that any errors can not only be detected but corrected.

They work by asking extra questions to make the answers as different as possible so we can still tell the right answer even if it has mistakes in it

They are widely used in




- CDs
- Digital TV and Radio
- Mobile phones
- Satellites


Invented in the 1940s by Hamming in the Bell Labs

Using very fancy maths (Galois theory)

And finally ….


This brief overview of what mathematicians have done for us is meant mainly to whet your appetite.

Later on in this series we will see many more applications of mathematics to the modern world, and I will explore both how the latest developments in maths are likely to lead to even newer technologies

And also what new maths I expect we might learn, stimulated by these technologies.