The 18th Century: Crossing Bridges

Robin Wilson
The Open University
and Gresham College

PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

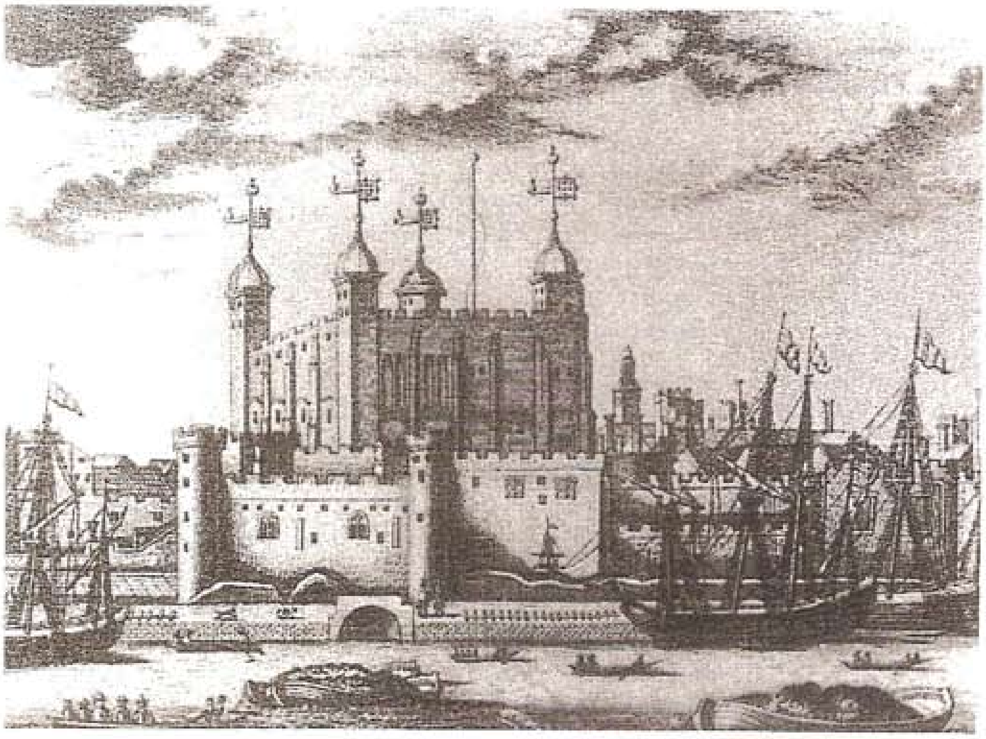
Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

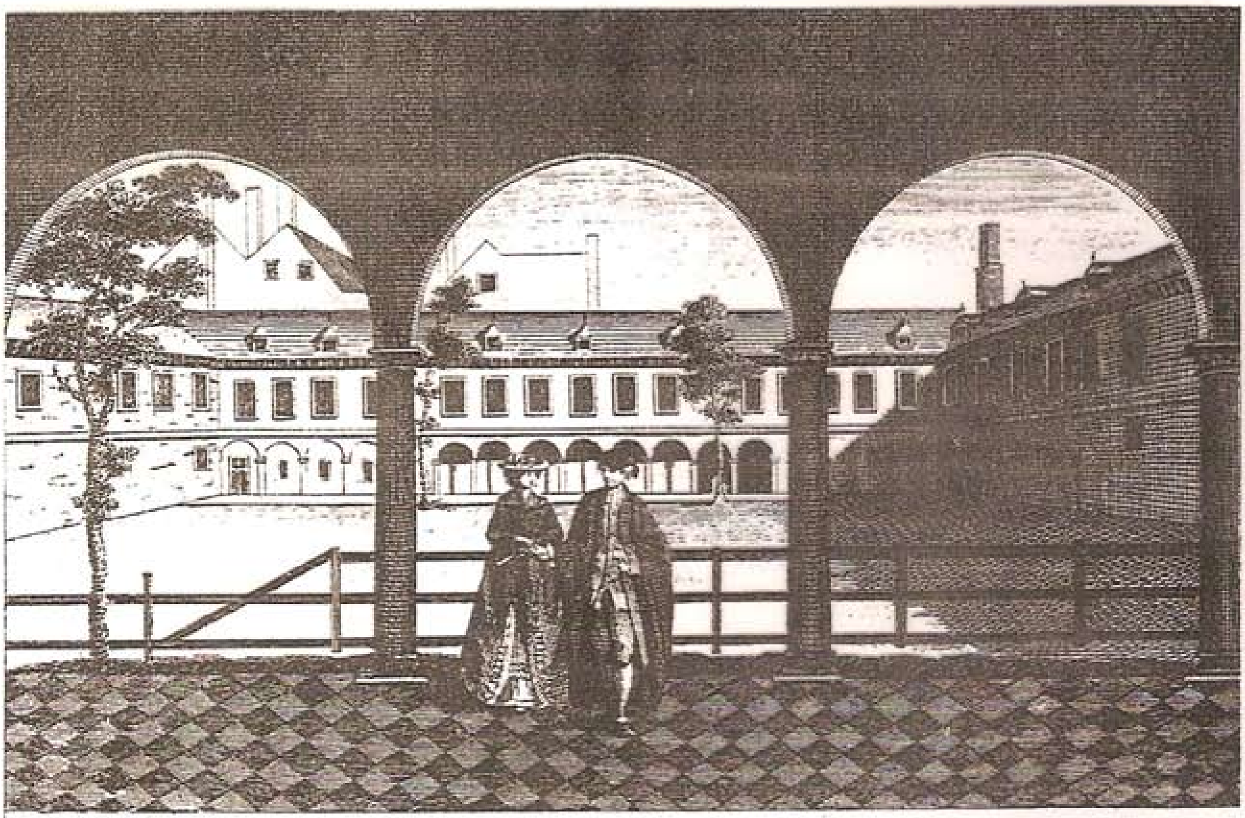
LONDINI,

Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

The Royal Mint

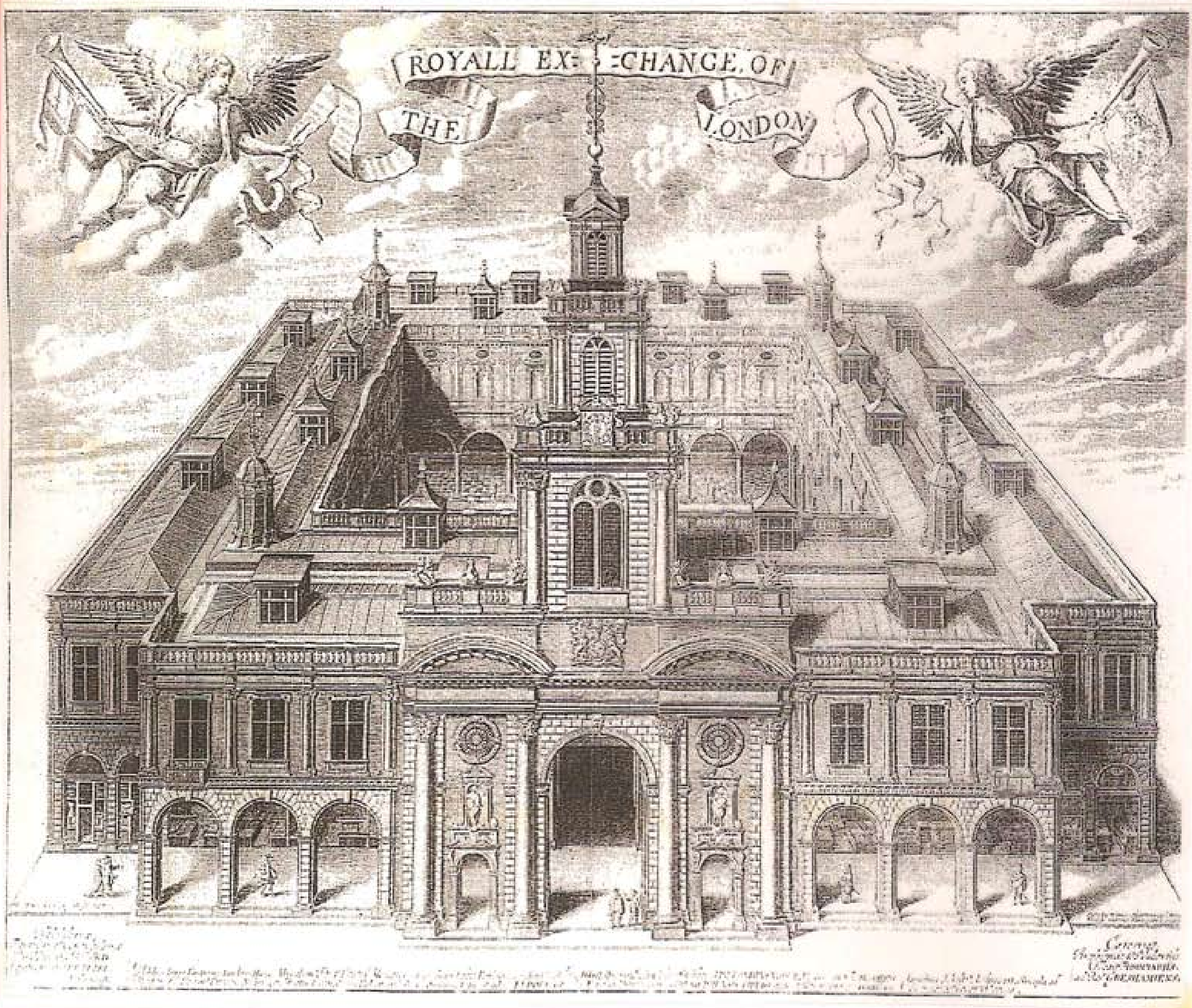


Gresham College in the 18th Century

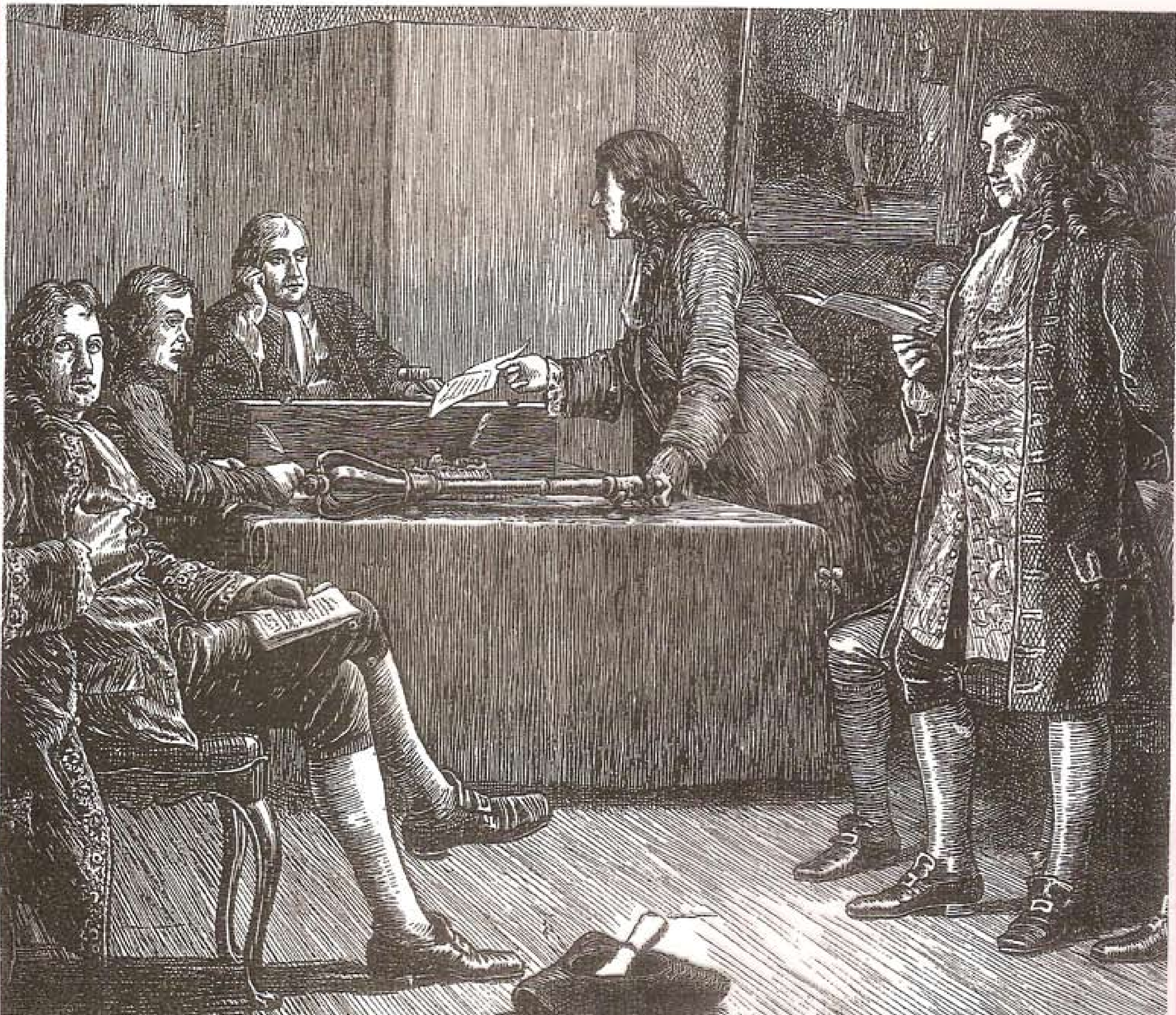


View of GRESHAM COLLEGE *as it appeared before it was taken down*

The Second Royal Exchange



Newton at the Royal Society



OPTICKS:

OR, A

TREATISE

OF THE

REFLEXIONS, REFRACTIONS, INFLEXIONS and COLOURS

OF

LIGHT.

ALSO

Two TREATISES

OF THE

SPECIES and MAGNITUDE

OF

Curvilinear Figures.

LONDON,
Printed for SAM. SMITH, and BENJ. WALFORD, Printers to the Royal Society, at the *Prince's Arms* in St. *Paul's* Church-yard. MDCCIV.

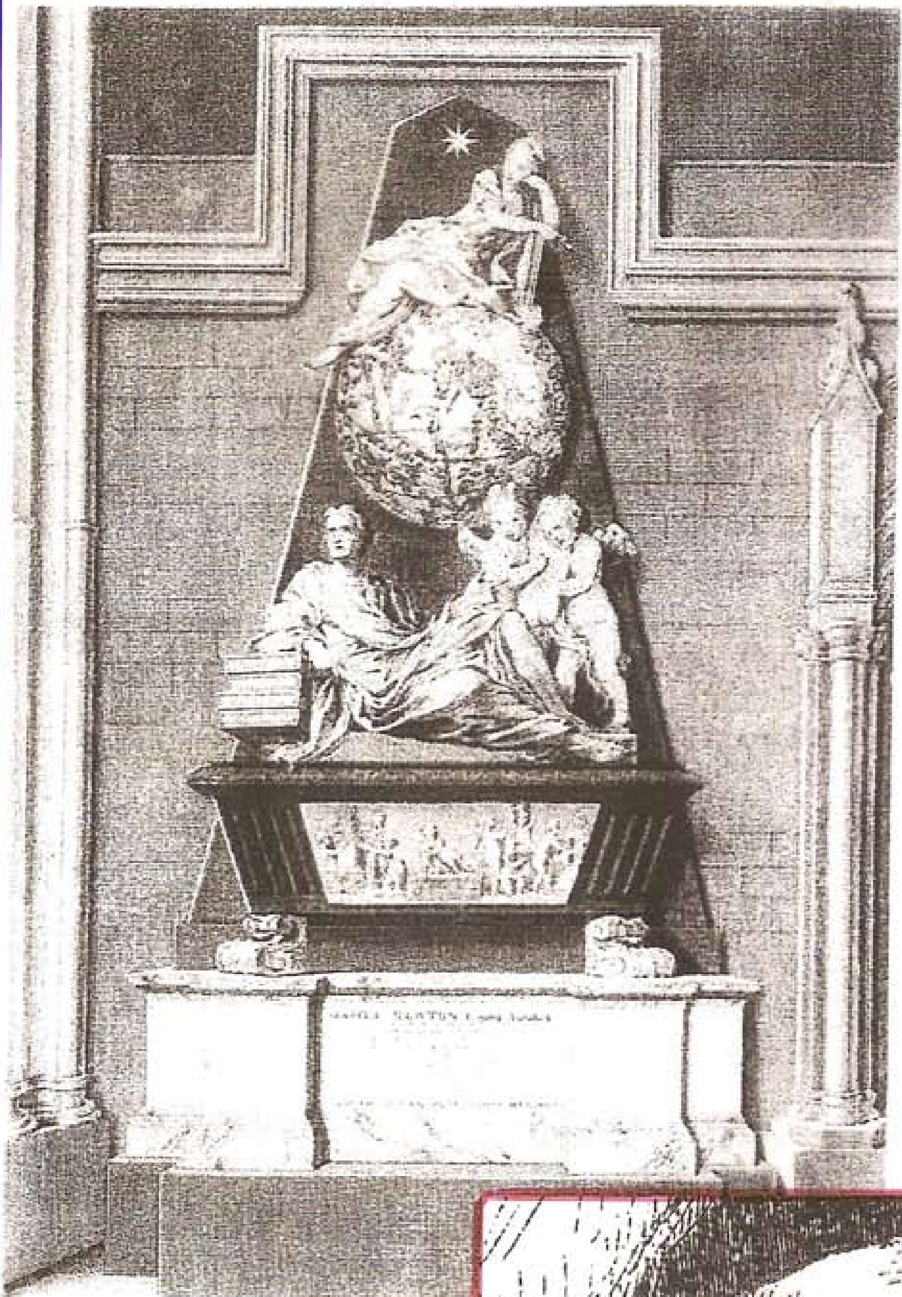
Gottfried Wilhelm Leibniz

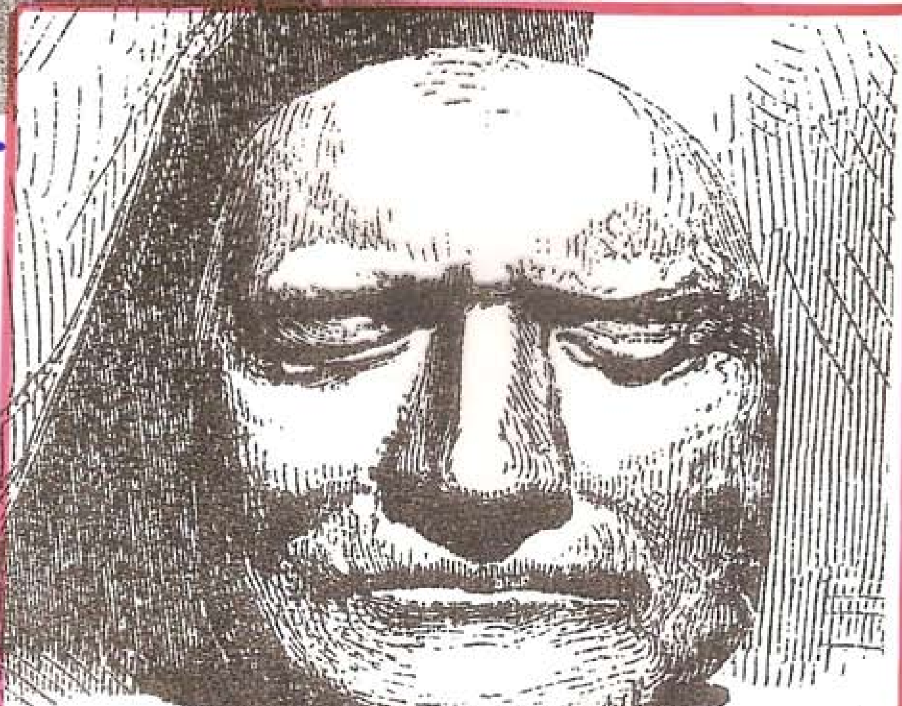
(1646 - 1716)

Britain vs. the Continent

Newton	Leibniz
Halley	The Bernoullis
Taylor	Euler
Maclaurin	d' Alembert
Whiston	Lagrange
Saunderson	Legendre
Bradley	Laplace
. . .	. . .

The Death of Isaac Newton, 1727





Newtonian Philosophy

EDMOND HALLEY (1656-1742)

Savilian Professor of Geometry

- while an undergraduate, sailed to St Helena to observe southern stars
- FRS at age 22
- persuaded Newton to write 'Principia'
- sea captain (52° south)
- edition of Apollonius' 'Conics'
- Astronomer Royal, 1720-1742
- predicted return of 'Halley's comet'

Newtonian philosophy advanced in the

OLD ASHMOLEAN (1683)

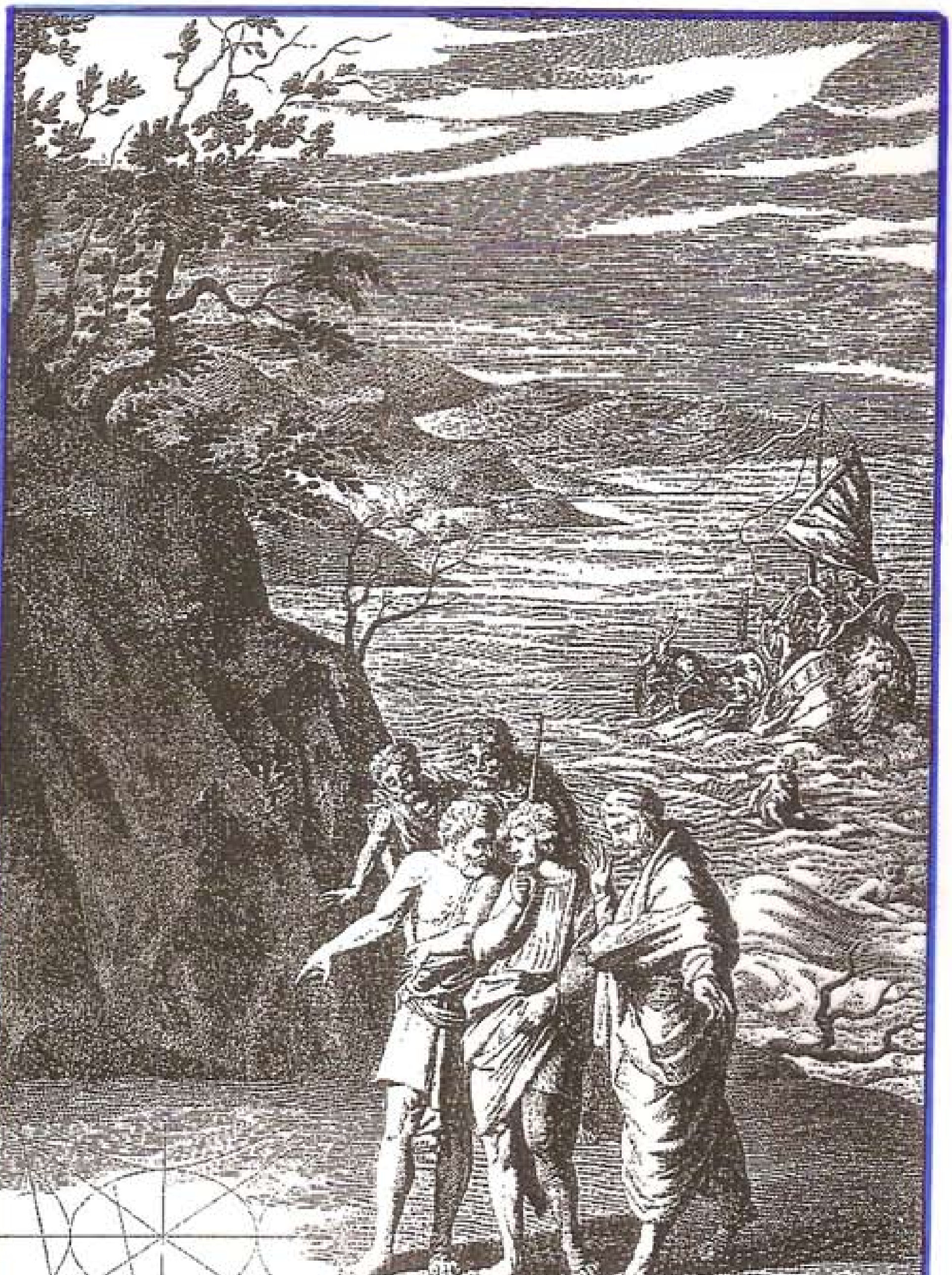
in the lectures by the Savilian professors

DAVID GREGORY, JAMES BRADLEY

Radcliffe Observatory built

Edmond Halley (1656-1742)

Halley's Edition of Apollonius's 'Conics' (1710)



Ariſtippus Philoſophus Socraticus, naufragio cum ejectus ad Rhodienſium litus animadvertiſſet Geometrica ſchemata deſcripta, exclamaviſſe ad comites ita dicitur; "Bene ſperemus, Hominum enim veſtigia video."
Vitruv. Architect. lib. 6. Praef.

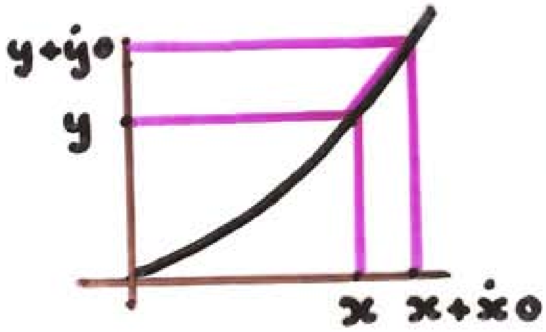
Brook Taylor (1685-1731)

Newton's calculus

Variables: changing with time
– 'flowing'

Derivatives: based on velocity
(tangents) – notation $\dot{x}$, $\dot{y}$

Example: $y = x^2$



Substitute $x + \dot{x}o$ for x
$y + \dot{y}o$ for y:

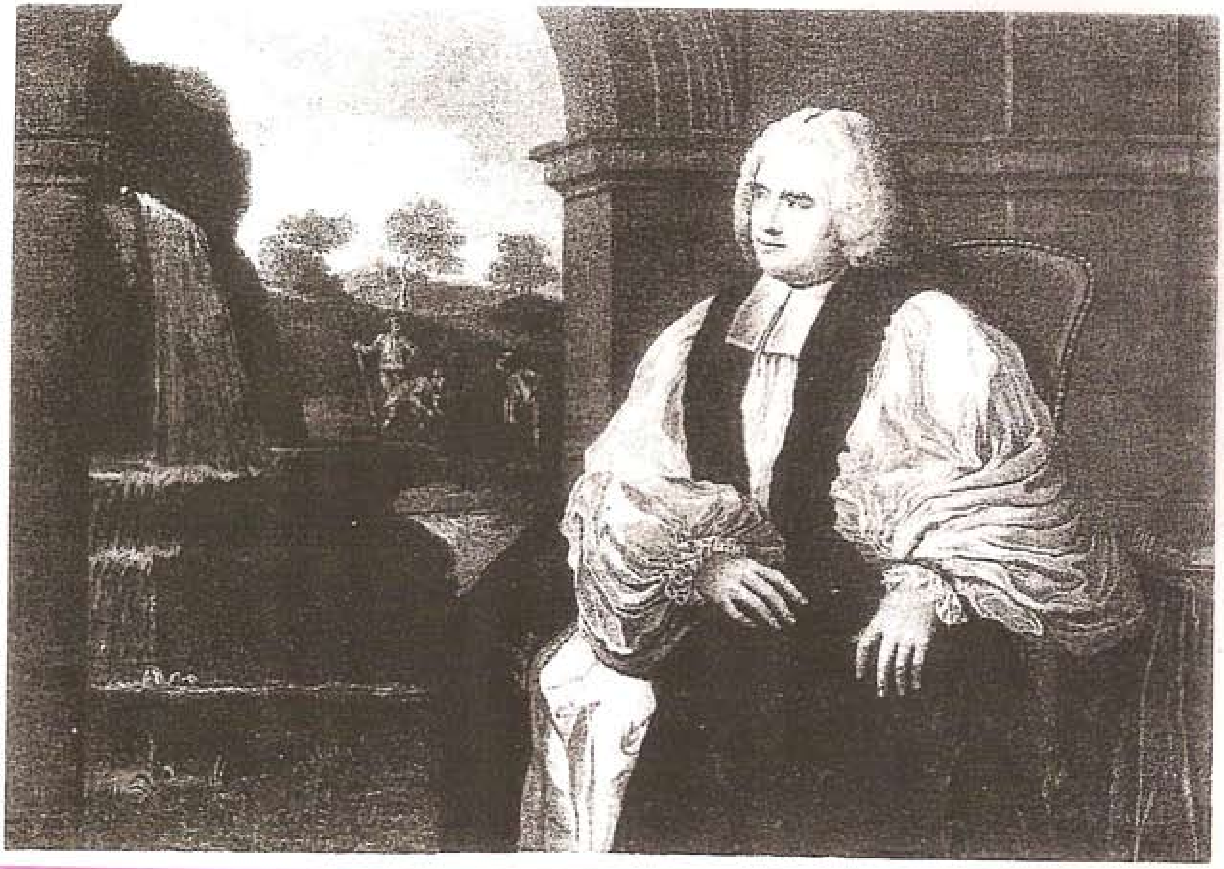
$$\cancel{y} + \dot{y}o = \cancel{x^2} + 2x\dot{x}o + \dot{x}^2 o^2$$

Cancel o: $\dot{y} = 2x\dot{x} + \dot{x}^2 o$

Ignore o: $\dot{y} = 2x\dot{x}$, or $\dot{y}/\dot{x} = 2x$

Integrals: find anti-derivatives
(areas) (fundamental theorem)

Bishop Berkeley's 'The Analyst'



THE

ANALYST;

OR, A

DISCOURSE

Addressed to an

Infidel MATHEMATICIAN.

WHEREIN

It is examined whether the Object, Principles, and Inferences of the modern Analysis are more distinctly conceived, or more evidently deduced, than Religious Mysteries and Points of Faith.

By the AUTHOR of *The Minute Philosopher*.

The SECOND EDITION,

First cast out the beam out of thine own Eye; and then shalt thou see clearly to cast out the mote out of thy brother's eye. S. Matt. c. vii. v. 5.

LONDON:

Printed for J. and R. TONSON and S. DRAPER in the *Strand*.

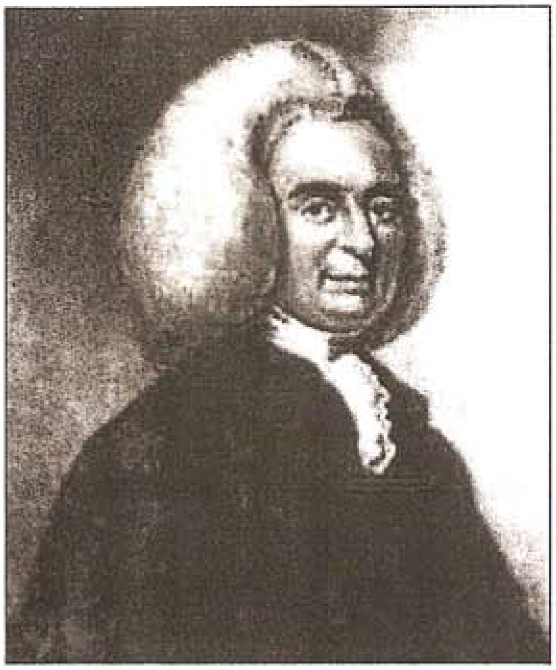
MDCCLIV.

Bishop Berkeley's *The Analyst* (1734)

The Analyst, or a discourse addressed to an infidel mathematician wherein it is examined whether the Object, Principles and inferences of the modern analysis are more distinctly conceived, or more evidently deduced, than Religious Mysteries and Points of Faith.

He used these fluxions like the scaffold of a building, as things to be laid aside or got rid of.
What are these fluxions?
The velocities of evanescent increments?
And what are these same evanescent increments?
They are neither finite quantities,
nor quantities infinitely small, nor yet nothing.
May we not call them the Ghosts of departed quantities?

Colin Maclaurin (1698-1746)



Principia Mathematica (1687)

Laws of Motion

Book I : The Motion of Bodies

- Inverse-square law of gravity
- Kepler's laws

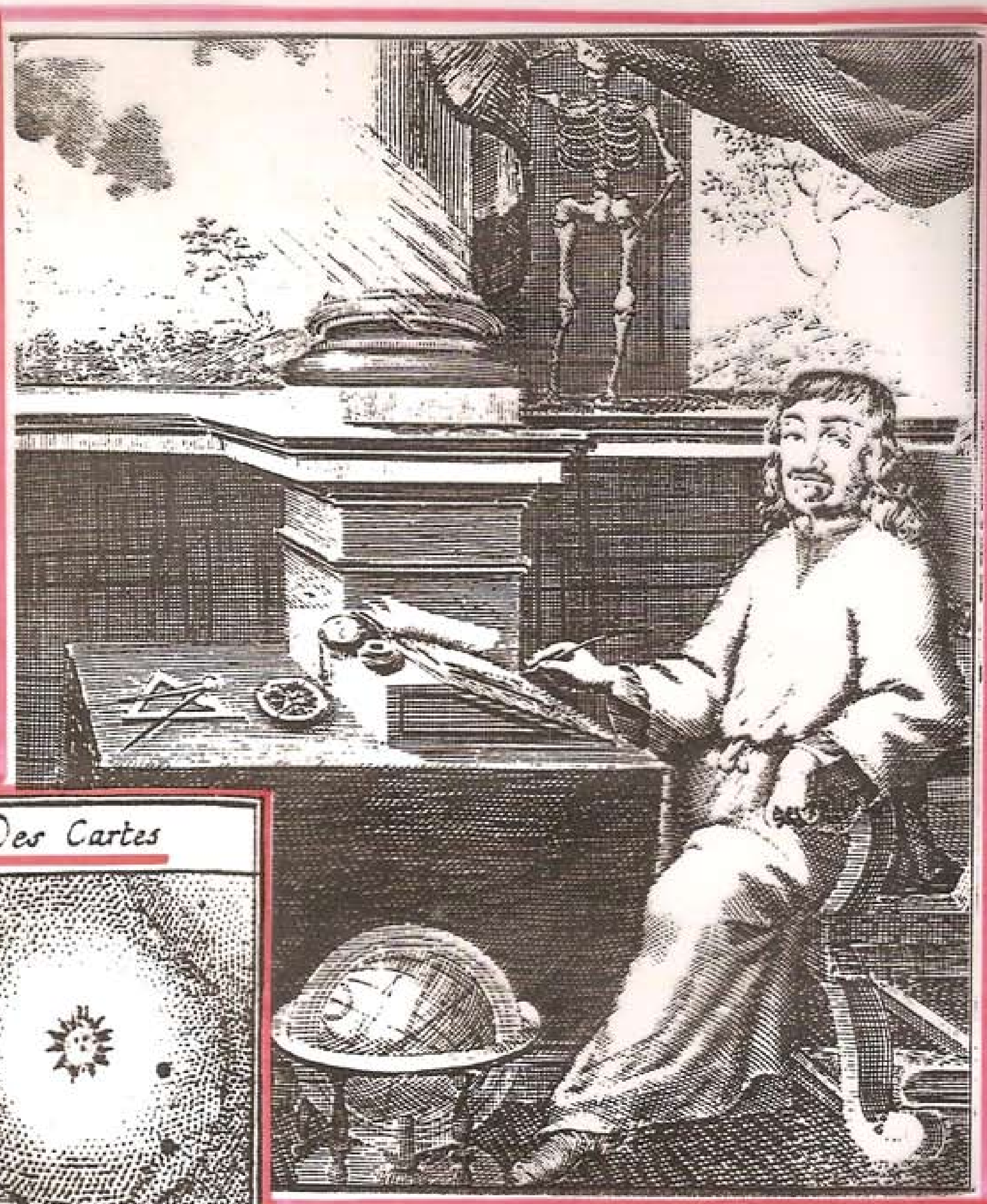
Book II : Motion in Resisting Media

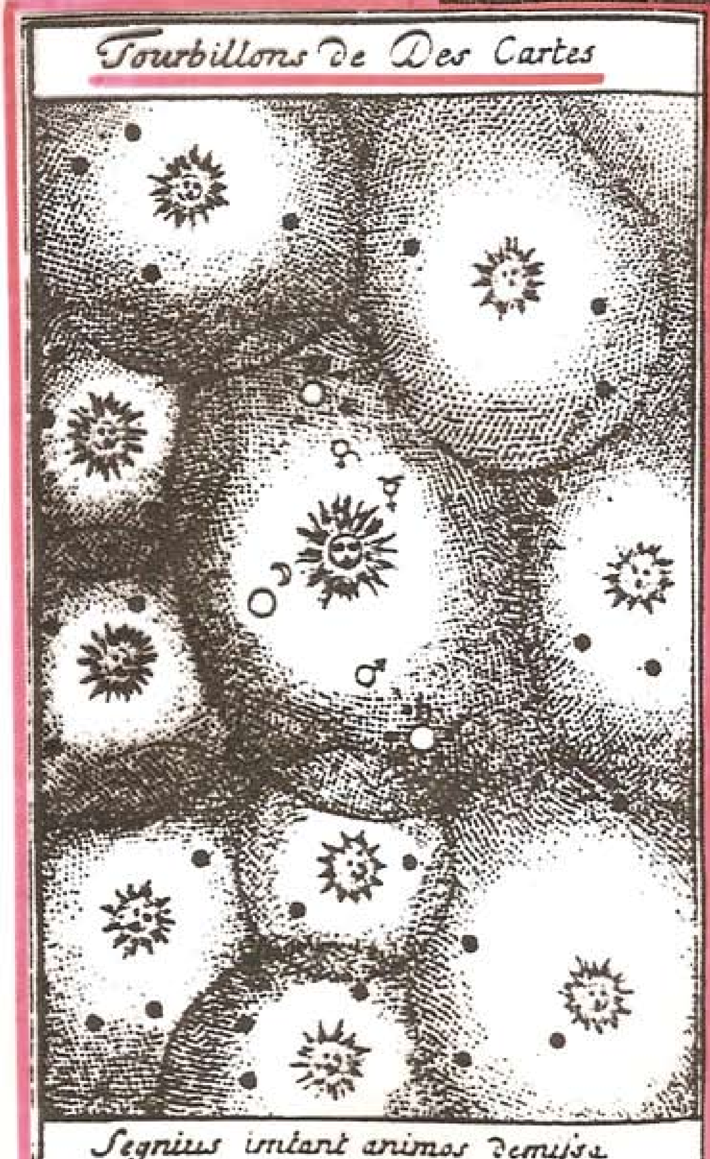
- Wave motion for light and sound
- Vortices : Descartes' theory

Book III : The System of the World

- Orbits of comets
- Motion of the moon
- Theory of tides
- Flattening of the Earth at the poles
- Precession of the equinoxes

René Descartes





Theory of vortices

The Shape of the Earth

Prop. XVIII, Theorem XVI:

That the axes of the planets are less than the diameters drawn perpendicular to the axes.

... if our earth was not higher about the equator than at the poles, the sea would subside about the poles, and, rising towards the equator, would lay all things there under water.

... the earth will be higher at the equator than at the poles by 85472 feet, or 17 1/10 miles. And its height at the equator will be about 19,658,600 feet, and at the poles 19,573,000 feet.

Success of the Principia

- Immediate in Britain and the Netherlands

'the greatest Discovery in Nature that ever was since the World's Creation'.

- France: de l'Hopital, Voltaire, Mme du Chatelet

- Maupertuis:

The shape of the Earth:

geodetic missions to Lapland and Peru to measure pendulum swings:

Newton correct: Earth is flatter at the poles.

What shape is the Earth?

In 1672, Mr Richer, in a voyage to Cayenna, near the equator, found that the pendulum of this clock no longer made its vibrations so frequently as in the latitude of Paris. In consequence of this it was discovered that, whereas the gravity of bodies is by so much the less powerful, as these bodies are farther removed from the centre of the earth, the region of the equator must absolutely be much more elevated than that of France; and so must be further removed from the centre; and, therefore, that the earth could not be a sphere.

Voltaire

It is evident in general that Sir Isaac Newton's Figure of a flat Spheroid, and Mr Cassini's of a long one, will give very different Distances of Places that have the same Longitude and Latitude. ... In a course of 100 Degrees Longitude, there might be a Mistake of more than two Degrees, if sailing really upon Sir Isaac Newton's Earth one should imagine himself to be upon Mr Cassini's. And how many Ships have perished by smaller mistakes?

Pierre Maupertuis

Maupertuis's Mission to Lapland

Letters between Euler and Clairaut

Clairaut to Euler : 11 Sept 1747

Pending something better, the law for all nature should be as $1/\text{dist}^2$ + a small function of distances detectable for the moon and almost zero for great distances ...

Euler to Clairaut : 30 Sept 1747

- disagreed - Mercury doesn't fit the explanation

Clairaut to Euler : 7 December 1747

Mercury can be dealt with by adding $l^2/357d^2$ (d = distance between earth and moon) - but this doesn't seem right - and $1/d^4$ is too strong

Clairaut - general announcement : 17 May 1749

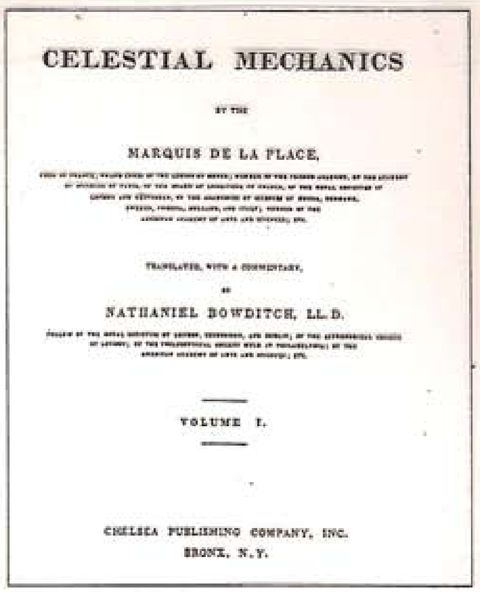
Having considered anew from a new viewpoint, I have been led to reconcile observations with a force $\propto$ the inverse square of the distance.

The Motion of the Moon

Euler

Clairaut

Laplace



CELESTIAL MECHANICS

BY THE

MARQUIS DE LA PLACE,

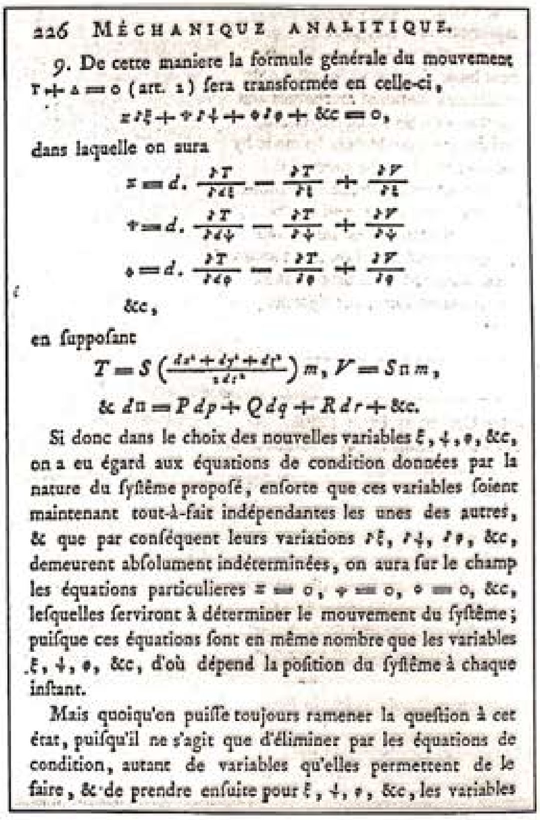
TRANSLATED, WITH A COMMENTARY,

BY

NATHANIEL BOWDITCH, LL.D.

VOLUME I.

CHELSEA PUBLISHING COMPANY, INC.
BRONX, N.Y.



226 MÉCHANIQUE ANALITIQUE.

9. De cette maniere la formule générale du mouvement $\Gamma + \Delta = 0$ (art. 1) sera transformée en celle-ci,

$$\Xi\,\delta\xi + \Psi\,\delta\psi + \Phi\,\delta\varphi + \&c = 0,$$

dans laquelle on aura

$$\Xi = d.\frac{\delta T}{\delta d\xi} - \frac{\delta T}{\delta \xi} + \frac{\delta V}{\delta \xi}$$

$$\Psi = d.\frac{\delta T}{\delta d\psi} - \frac{\delta T}{\delta \psi} + \frac{\delta V}{\delta \psi}$$

$$\Phi = d.\frac{\delta T}{\delta d\varphi} - \frac{\delta T}{\delta \varphi} + \frac{\delta V}{\delta \varphi}$$

&c,

en supposant

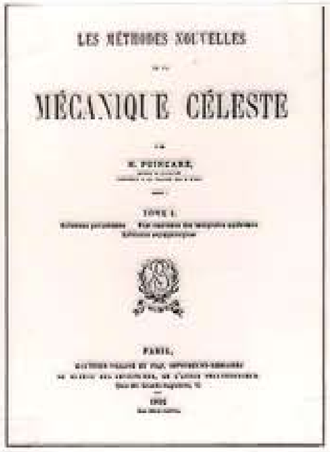
$$T = S\left(\frac{dx^2 + dy^2 + dz^2}{2dt^2}\right)m, \quad V = S\Pi m,$$

$$\& \; d\Pi = P\,dp + Q\,dq + R\,dr + \&c.$$

Si donc dans le choix des nouvelles variables ξ, ψ, φ, &c, on a eu égard aux équations de condition données par la nature du systême proposé, ensorte que ces variables soient maintenant tout-à-fait indépendantes les unes des autres, & que par conséquent leurs variations $\delta\xi$, $\delta\psi$, $\delta\varphi$, &c, demeurent absolument indéterminées, on aura sur le champ les équations particulieres $\Xi = 0$, $\Psi = 0$, $\Phi = 0$, &c, lesquelles serviront à déterminer le mouvement du systême; puisque ces équations sont en même nombre que les variables ξ, ψ, φ, &c, d'où dépend la position du systême à chaque instant.

Mais quoiqu'on puisse toujours ramener la question à cet état, puisqu'il ne s'agit que d'éliminer par les équations de condition, autant de variables qu'elles permettent de le faire, & de prendre ensuite pour ξ, ψ, φ, &c, les variables

Poincaré



LES MÉTHODES NOUVELLES

MÉCANIQUE CÉLESTE

H. POINCARÉ,

TOME I.

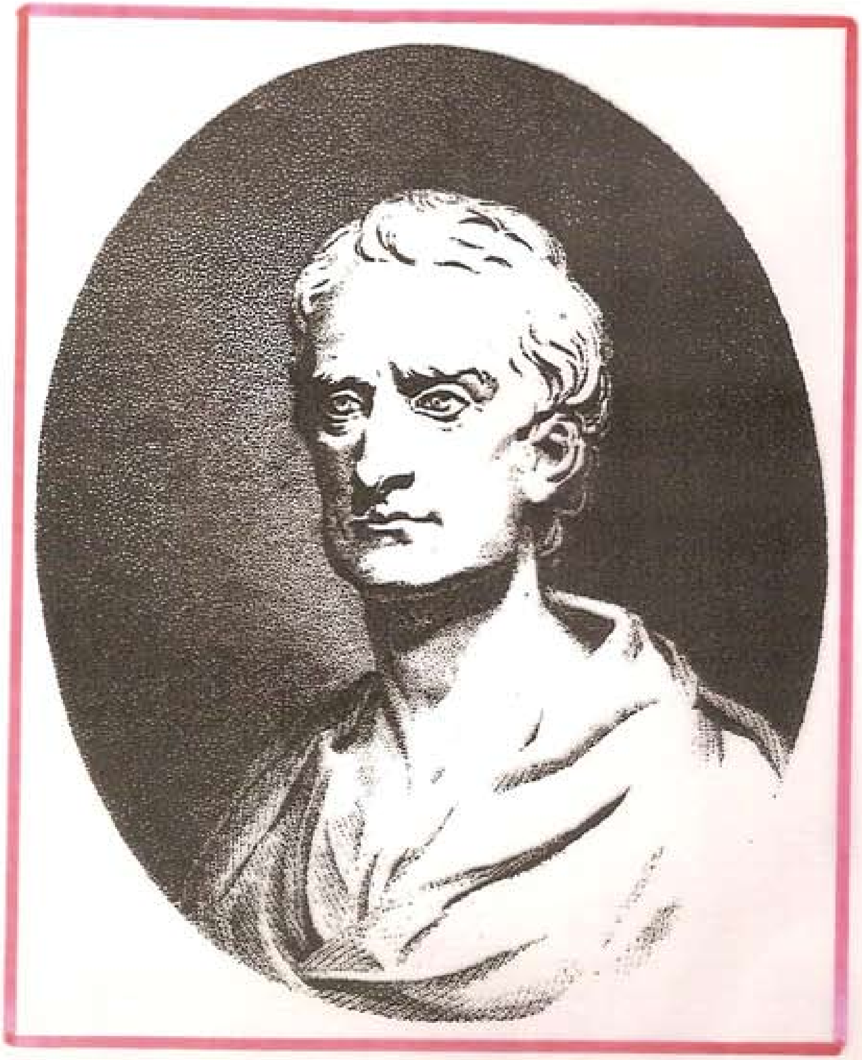
PARIS,

Lagrange

Nature, and Nature's Laws
lay hid in Night.

God said, Let Newton be!
and All was Light.

A. POPE



LINEÆ TERTII ORDINIS *NEUTONIANÆ*,

SIVE

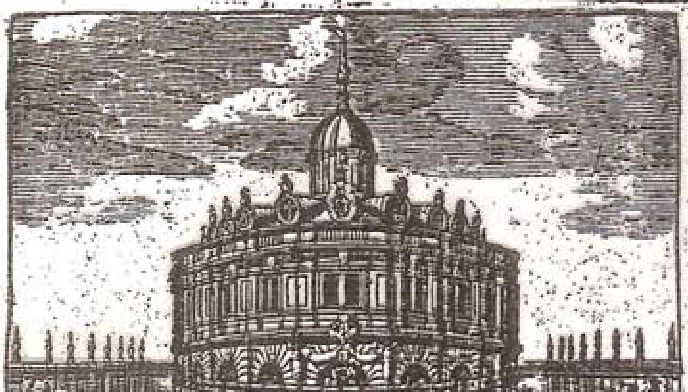
Illuſtratio Tractatus D. *Neutoni* De Enumeratione *Linearum Tertii Ordinis*.

Cui Subjungitur,

Solutio Trium Problematum.

R Bates A M e coll [illegible]

Authore JACOBO STIRLING, è Coll. Ball. *Oxon.*

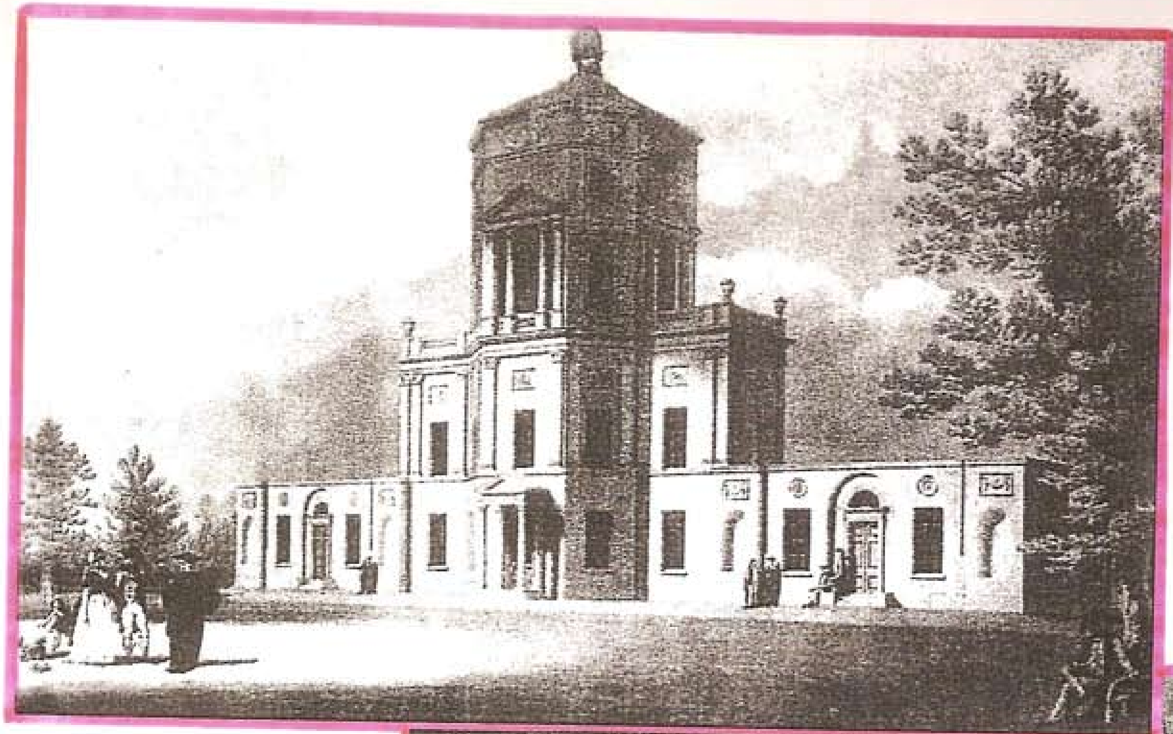


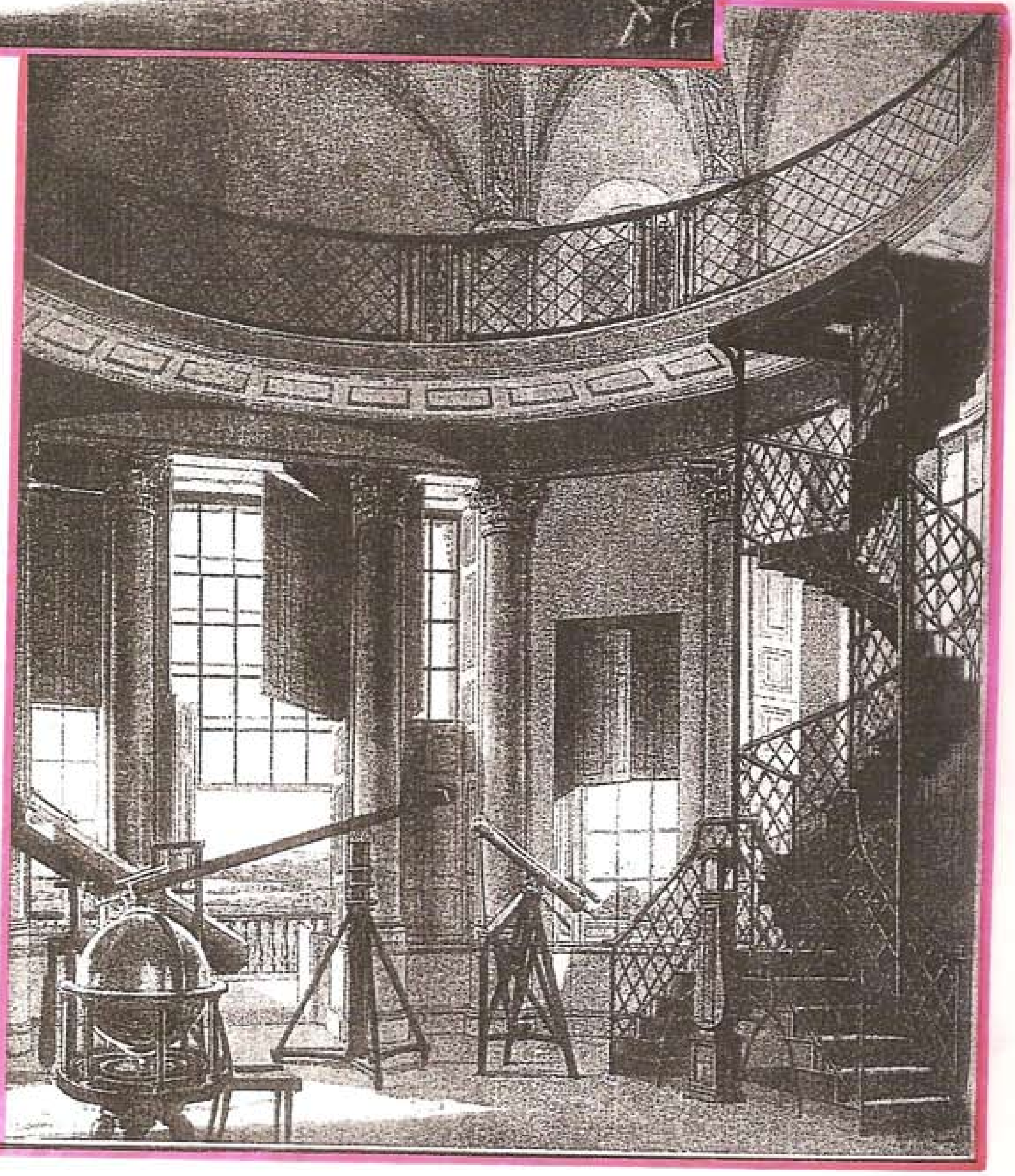
OXONIÆ,

E THEATRO SHELDONIANO, Impenſis *Edvardi Whiſtler* Bibliopolæ *Oxonienſis*, MDCCXVII.

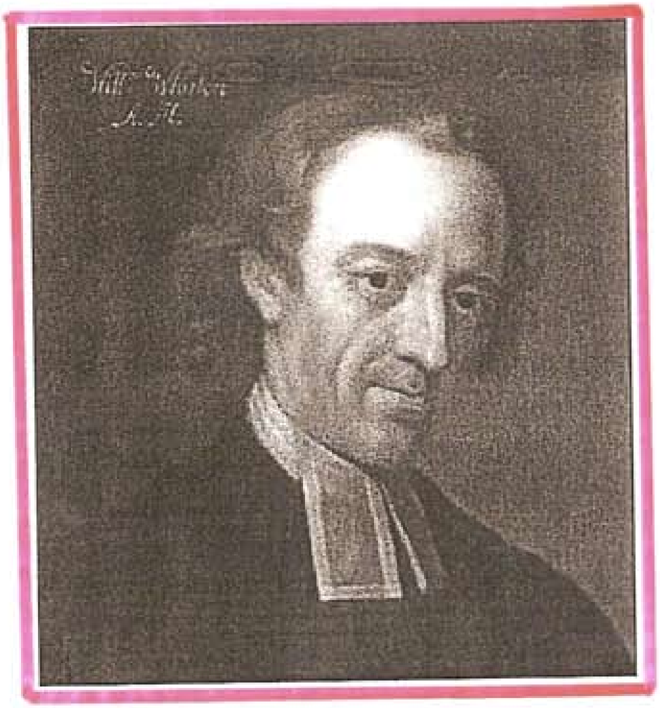
Publish'd by W Hogarth March
Price Six pence

Hornsby's Radcliffe Observatory





William Whiston



R. Paul

A

New METHOD

For Discovering the

LONGITUDE

BOTH AT

SEA and LAND,

Humbly Proposed to the Consideration of the PUBLICK.

BY

William Whiston, M. A. sometime Professor of the Mathematicks in the University of *Cambridg*. and *Humphry Ditton*, Master of the New Mathematick School in *Christ's Hospital*, London.

LONDON:

Printed for JOHN PHILLIPS, at the *Black Bull* in *Cornhill*. 1714. *July 15*

Nicholas Saunderson

Mr Sanderson's Mechanicks

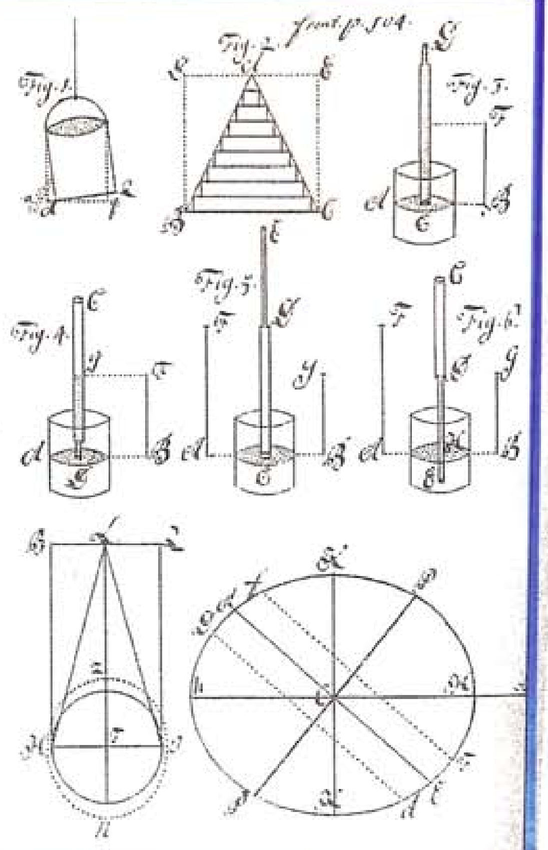
1. Mechanicks is that Part of Natural Philosophy, wch treats of ye Nature & Laws of Motion.

2. What we have to say on this subject, shall be reduced to three Heads. Under ye 1st we shall explain ye Nature of Machines, considering ye Motions, & Powers of Weights acting continually against Obstacles, & other Resistances. Under ye 2nd we shall speak of those Laws, wch relate to ye Effects of moving Force upon such Bodies, as are at Liberty to move without any Obstacle, or Resistance. In ye 3rd we shall apply ye fore-mentioned Laws to ye Motion of Heavenly Bodies, & show ye the whole Planetary System is govern'd by 'em.

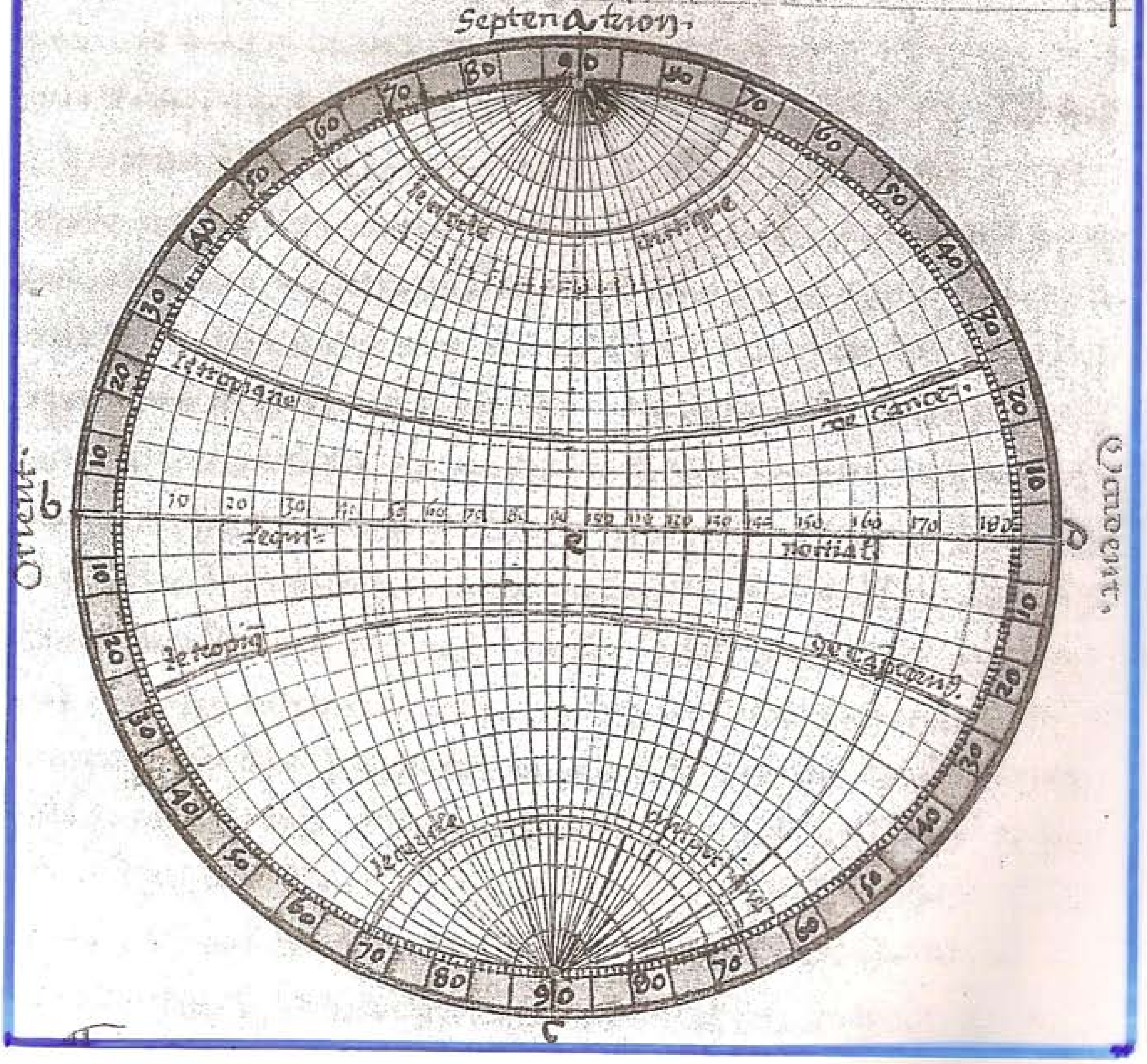
Part 1st of Motion in general.

3. Motion is a continual shifting out of one place into another, without continuing in ye same place for any Time; that affection of Motion, by wch a Body runs through a certain space in a certain Time is called ye Celerity, or Velocity of ye Motion. The greater ye force is impressed upon a Body to make it change it's Place, ye greater is it's Motion, (for ye Effect is always proportionable to its Cause) & that Force is called ye Momentum or Quantity of Motion.

17



De lesphere



Septentrion
Orient
Occident
le tropique
de cancer
le tropiq
de capricorn

The Ballad of Gresham College

If to be rich and to be learn'd

Be every Nation's cheifest glory,

How much are English men concern'd,

Gresham to celebrate thy story

Who built th'Exchange t'enrich the Citty

And a Colledge founded for the witty.

. . .

The College will the whole world measure;

Which most impossible conclude,

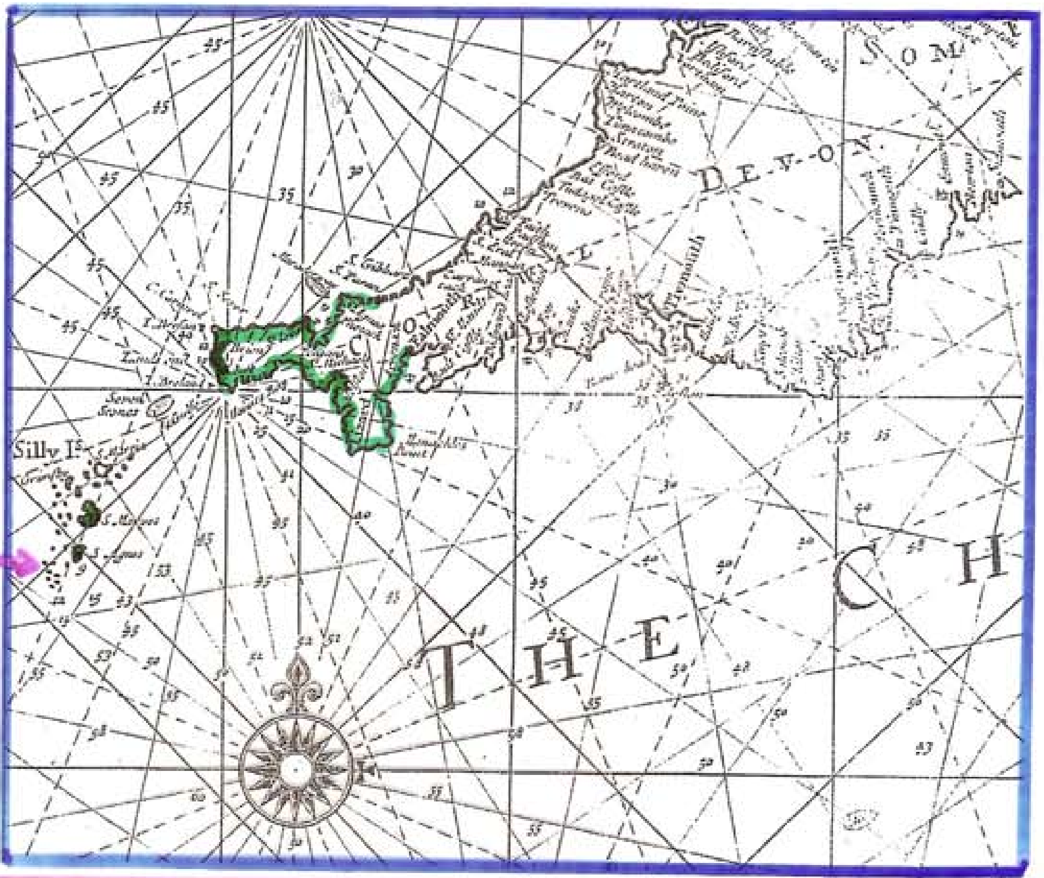
And Navigation make a pleasure

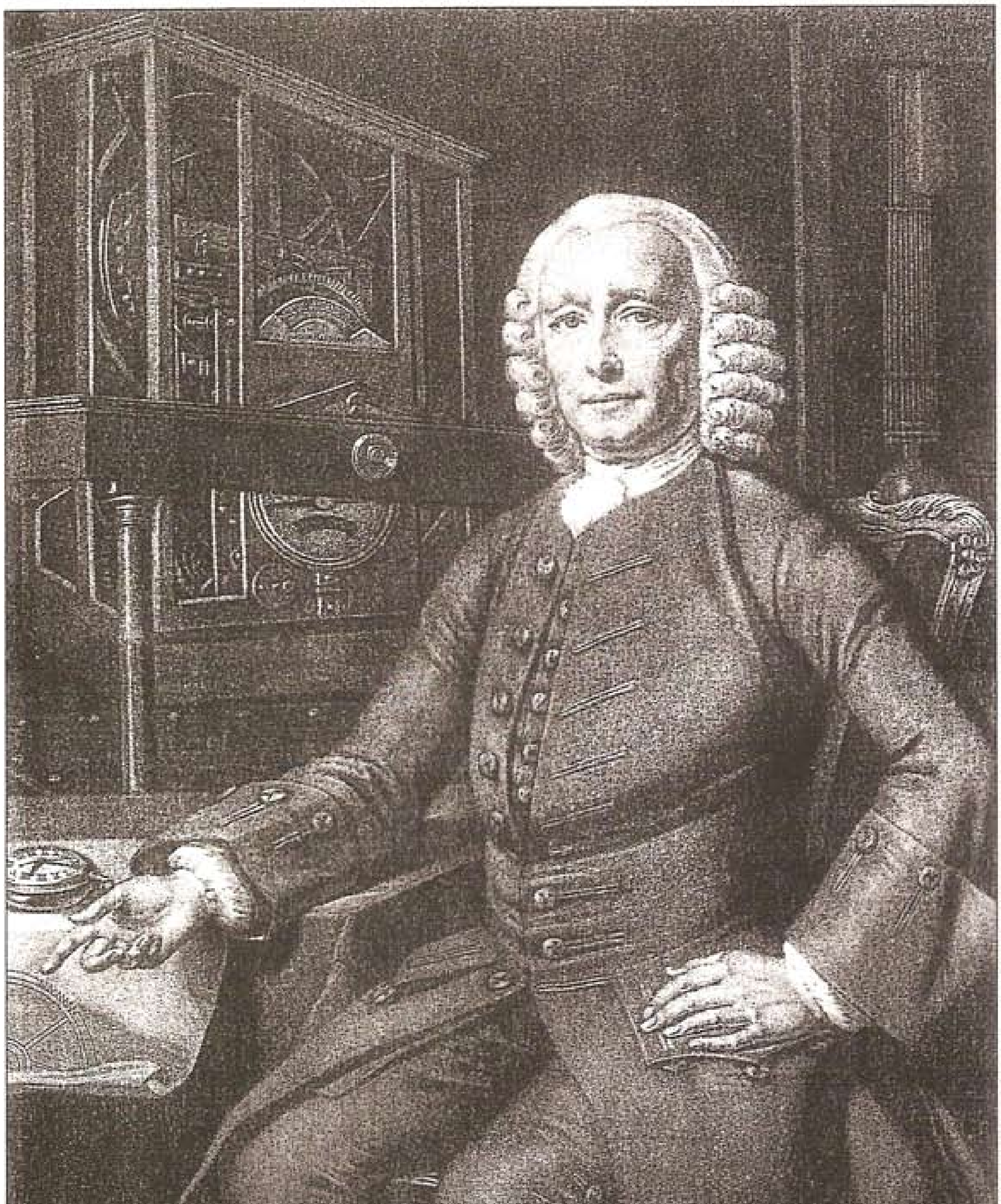
By finding out the Longitude.

Every Tar shall then with ease

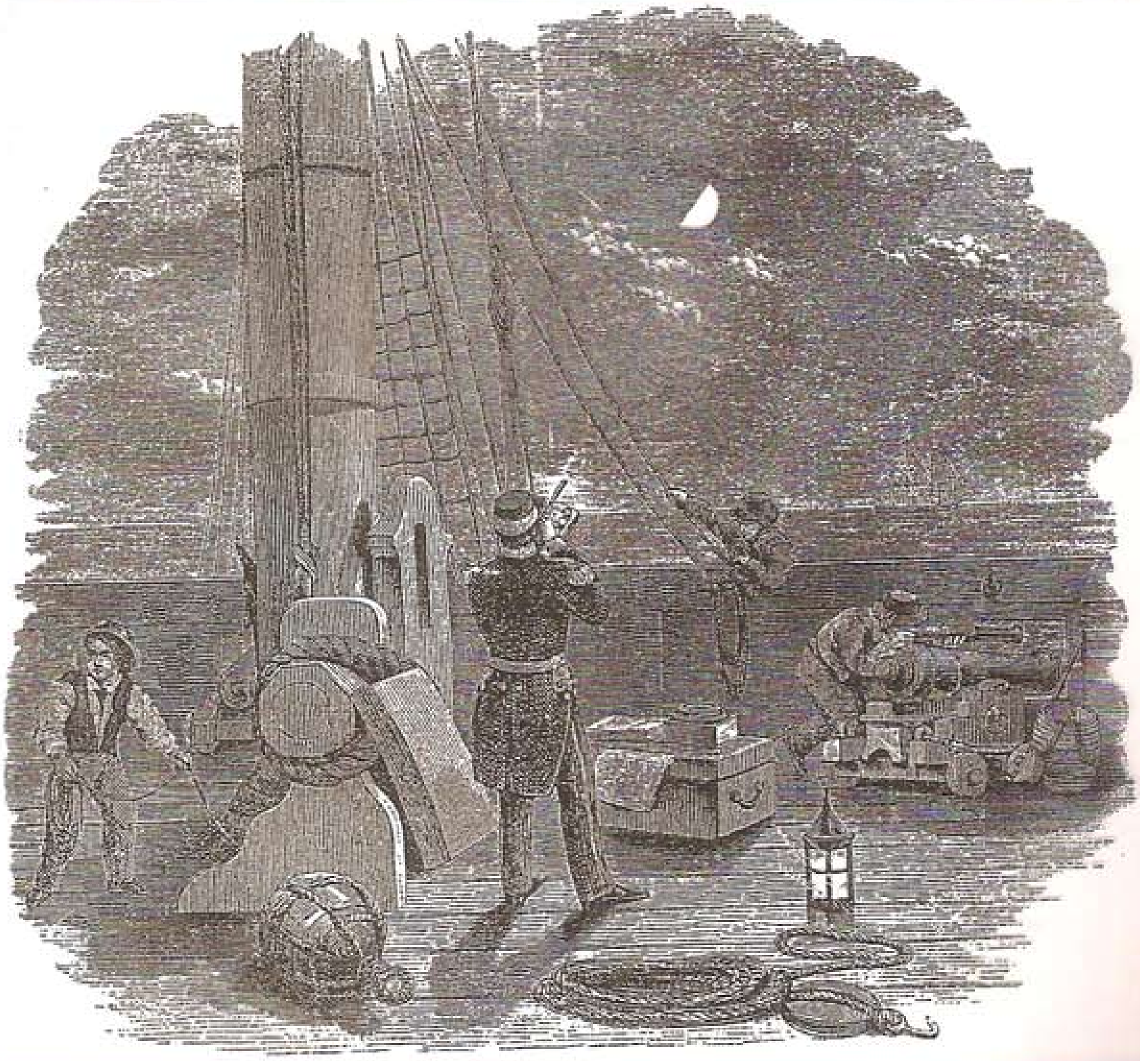
Sayle any ship to the Antipodes.

The Wreck of the 'Association' (1707)





John Harrison



Leibniz's calculus

Variables: no concept of motion

- sequences of close values

Derivatives (tangents): differences of successive values - infinitely small

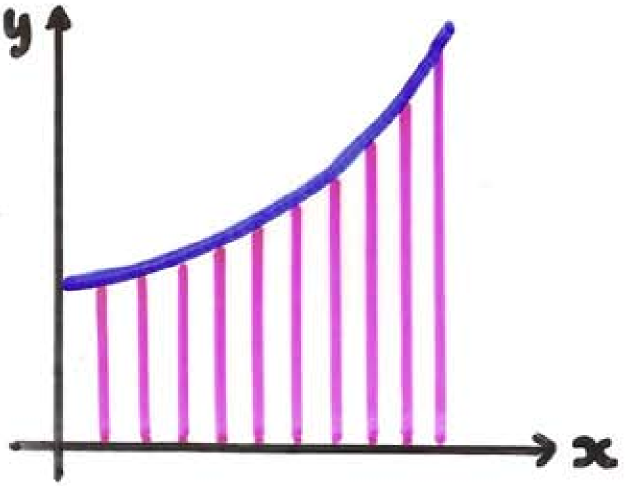
- notation dy/dx

Integrals (areas): area = sum of lines

notation:

all lines = omn. l

$= \int l$



Jakob Bernoulli (1654-1705)

polar coordinates

catenary

logarithmic spiral

probability ('Ars Conjectandi')

word "integral"

Jakob Bernoulli's 'Ars Conjectandi' (1713)

JACOBI BERNOULLI,
Profess. Basil. & utriusque Societ. Reg. Scientiar.
Gall. & Pruss. Sodal.
MATHEMATICI CELEBERRIMI,

ARS CONJECTANDI,

OPUS POSTHUMUM.

Accedit

TRACTATUS
DE SERIEBUS INFINITIS,

Et EPISTOLA Gallicè scripta
DE LUDO PILÆ
RETICULARIS.

BASILEÆ,
Impensis THURNISIORUM, Fratrum.
cIↃ IↃcc XIII.
(1713)

probability:

'law of large numbers'

limit theorems

binomial distribution

combinatorics:

figurate numbers

combinations

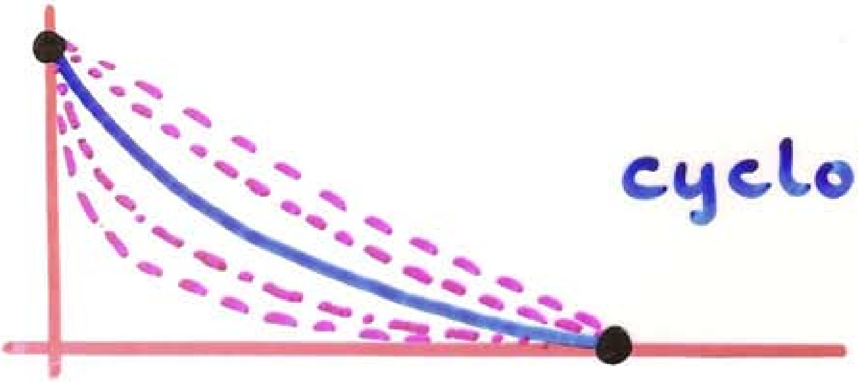
'Bernoulli numbers'

preceded by a hymn on the infinite variety of nature – this variety stems from the combinations and arrangements of its parts, and the combinatorial art helps us to enumerate these.

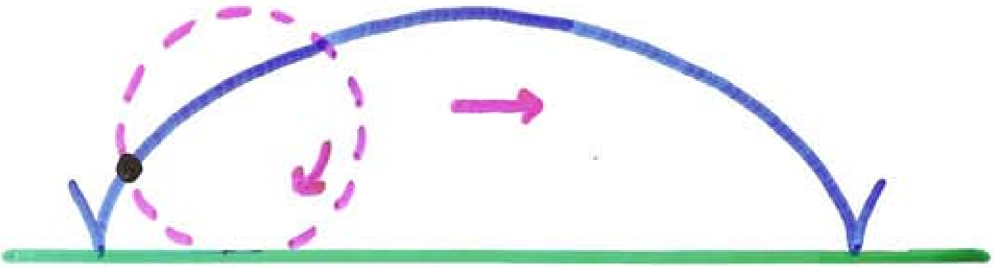
Brachistochrone Problem

1697 : Johann Bernoulli :

Find the 'curve of quickest descent'



cycloid



'I recognize the lion by his claws'

Johann
Bernoulli
(1667-1748) →

calculus of variations
brachistochrone
teacher of L'Hôpital,...

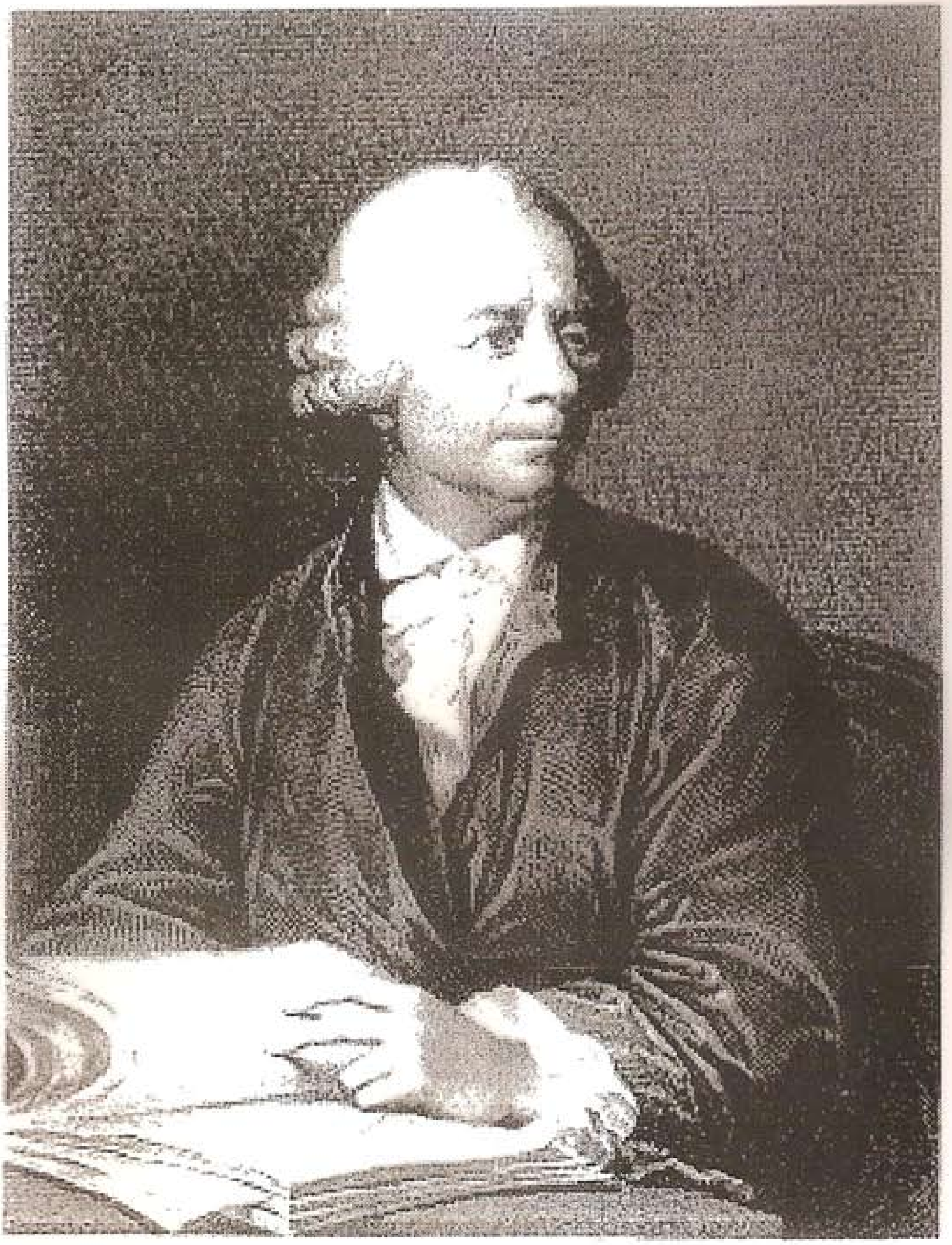
Daniel
Bernoulli
← (1700-1782)

probability
hydrodynamics

Marquis de l'Hôpital (1661-1704)

Published the first book on the calculus (1696):

'Analyse des infiniment petits'



Leonhard Euler

(1707 - 1783)

INSTITUTIONES CALCULI DIFFERENTIALIS

CUM EIUS VSU

IN ANALYSI FINITORUM

AC

DOCTRINA SERIERUM

AUCTORE

LEONHARDO EULERO

ACAD. REG. SCIENT. ET ELEG. LITT. BORUSS. DIRECTORE
PROF. HONOR. ACAD. IMP. SCIENT. PETROP. ET ACADEMIARUM
REGIARUM PARISINAE ET LONDINENSIS
SOCIO.

IMPENSIS
ACADEMIAE IMPERIALIS SCIENTIARUM
PETROPOLITANAE
1755.

INSTITVTIONVM CALCVLI INTEGRALIS

VOLVMEN PRIMVM

IN QVO METHODVS INTEGRANDI A PRIMIS PRINCIPIIS VSQVE AD INTEGRATIONEM AEQVATIONVM DIFFERENTIALIVM PRIMI GRADVS PERTRACTATVR.

AVCTORE

LEONHARDO EVLERO

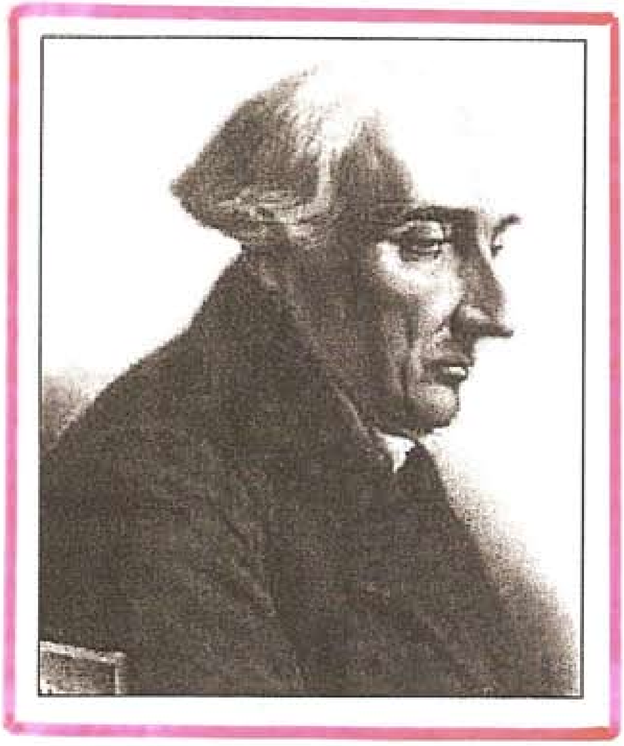
ACAD. SCIENT. BORVSSIAE DIRECTORE VICENNALI ET SOCIO ACAD. PETROP. PARISIN. ET LONDIN.

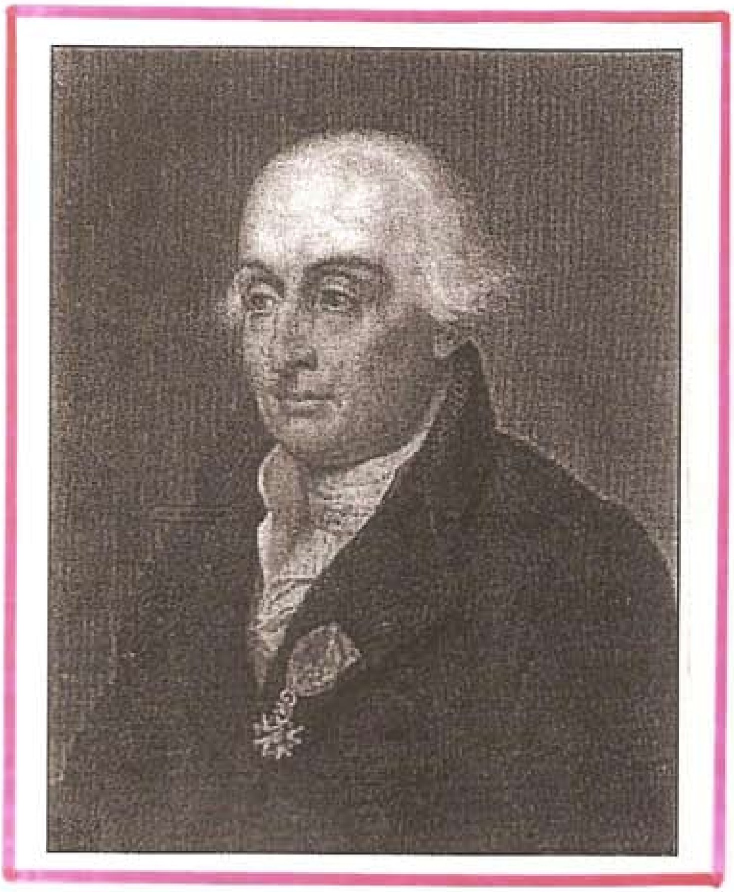
PETROPOLI

Impenſis Academiae Imperialis Scientiarum

1768.

Joseph-Louis Lagrange (1736-1813)





$$f(x) = a + bx + cx^2 + dx^3 + \dots$$

$$f'(x) = b + 2cx + 3dx^2 + \dots$$

$$\sin x = x - x^3/6 + x^5/120 - \dots$$

$$\text{so } [\cos x =]\ 1 - x^2/2 + x^4/24 - \dots$$

MAP OF KÖNIGSBERG

from M. Zeiller, Topographia Prussiae et Pomerelliae, Frankfurt, c. 1650.

SOLUTIO PROBLEMATIS AD GEOMETRIAM SITUS PERTINENTIS

Commentatio 53 indicis Enestroemiani
Commentarii academiae scientiarum Petropolitanae 8 (1736), 1741, p. 128—140

1. Praeter illam geometriae partem, quae circa quantitates versatur et omni tempore summo studio est exculta, alterius partis etiamnum admodum ignotae primus mentionem fecit Leibnitzius[1]), quam *Geometriam situs* vocavit. Ista pars ab ipso in solo situ determinando situsque proprietatibus eruendis occupata esse statuitur; in quo negotio neque ad quantitates respiciendum neque calculo quantitatum utendum sit. Cuiusmodi autem problemata ad hanc situs geometriam pertineant et quali methodo in iis resolvendis uti oporteat, non satis est definitum. Quamobrem, cum nuper problematis cuiusdam mentio esset facta, quod quidem ad geometriam pertinere videbatur, at ita erat comparatum, ut neque determinationem quantitatum requireret neque solutionem calculi quantitatum ope admitteret, id ad geometriam situs referre haud dubitavi, praesertim quod in eius solutione solus situs in considerationem veniat, calculus vero nullius prorsus sit usus. Methodum ergo meam, quam ad huius generis problemata solvenda inveni, tanquam specimen Geometriae situs hic exponere constitui.

2. Problema autem hoc, quod mihi satis notum esse perhibebatur, erat sequens: Regiomonti in Borussia esse insulam *A*, *der Kneiphof* dictam, fluviumque eam cingentem in duos dividi ramos, quemadmodum ex figura (Fig. 1) videre licet; ramos vero huius fluvii septem instructos esse pontibus *a*, *b*, *c*, *d*, *e*, *f* et *g*. Circa hos pontes iam ista proponebatur quaestio, num quis cursum ita instituere queat, ut per singulos pontes semel et non plus quam semel transeat. Hocque fieri posse, mihi dictum est, alios negare alios dubitare; neminem vero affirmare. Ego ex hoc mihi sequens maxime generale formavi problema: quaecunque sit fluvii figura et distributio in ramos atque quicunque fuerit numerus pontium, invenire, utrum per singulos pontes semel tantum transiri queat an vero secus.

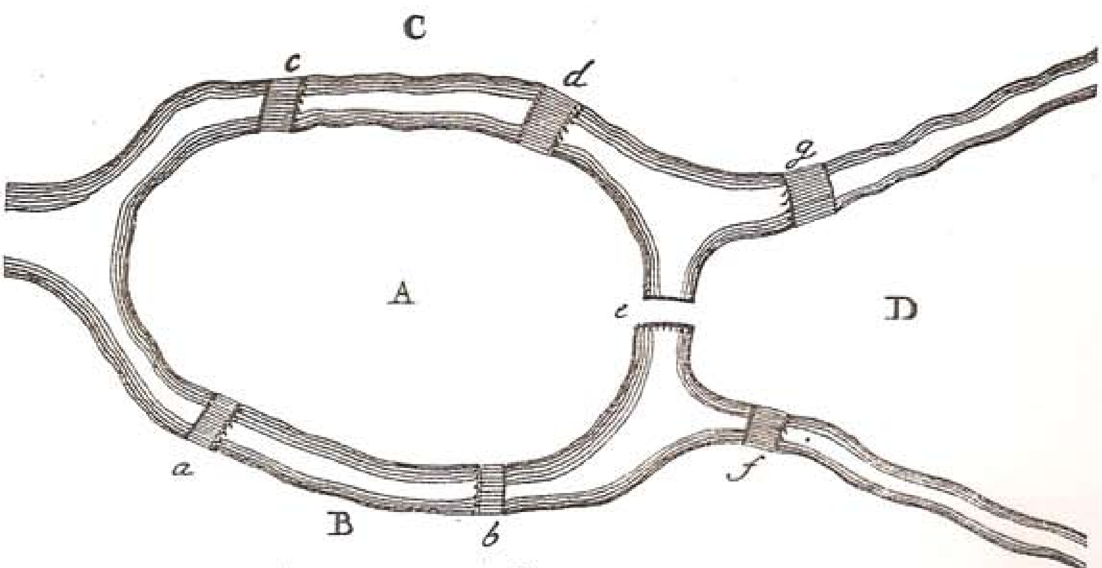


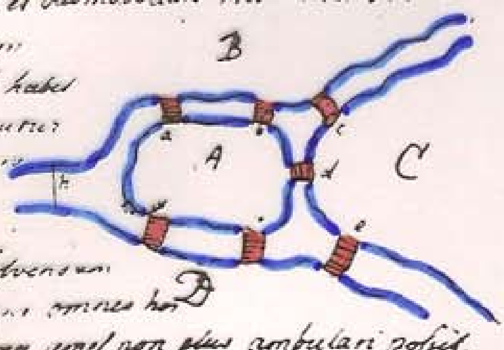
Fig. 1.

Letter from Leonhard Euler to Giovanni Marinoni (Vienna)

13 March 1736

Lfg

[illegible] uaidam meditationes Tecum communicare quas ut
benevole accipias, tuumq de iis judicium ... ibas etiam atq
[illegible]. Quaestio mihi aliquando proponebatur circa insulam
in urbe Regiomonti sitam fluvio [illegible] pontibus trajecto cum
[illegible], quaerebaturq num quis per singulos pontes ~~[illegible]~~ continuo
cursu semel ambulare possit, simul [illegible] nimenen
[illegible] hac lege cursum instituere potuisset. Haec quaestio
esti ~~[illegible]~~ videatur tamen mihi [illegible] ~~[illegible]~~ maxime
contemnatu [illegible] solvendam [illegible] geometria neq
[illegible] essent idonea. [illegible] nam
ea forte ad geometriam situs, quam [illegible]
[illegible]. Cum itaq [illegible] de re [illegible] meditatus, facilem adeptus
sum regulam firmissima demonstratione munitam qua in hu-
jusmodi quaestionibus statim discernere licet utrum hujus
modi [illegible] quotcunq et quomodocunq sitos [illegible]



[illegible]. Situs pontium
[illegible] habet
uti in figura [illegible]
in qua [illegible] insulam [illegible]
C, et D [illegible]
fluvii ramis a se [illegible]
discerptas designant. Ad solvendam
nunc quaestionem ~~[illegible]~~ per omnes hos
[illegible] per unumquemq semel non plus ambulari possit,
ante omnia est [illegible] sint regiones aqua disjunc-
[illegible] hujusmodi regiones [illegible] A, B, C, D
[illegible]. Deinde videndum est quot pontes in unamquamq regionem
[illegible] numerus pontium eo ducentium [illegible]
[illegible]. Sic in nostro exemplo ad A quinq pontes, ad reliqua
[illegible] pontes [illegible], seu numerus
pontium ad singulas ducentium est impar. quod ad quaestionem

19. Sint duae insulae A et B aqua circumdatae, qua cum aqua communicent quatuor fluvii, quemadmodum figura (Fig. 3) repraesentat. Traiecto porro sint super aquam insulas circumdantem et fluvios quindecim pontes a, b, c, d etc. et quaeritur, num quis cursum ita instituere queat, ut per

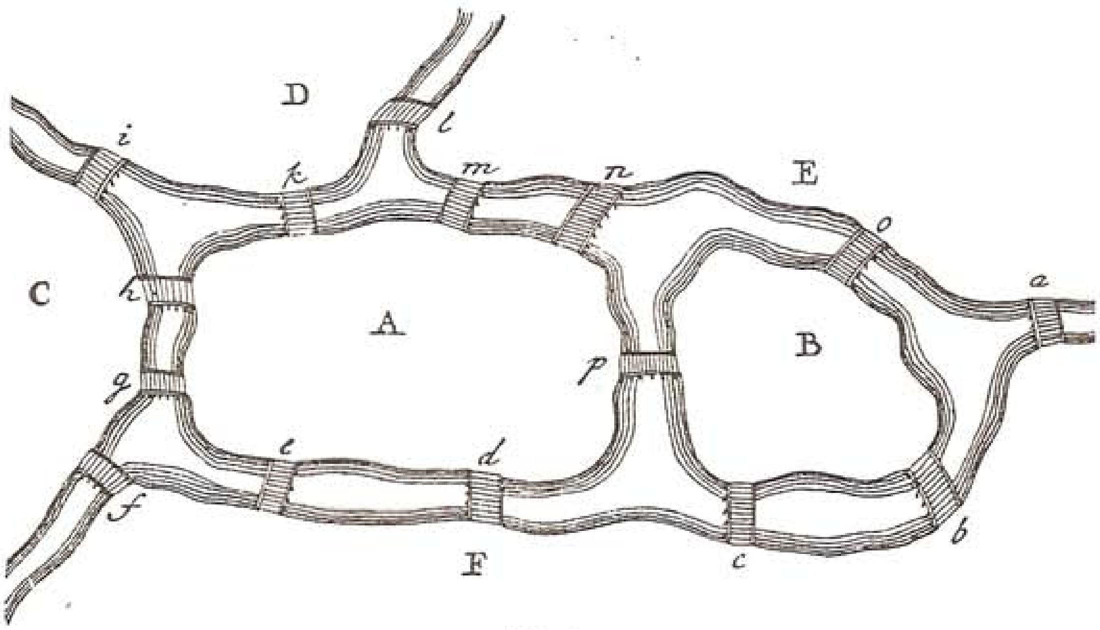


Fig. 3.

omnes pontes transeat, per nullum autem plus quam semel. Designo ergo primum omnes regiones, quae aqua a se invicem sunt separatae, litteris A, B, C, D, E, F, cuiusmodi ergo sunt sex regiones. Dein numerum pontium 15 unitate augeo et summam 16 sequenti operationi praefigo.

		16
A^*,	8	4
B^*,	4	2
C^*,	4	2
D,	3	2
E,	5	3
F^*,	6	3
		16

20. Casu ergo quocunque proposito statim facillime poterit cognosci, utrum transitus per omnes pontes semel institui queat an non, ope huius regulae:

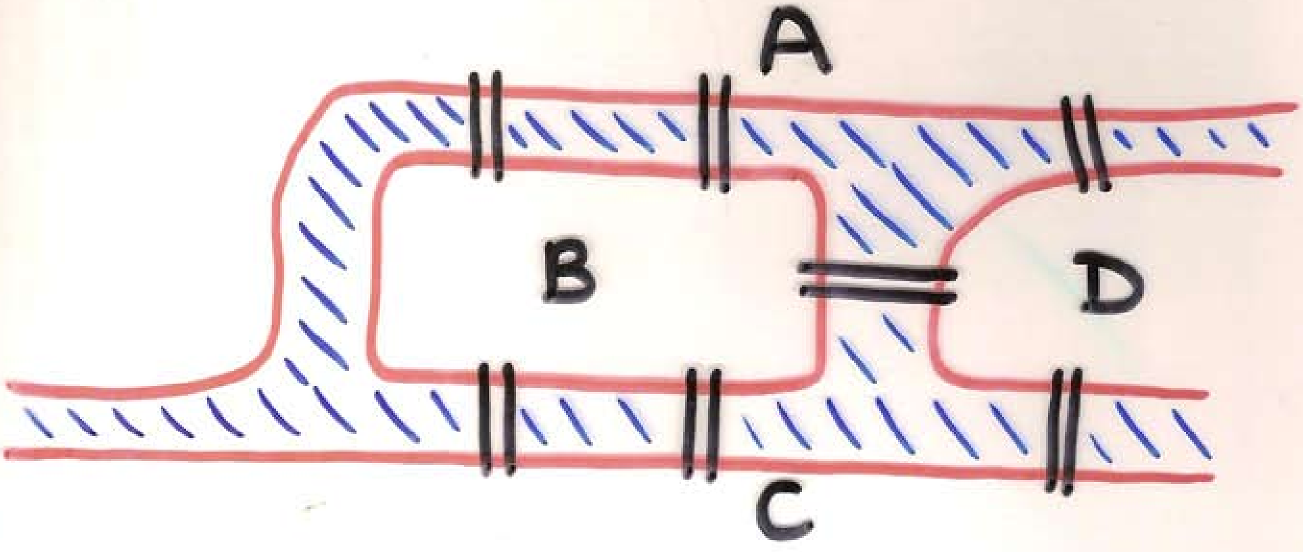
Si fuerint plures duabus regiones, ad quas ducentium pontium numerus est impar, tum certo affirmari potest talem transitum non dari.

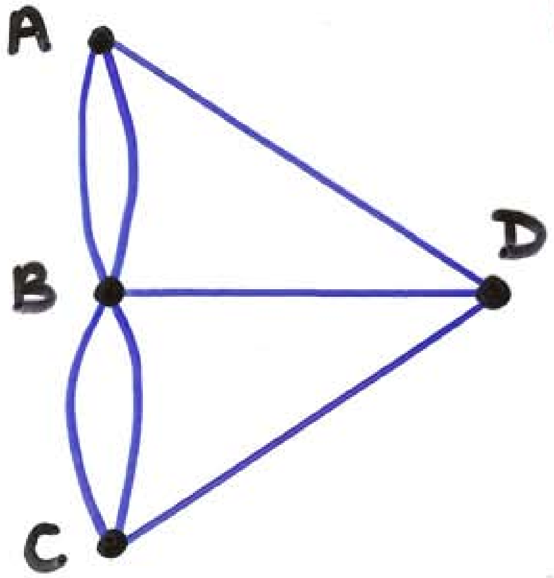
Si autem ad duas tantum regiones ducentium pontium numerus est impar, tunc transitus fieri poterit, si modo cursus in altera harum regionum incipiatur.

Si denique nulla omnino fuerit regio, ad quam pontes numero impares conducant, tum transitus desiderato modo institui poterit, in quacunque regione ambulandi initium ponatur.

Hac igitur data regula problemati proposito plenissime satisfit.

The BRIDGES of KÖNIGSBERG

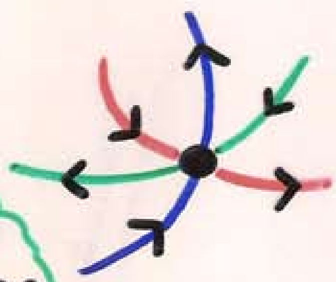




solved by EULER in 1736

whenever you enter a vertex,

you must be able to leave it

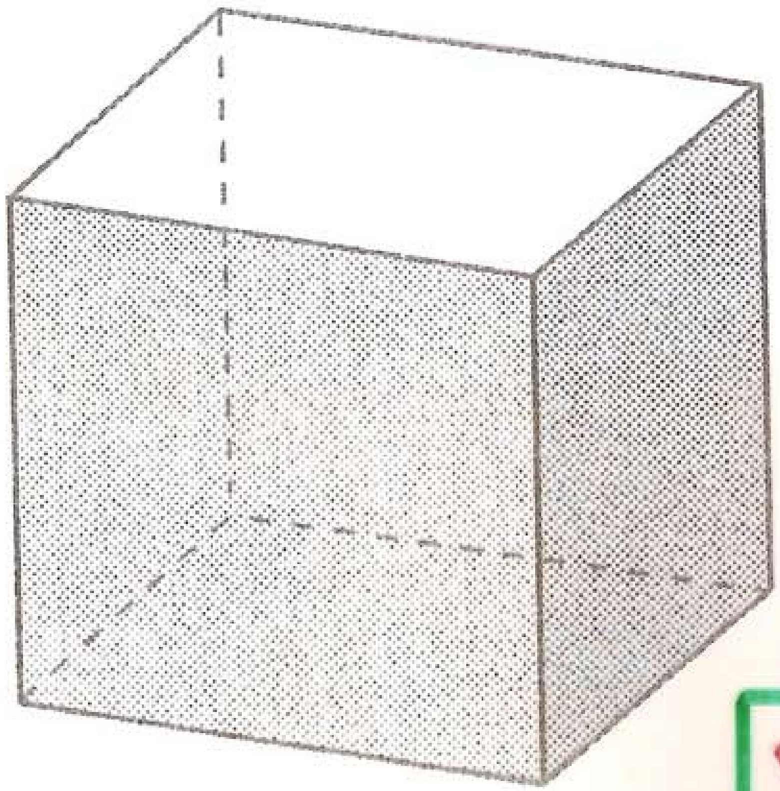


So 0 or 2 odd degrees. even degree

Königsberg has degrees 3, 3, 3, 5,

and so is impossible.

Euler's Polyhedron Formula



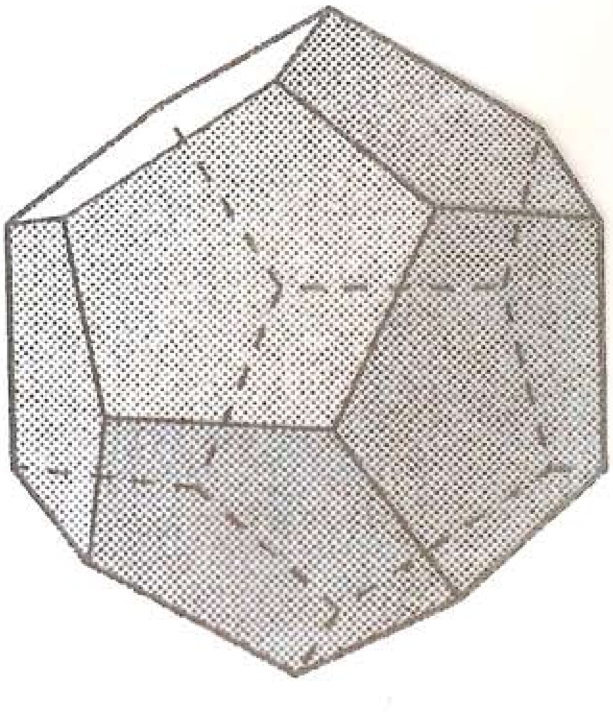
8 vertices

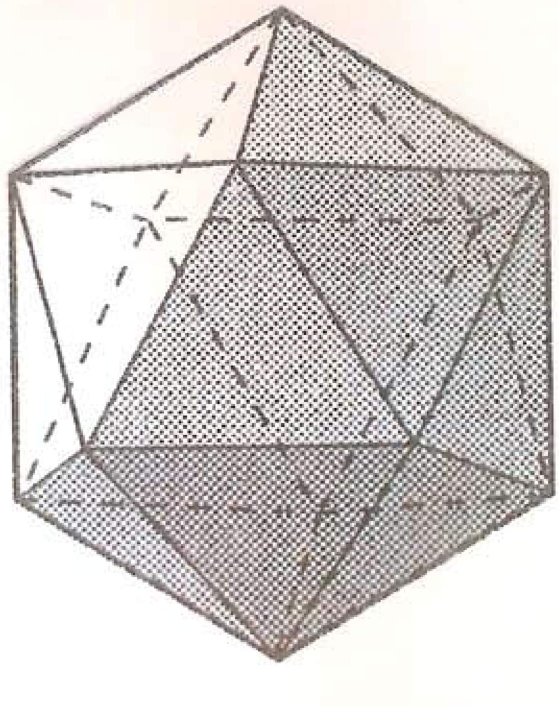
6 faces

12 edges

$8 + 6 = 12 + 2$

$$\boxed{V + F = E + 2}$$





$20 + 12 = 30 + 2$

$12 + 20 = 30 + 2$

Euler on Polyhedra

Berlin d. 14. November 1750.

Neulich kam mir in Sinn die allgemeinen Eigenschaften der Cörper, welche hedris planis eingeschlossen sind, zu bestimmen, weil kein Zweifel ist, dass sich in denselben nicht eben dergleichen allgemeine Eigenschaften finden sollten, als in den figuris planis rectilineis, deren Eigenschaften darin bestehen, dass 1. in einer jeglichen figura plana der numerus laterum dem numero angulorum gleich ist, hernach 2. dass die summa angulorum omnium gleich ist bis tot rectis quot sunt latera, demtis quatuor. Wie aber in den figuris planis nur latera und anguli zu betrachten vorkommen, so müssen bei den Cörpern mehr Stücke in Betrachtung gezogen werden, als

I. die hedrae, deren Anzahl sey $= H$;

II. die anguli solidi, deren Anzahl sey $= S$;

III. die Fügungen, wo zwey hedrae secundum latera zusammenkommen, so ich aus Mangel eines recipirten Worts, *acies* nenne, deren Anzahl sey $= A$;

IV. die latera singularum hedrarum, quorum omnium simul sumtorum numerus sit $= L$;

V. die anguli plani singularum hedrarum, quorum omnium numerus sit $= P$.

1. Bei diesen fünf Stücken ist nun erstlich klar, dass $P = L$, weil in allen hedris der numerus angulorum $=$ numero laterum.

2. Ist auch immer $A = \frac{1}{2} L$, oder $A = \frac{1}{2} P$, weil immer zwey latera diversarum hedrarum zusammenkommen, um eine aciem zu formiren.

3. Dahero ist der numerus laterum seu angulorum planorum omnium hedrarum corpus includentium allzeit par.

4. Semper est vel $L = 3H$ vel $L > 3H$
5. Semper est vel $P = 3S$ vel $P > 3S$

$\left.\begin{array}{l} \\ \end{array}\right\}$ at est $P = L$.

Dieses ist klar, weil keine hedra aus weniger als drey Seiten, und kein angulus solidus aus weniger als drey angulis planis bestehen kann. Folgende Proposition aber kann ich nicht recht rigorose demonstriren:

6. In omni solido hedris planis incluso aggregatum ex numero hedrarum et numero angulorum solidorum binario superat numerum acierum, seu est $H + S = A + 2$, seu $H + S = \frac{1}{2} L + 2 = \frac{1}{2} P + 2$.

Euler's Polyhedron Letter (1750)

2. Ist auch immer $A = \frac{1}{2}L$ oder $A = \frac{1}{2}P$, weil immer zwey Latera oder zwey anguli plani zusammen eine aciem formiren.

3. Daher ist der numerus laterum seu angulorum planorum omnium hedrarum corpus includentium allzeit par.

4. Semper est vel $L = 3H$ vel $L > 3H$ } et est $P = L$.

5. Semper est vel $P = 3S$ vel $P > 3S$ }

Dieses ist klar, weil keine Hedra aus weniger als 3 Seiten, und kein angulus solidus aus weniger als 3 angulis planis bestehen kann.

Folgende proposition aber kann ich noch nicht recht rigorose demonstriren:

6. In omni solido hedris planis incluso aggregatum ex numero hedrarum et numero angulorum solidorum binario superat numerum acierum, seu est $H + S = A + 2$ seu $H + S = \frac{1}{2}L + 2 = \frac{1}{2}P + 2$.

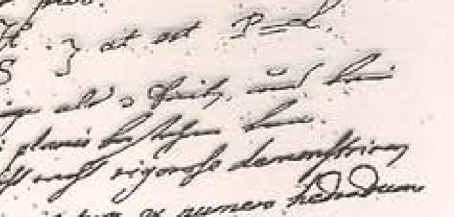
7. Impossibile est ut sit $A + 6 > 3H$ vel $A + 6 > 3S$

8. Impossibile est ut sit $H + 4 > 2S$ vel $S + 4 > 2H$

9. Nullum formari potest solidum cujus omnes hedrae sint 6 pluriumve laterum, nec cujus omnes anguli solidi ex sex pluribusve angulis planis sint conflati.

10. Summa omnium angulorum planorum, qui in ambitu solidi existunt, aequatur quot sunt unitates in $4A - 4H$ tot angulis rectis.

11. Summa omnium angulorum planorum aequatur quater tot angulis rectis, quot sunt anguli solidi demtis octo, seu est $= 4S - 8$ rectis.

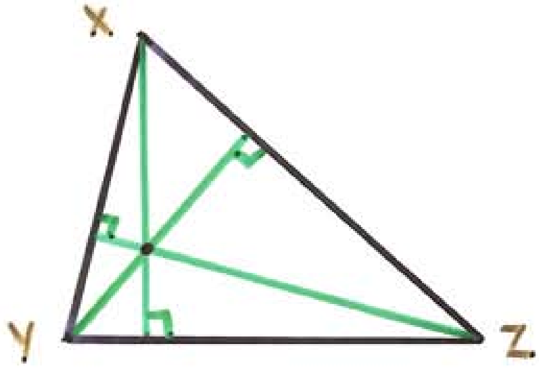


Exempli gratia sit prisma triangulare ubi est

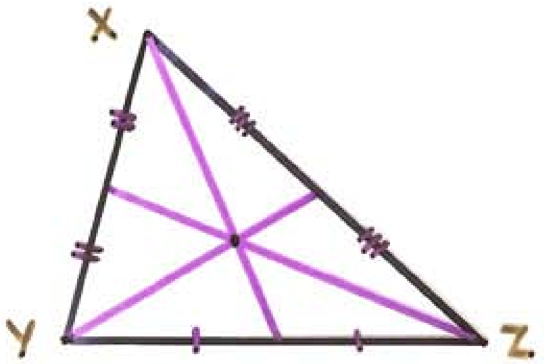
1. numerus hedrarum $H = 5$
2. numerus ang. sol. $S = 6$
3. numerus acierum (ab, ac, bc, ad, be, cf, de, df, ef) $A = 9$
4. numerus laterum et angulorum planorum $L = P = 18$. Includitur enim corpus duobus triangulis et tribus quadrilateris, unde $L = P = 2.3 + 3.4 = 18$.

Hier ist nun der Theor. 6: $H + S\ (11) = A + 2\ (11)$, und dann summa omnium angulorum planorum (weil der beyden $\Delta = 4$ rectis, und der dreyen $\square = 12$ rectis) erit $= 16$ rectis $= 4(A - H) = 4S - 8$ rectis.

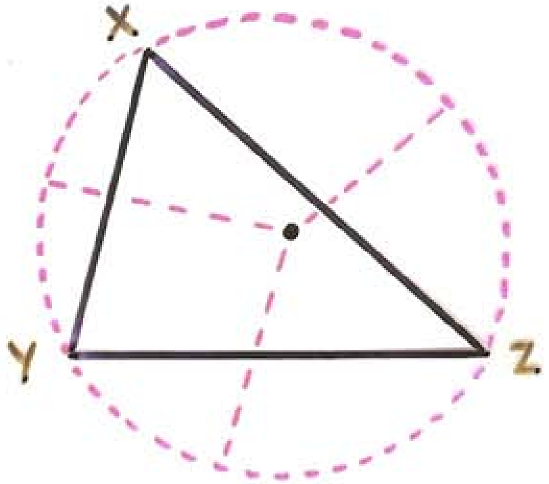
Geometry of a Triangle



orthocentre O



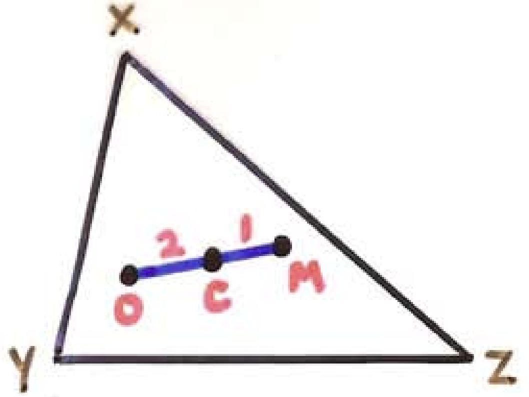
centroid C



circumcentre M

Euler line

(OC = 2CM)



Fermat Prime Numbers

Fermat : $F_n = 2^{2^n} + 1$ is prime (?)

$F_0 = 2^1 + 1 = 3$, $F_1 = 2^2 + 1 = 5$,

$F_2 = 2^4 + 1 = 17$, $F_3 = 2^8 + 1 = 257$,

$F_4 = 2^{16} + 1 = 65,537$

$F_5 = 4,294,967,297$? ← $2^{32} + 1$

Euler : F_5 is divisible by 641

Proof : $641 = 5^4 + 2^4 = (5 \times 2^7) + 1$.

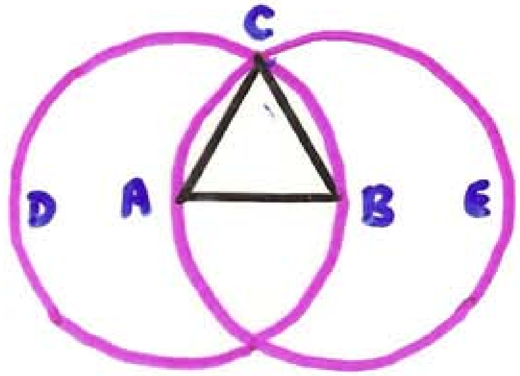
So $2^{32} + 1 = 2^{28}(5^4 + 2^4) - (5.2^7)^4 + 1$

$= 2^{28}.641 - (641-1)^4 + 1$

$= 641 \times K$, so $641 \mid 2^{32} + 1$.

Euclid Book I, Prop. I

On a given straight line
to construct an equilateral triangle



Let AB be the line.

With centre A and
distance AB, draw the
circle BCD. [Post. 3]

With centre B and distance BA,
draw the circle ACE. [Post. 3]

Join AC and BC. [Post. 1]

Then the triangle ABC is equilateral.

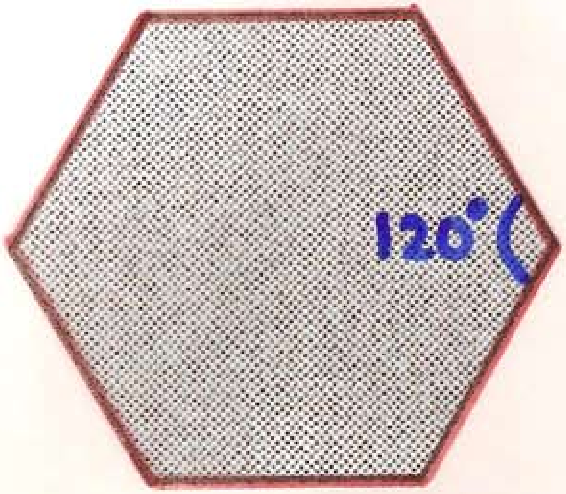
Proof . . .

Regular Polygons

triangle

square

pentagon



hexagon

heptagon

octagon

Constructing Polygons

Gauss: A regular polygon with n sides can be constructed with straight-edge and compasses if and only if n has the form

$$n = 2^k \times p_1 \times p_2 \times \cdots .$$

distinct Fermat primes

3, 4, 5, 6, 8, 10, 12, 15, 16,

17, 20, 24, 30, 32, 34, 40, 48,

51, 60, 64, 68, 80, 85, 96,

102, ..., 257, ..., 65537, ...