

The 18th Century :

Crossing Bridges

Robin Wilson

The Open University

and Gresham College

PHILOSOPHIAE  
NATURALIS  
PRINCIPIA  
MATHEMATICA.

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Autore J S. NEWTON, *Trin. Coll. Cantab. Soc. Mathefeos*  
Professore Lucasiano, & Societatis Regalis Sodali.

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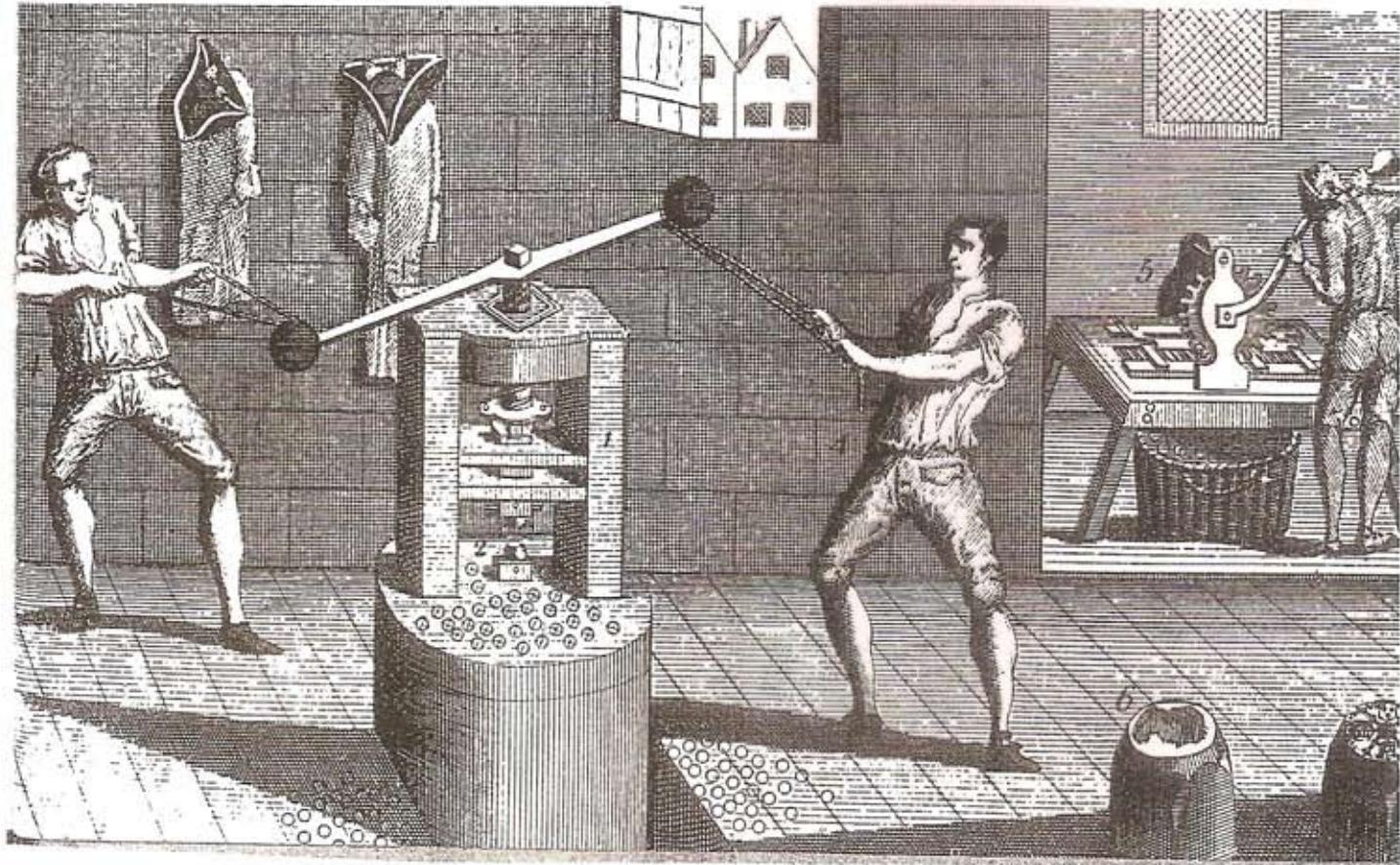
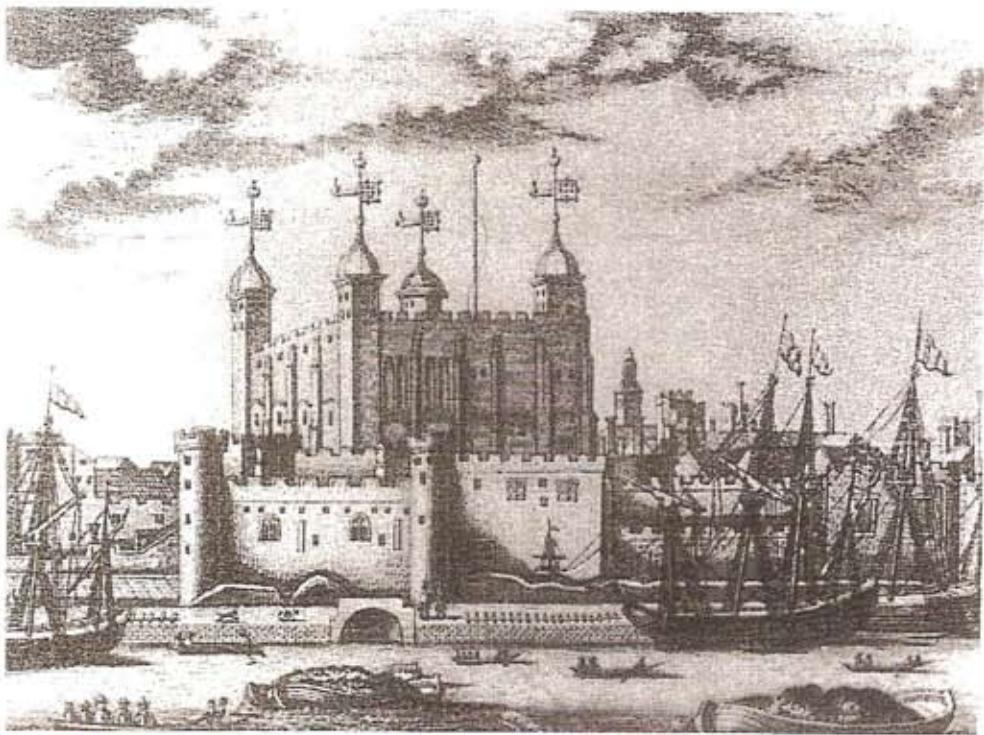
IMPRIMATUR.  
S. P E P Y S, Reg. Soc. PRÆSES.  
Julii 5. 1686.

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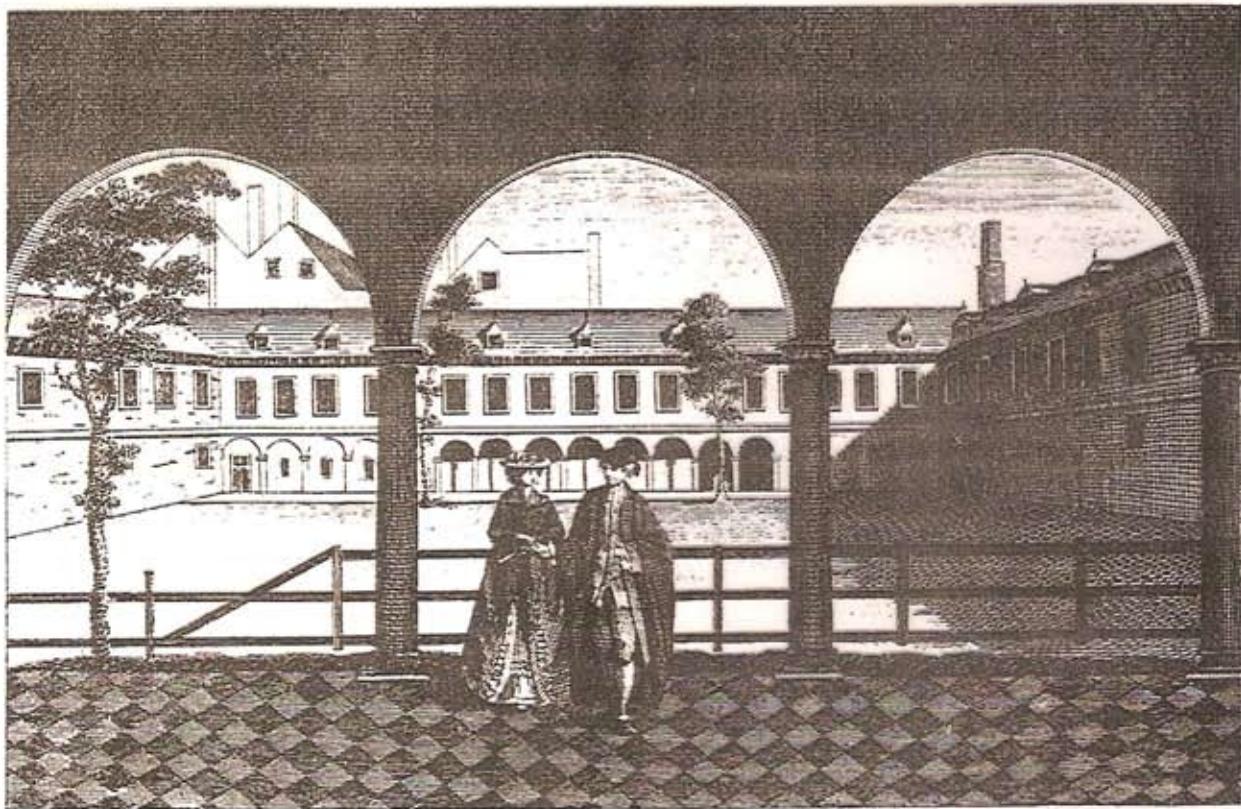
LONDINI,

Jussu Societatis Regiae ac Typis Josephi Streater. Prostat apud  
plures Bibliopolias. Anno MDCLXXXVII.

# The Royal Mint

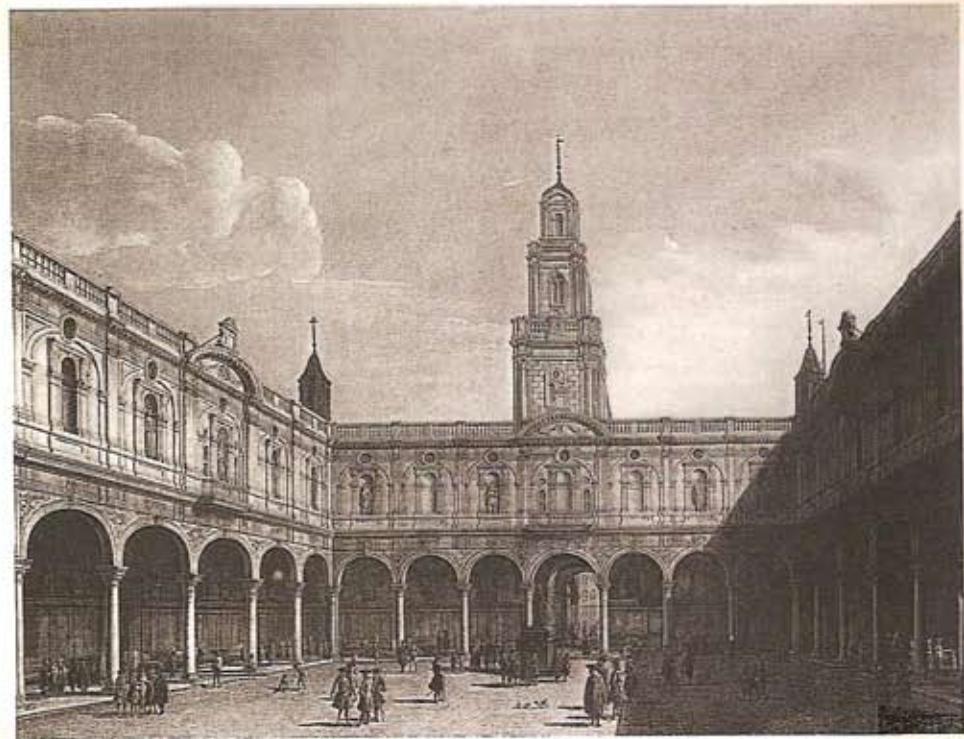
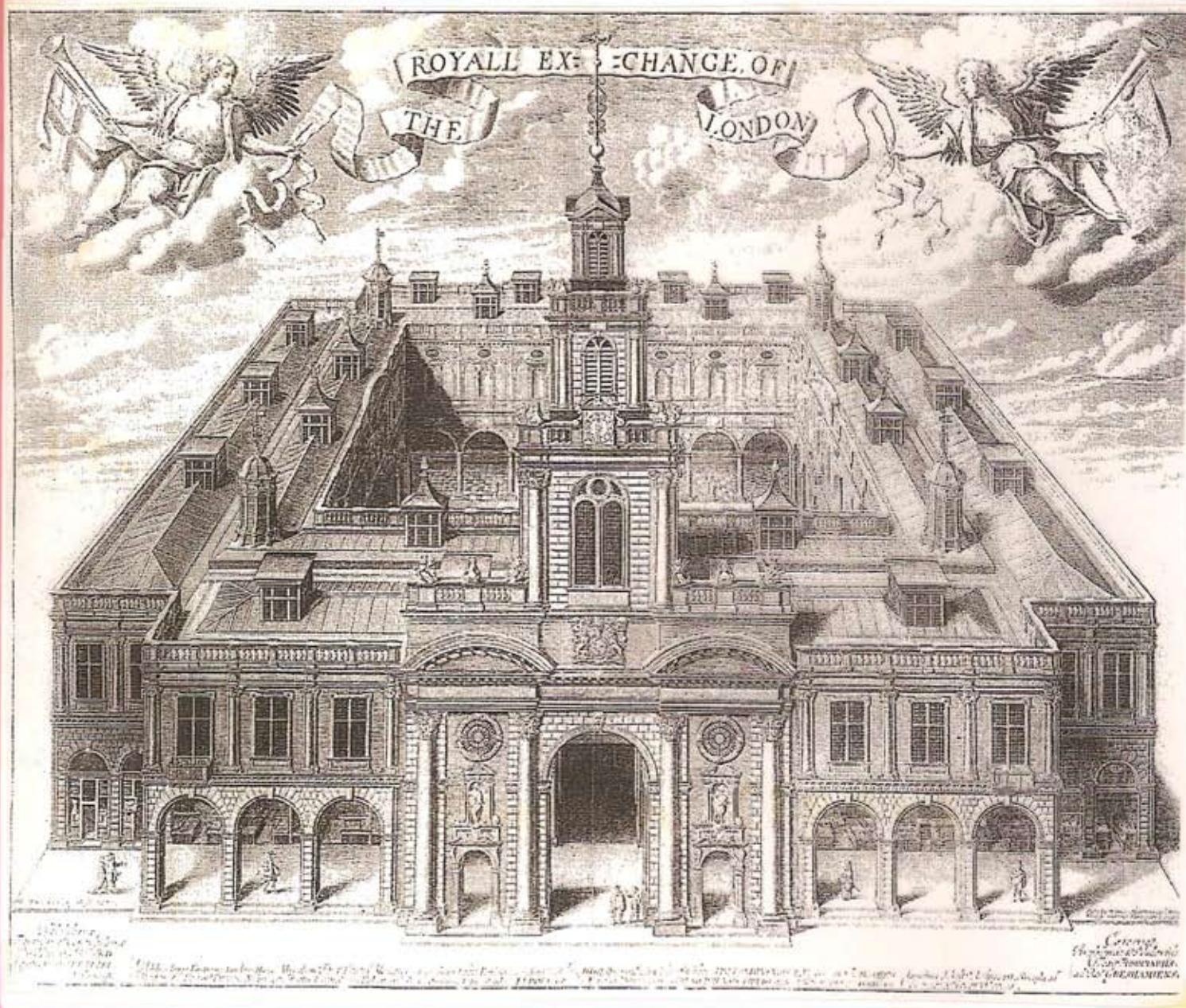


# Gresham College in the 18th Century



*View of GRESHAM COLLEGE as it appeared before it was taken down*

# The Second Royal Exchange



# Newton at the Royal Society



**OPTICKS:**

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OR, A  
TREATISE  
OF THE  
REFLEXIONS, REFRACTIONS,  
INFLEXIONS and COLOURS  
OF  
**L I G H T.**

ALSO  
Two TREATISES  
OF THE  
SPECIES and MAGNITUDE  
OF  
**Curvilinear Figures.**

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LONDON,  
Printed for SAM. SMITH, and BENJ. WALFORD,  
Printers to the Royal Society, at the Prince's Arms in  
St. Paul's Church-yard. MDCCIV.



Gottfried Wilhelm Leibniz  
(1646 - 1716)

# Britain vs. the Continent

## Newton

Halley

Taylor

MacLaurin

Whiston

Saunders

Bradley

...

## Leibniz

The Bernoullis

Euler

d'Alembert

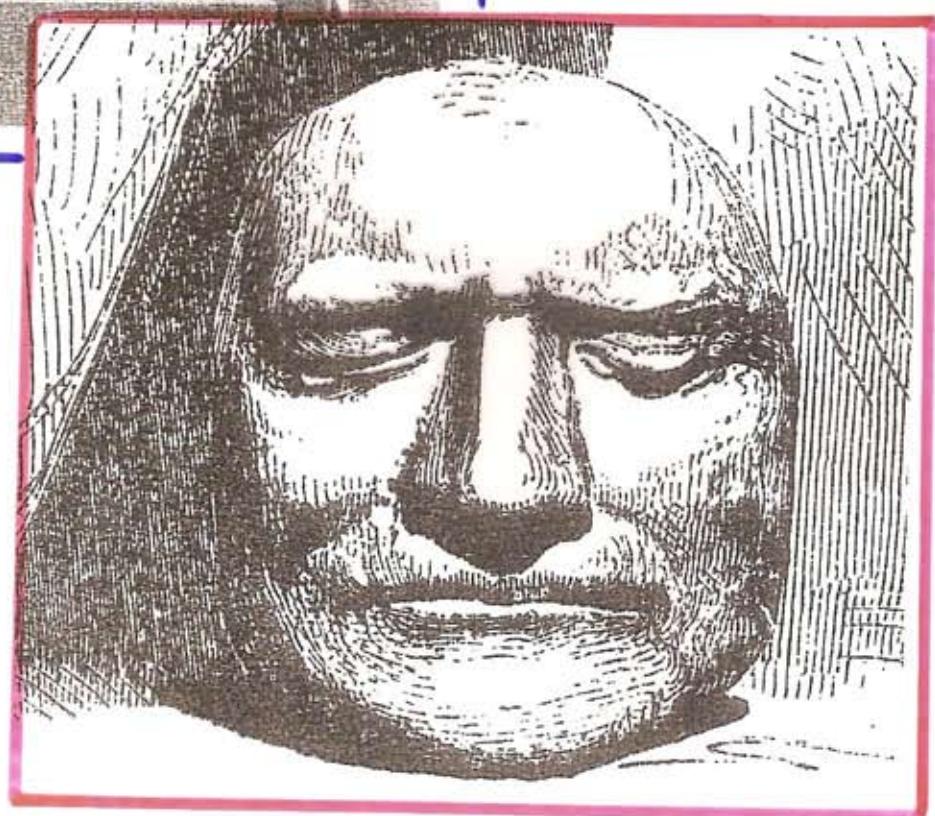
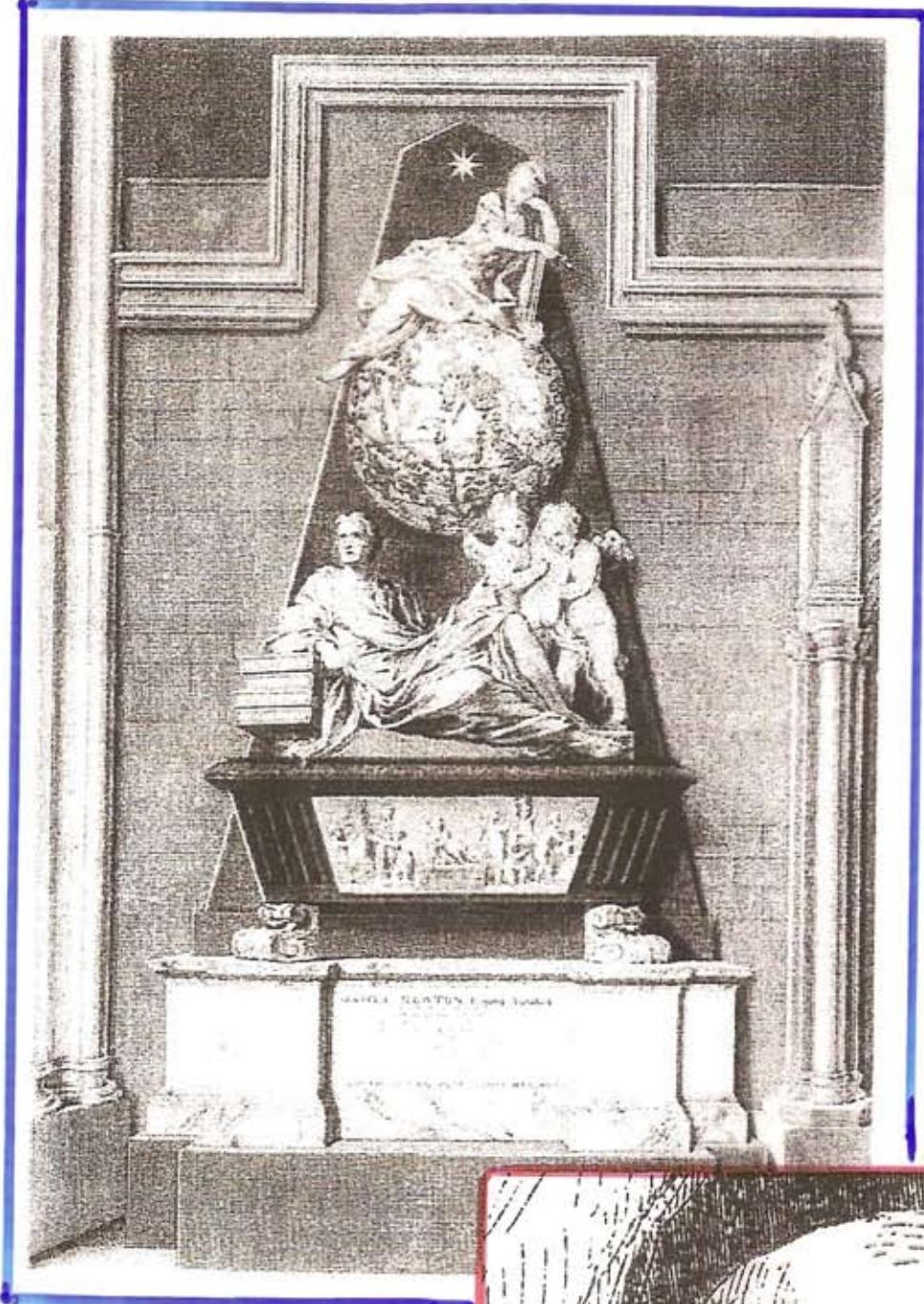
Lagrange

Legendre

Laplace

...

# The Death of Isaac Newton, 1727



# Newtonian Philosophy

## EDMOND HALLEY (1656-1742)

Savilian Professor of Geometry

- while an undergraduate, sailed to St Helena to observe southern stars
- FRS at age 22
- persuaded Newton to write 'Principia'
- sea captain ( $52^{\circ}$  south)
- edition of Apollonius' 'Conics'
- Astronomer Royal, 1720-1742
- predicted return of 'Halley's comet'

Newtonian philosophy advanced in the

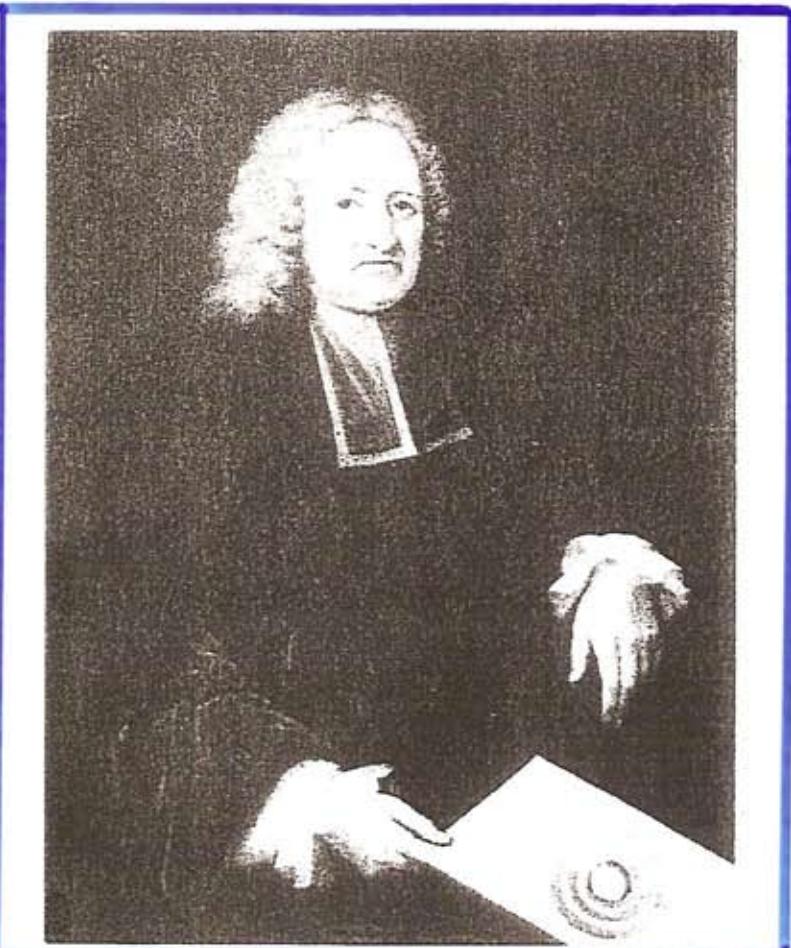
## OLD ASHMOLEAN (1683)

in the lectures by the Savilian professors

DAVID GREGORY, JAMES BRADLEY

Radcliffe Observatory built

# Edmond Halley (1656-1742)



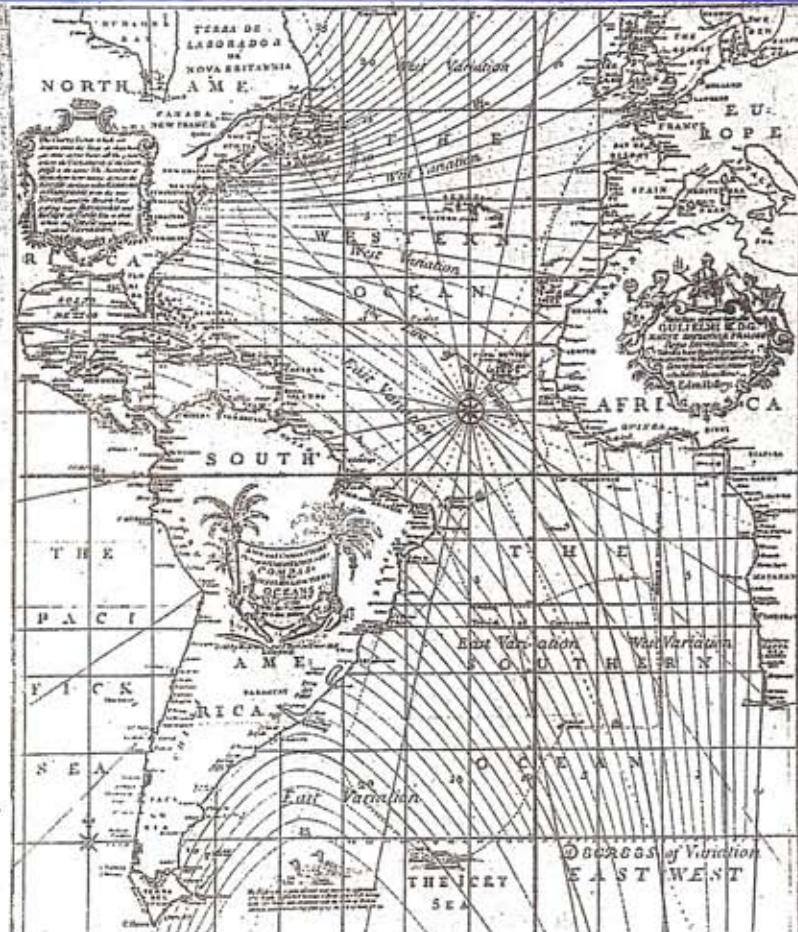
### Description

U S E S  
of a Bear and Compass  
S E A - C H A R T  
of the Western and Southern  
O C E A N,  
Showing the Variations of the  
C O M P A S S.

[11] [E.] Production of the Glass  
[2] which a company called  
Liberty's has from its premises  
in New-York, ought to be  
done by the Legislature, as  
soon as and sufficient  
time will be given for it to  
be disposed of, that is,  
such as to leave Glass in  
the hands of the people  
as little as possible, or at most in  
so far as may be necessary  
therefore. It shall only seize  
the Liberty to offer it to the  
people at public sale, as  
and from time to time, the  
Chairman of the Committee  
on Journals, so often as the  
Committee of Revision  
and of Finance, or, as he  
may direct, the Committee  
of the General Assembly  
desire it, shall be sent to  
the Legislature, accompanied  
with a bill to appropriate  
what has been so seized. Now, is the  
Chair-Liber's about to do what  
she has said?—I am sorry to say  
that I have no knowledge of the  
Proceedings of the Assembly, though  
as Consy [which was then defiled  
mentioning on what day it had been in  
the hands of the Legislature]—I am  
not very greatly surprised at the  
Patriot's saying on the Year of our  
Lord 1776.

That this may be the older one,  
indeed, with South America. As  
above the *Puntius* is 1 and 1/2  
inches at Madras (11 feet), so the  
native pair will be at Cape Town as  
Hypophthalmus and will be in  
the Benthos from time to time. And this may suffice by way of  
digression.

As in the U.S. of old, duty will only be undertaken, probably by each as are equipped with the Ammunition Comptroller, radio, or correct the Courts of Justice, etc. For if the Veterans of the Comptroller is not allowed, all 2nd Lieutenants must be held responsible; and it is recommended General Wadsworth, as where the Major is not provided to direct the Veterans duty, the Sheriff will satisfy them him what allowances



In good roads for the Protection of the  
Country, and thereby help the  
Inhabitants.

But this Committee of the Council is not up to scratch; as is now being shown by a Special Order of the House of Commons, dated 2nd March, 1867, in the name of the Committee of the Ecclesiastical Commission, to the effect that, "in view of the Report of the Select Committee on Ecclesiastical Affairs, you will not be Varying the Statute of 1851, by giving a power to the Commissioners which shall fit in with the words or intendment of your Statute." The Report of the Select Committee on Ecclesiastical Affairs, which may perhaps give you some idea, shows, that the Testimony it contains, is that the Statute of 1851, in so far as it relates to the Church of Ireland, in Ireland, was intended to give a power to the Commissioners to make Laws without the King's Consent, in the same manner as the Duke of Wellington's Government did in the Disraeli Bill. Whether this Statute was intended by the Commissioners to give such a power, or whether it was given by the King, I do not know. But I do know, that when I was in Ireland, in 1851, I heard a good deal of talk about the Statute of 1851, at Limerick, in the South of Ireland, and I also heard a good deal to do, that the Statute of 1851 will be more or less amended, as the Commissioners will do in the Trial of the Bishop of Limerick, in the year 1867.

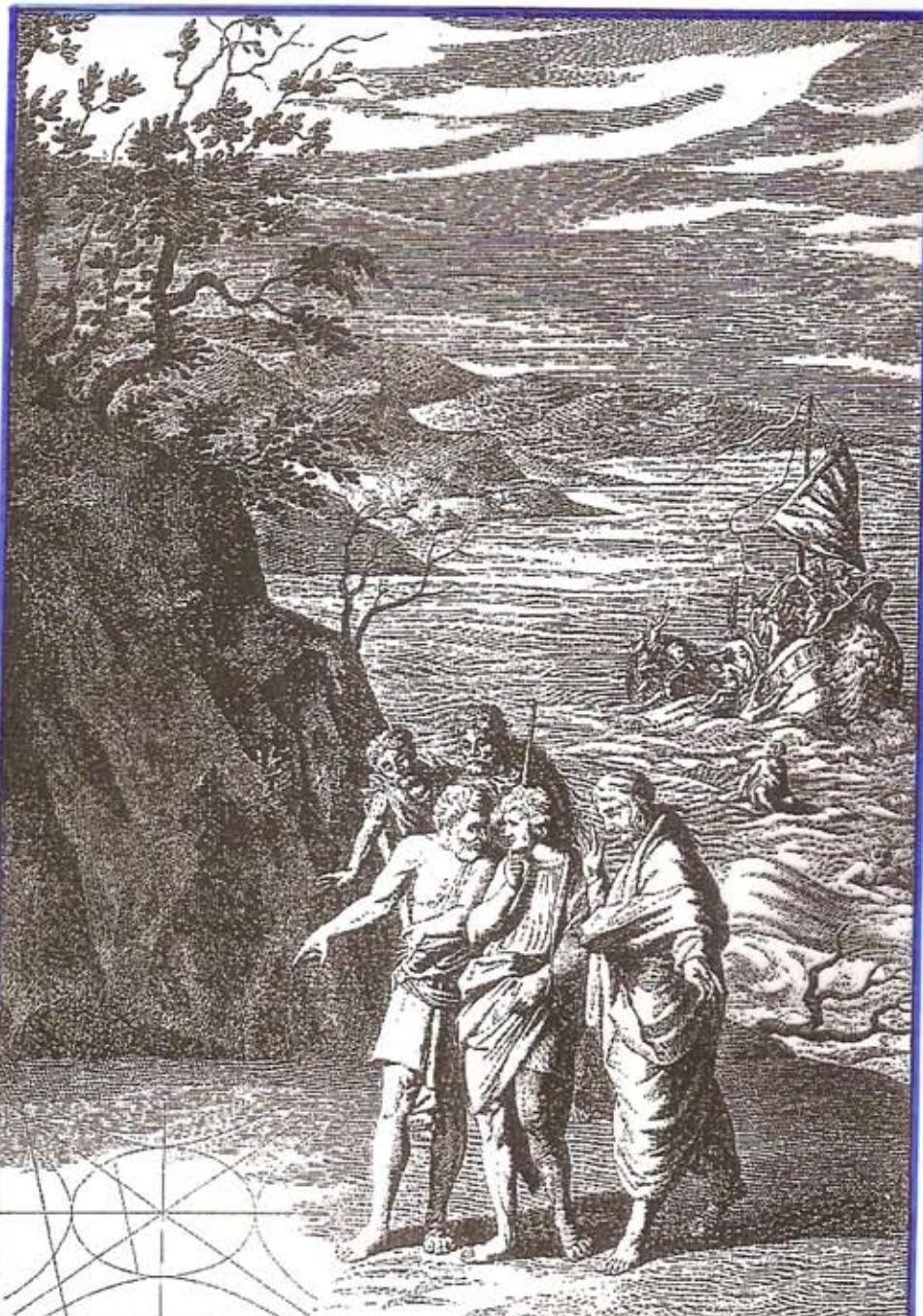
A slender 6 ft. 6 in. monkey called an *Uromyscus* is found in the low grounds of the Ganges valley, from the Ganges to the Brahmaputra, and in thick vegetation, as along Ganga River. It is giving a very good illustration of the Diffusion of the Land to Man over India; for down the Volga it shows a Degree or such poor Degree of Longevity as may be found in the Uromyscus. But the Uromyscus shows a long history and the North American *Cavia* living nearly East and West, cannot be satisfactorily for this Pug-  
pall.

This Chart, at 1/400, was made by G. L. Johnson of the U. S. Survey, but it must be used with care, as it is a sketch of the 1st Order of the U. S. Survey. The Variations show it very well, which will make it necessary to use one or two whole Sections to project it over sufficient area to determine chart C. Since Ellipticity, the 1st Order Variations, are made in the form of a Section, a Degree is about 10 miles, so that the distance from one end of a Section to the other is about 10 miles. The 1st Order Variations are made in the form of a Section, so that the distance from one end of a Section to the other is about 10 miles. The 1st Order Variations are made in the form of a Section, so that the distance from one end of a Section to the other is about 10 miles.

I shall need to be as ready about it, but let it remain until all the following Ministers are defined so that their Addressees and Interpreters, research the possibility of additional Work. And if by probability of Otherwise, it be found, in any Part defective, the Means of

The CHART is made and sold by  
William Morris, and George Page  
of Boston.

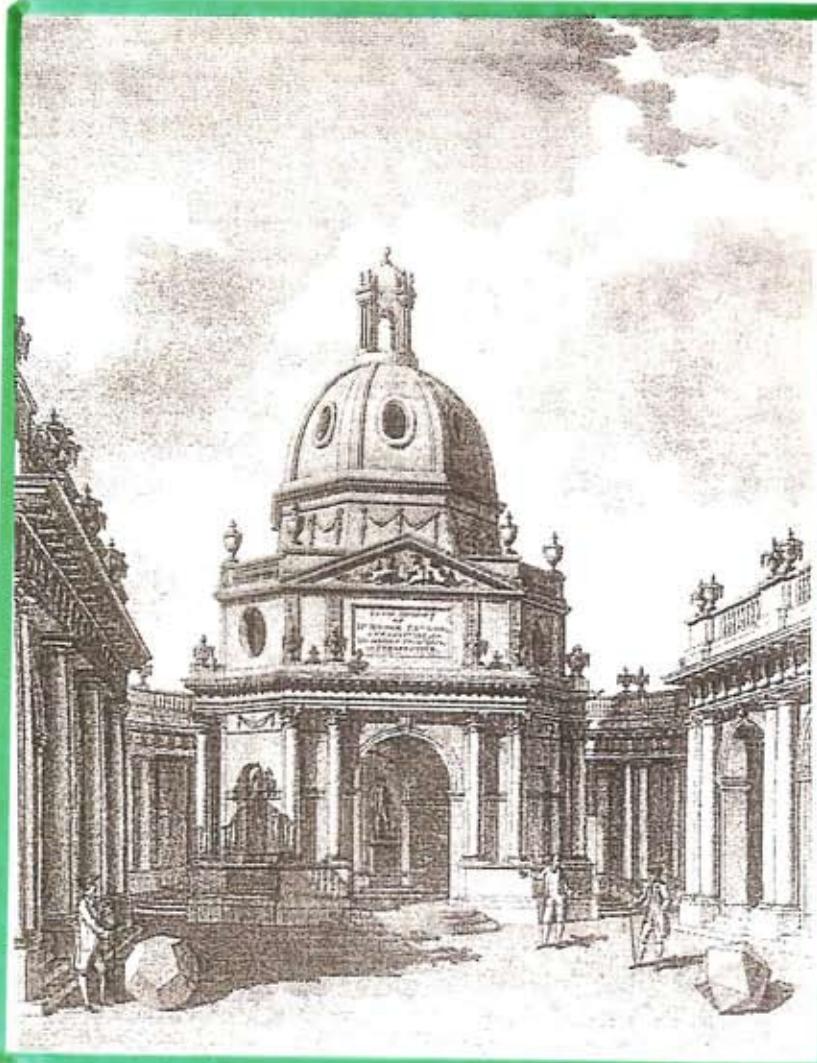
# Halley's Edition of Apollonius's 'Conics' (1710)



Aristippus Philosophus Socraticus, naufragio cum ejectus ad Rhodienium libus animadverissim Geometrica schemata descripta, exclamavisse ad comites ita dicitur. Bene speremus, Hominum enim vestigia video.

Vitruvii Architecti lib. 10. Pref.

# Brook Taylor (1685-1731)

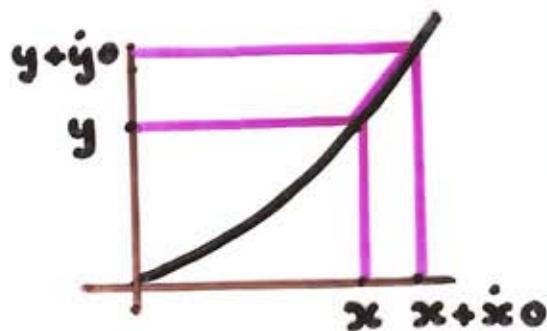


# Newton's calculus

Variables : changing with time  
- 'flowing'

Derivatives: based on velocity  
(tangents) - notation  $\dot{x}$ ,  $\dot{y}$

Example :  $y = x^2$



Substitute  $x + \dot{x}_0$  for  $x$   
 $y + \dot{y}_0$  for  $y$ :

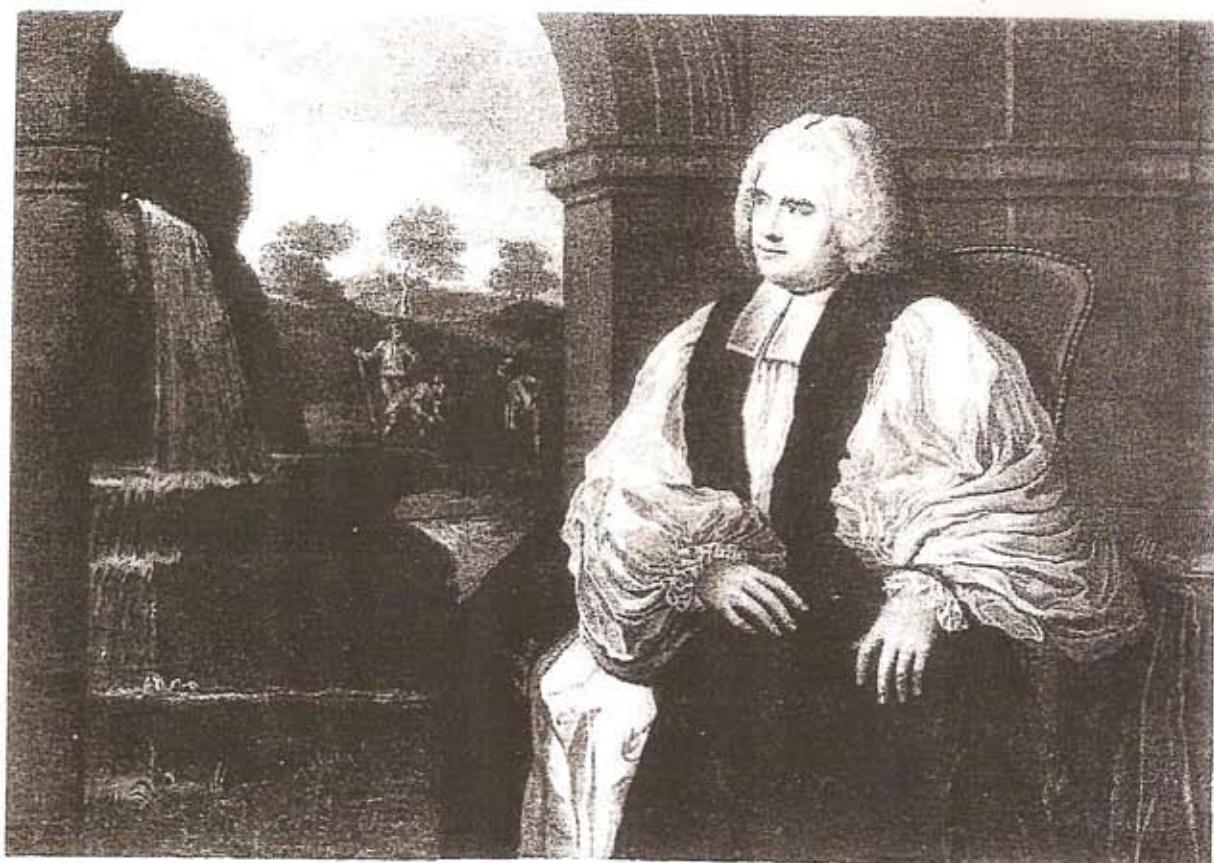
$$y + \dot{y}_0 = x^2 + 2x\dot{x}_0 + \dot{x}^2_0$$

Cancel  $o$ :  $\dot{y} = 2x\dot{x} + \dot{x}^2_0$

Ignore  $o$ :  $\dot{y} = 2x\dot{x}$ , or  $\dot{y}/\dot{x} = 2x$

Integrals : find anti-derivatives  
(areas) (fundamental theorem)

# Bishop Berkeley's 'The Analyst'



THE  
ANALYST;  
OR, A  
DISCOURSE

Addressed to an  
Infidel MATHEMATICIAN.

WHEREIN

It is examined whether the Object, Principles, and Inferences of the modern Analysis are more distinctly conceived, or more evidently deduced, than Religious Mysteries and Points of Faith.

By the AUTHOR of *The Minute Philosopher*.

The SECOND EDITION,

*First cast out the beam out of thine own Eye; and then  
jhalst thou see clearly to cast out the mote out of thy  
brother's Eye.*  
S. Matt. c. viii. v. 5.

L O N D O N:

Printed for J. and R. TONSON and S. DRAPER  
in the Strand.

M DCC LIV.

## **Bishop Berkeley's *The Analyst* (1734)**

The Analyst, or a discourse addressed to an infidel mathematician wherein it is examined whether the Object, Principles and inferences of the modern analysis are more distinctly conceived, or more evidently deduced, than Religious Mysteries and Points of Faith.

*He used these fluxions like the scaffold of a building, as things to be laid aside or got rid of.*

*What are these fluxions?*

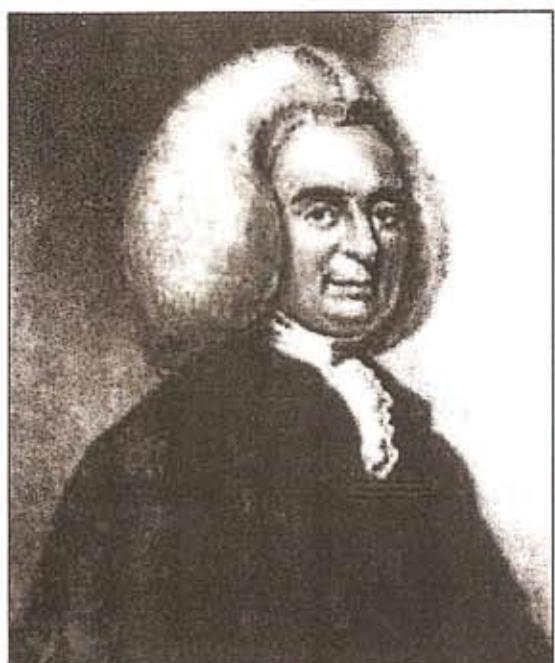
*The velocities of evanescent increments?*

*And what are these same evanescent increments?*

*They are neither finite quantities, nor quantities infinitely small, nor yet nothing.*

*May we not call them the Ghosts of departed quantities?*

# Colin MacLaurin (1698-1746)



# Principia Mathematica (1687)

## Laws of Motion

### Book I : The Motion of Bodies

Inverse-square law of gravity

Kepler's laws

### Book II: Motion in Resisting Media

Wave motion for light and sound

Vortices : Descartes' theory

### Book III : The System of the World

Orbits of comets

Motion of the moon

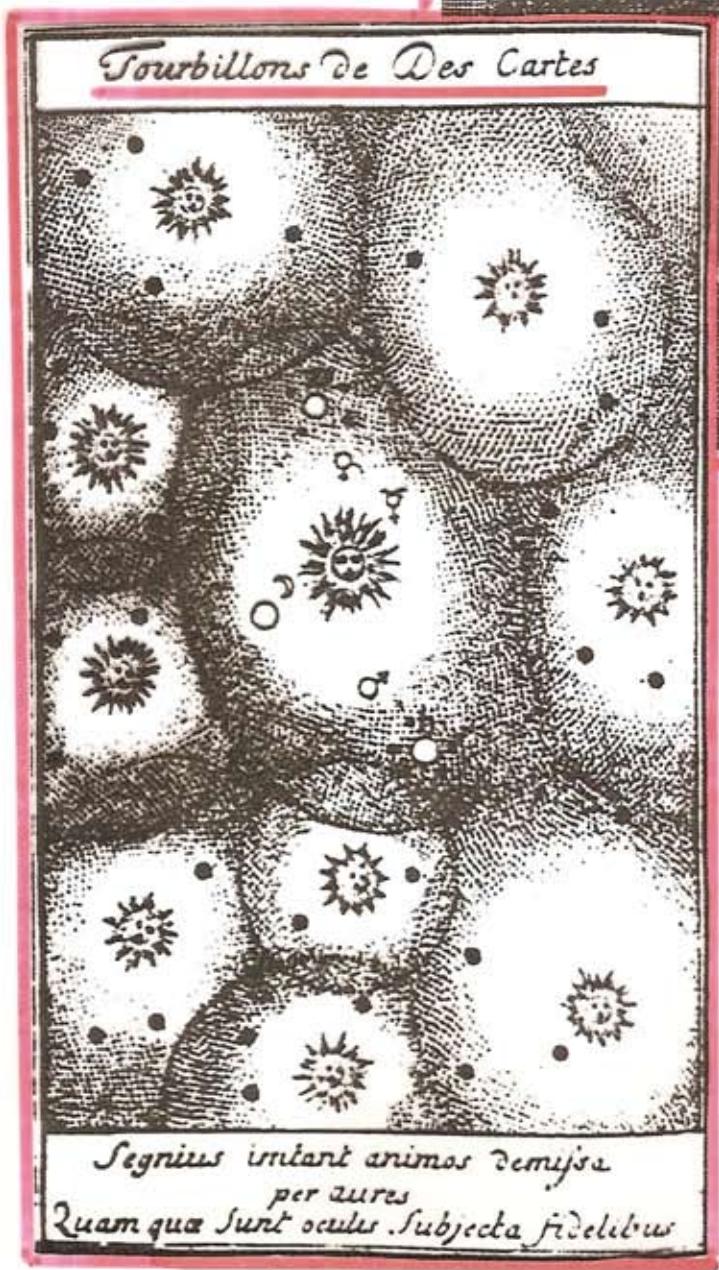
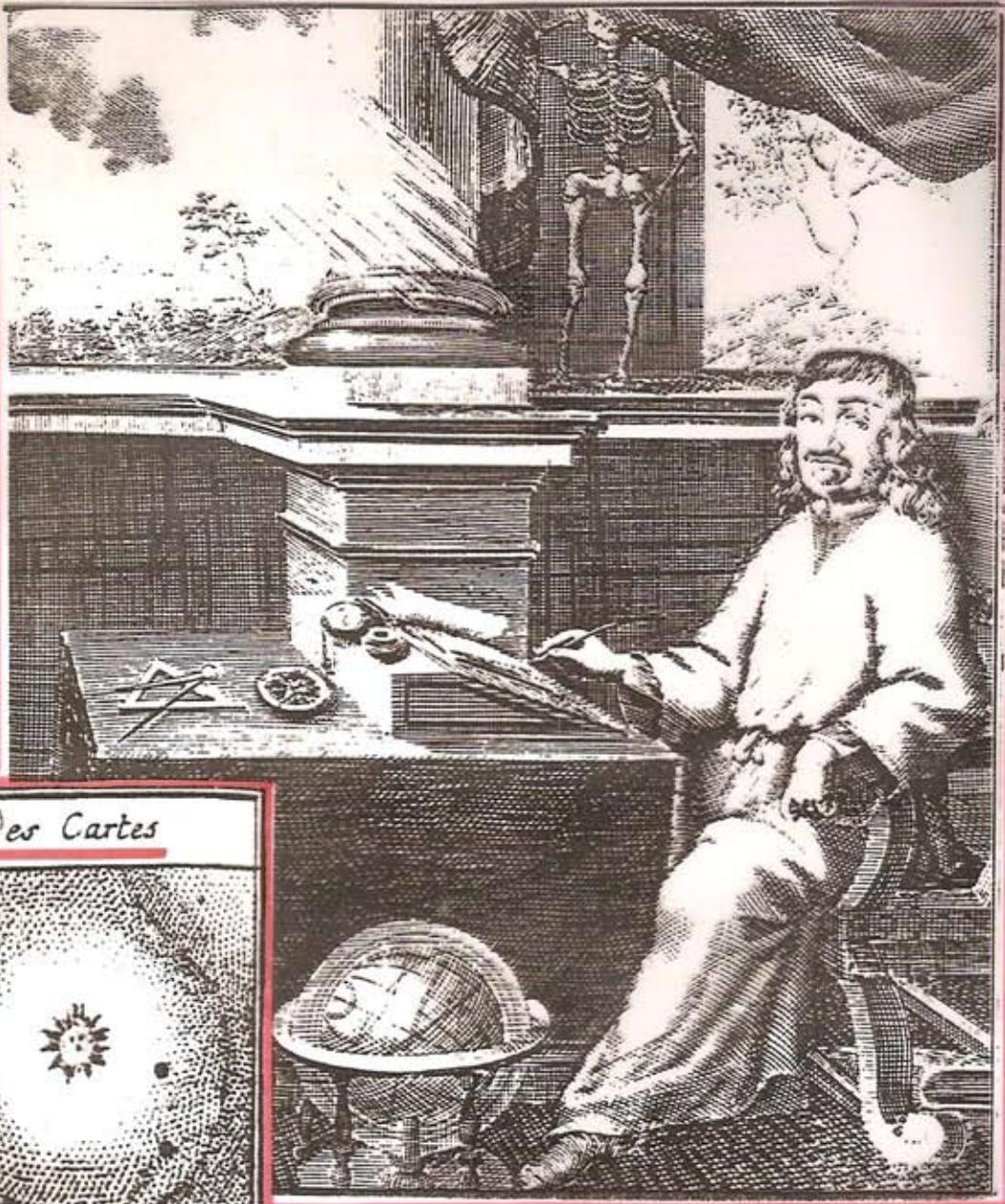
Theory of tides

Flattening of the Earth at the poles

Precession of the equinoxes

... - 100% of energy

# René Descartes



Theory of  
vortices

*Segnius imitari animos demissa  
per aures  
Quam quæ sunt oculis. Subjecta fidelibus*

## The Shape of the Earth

### Prop. XVIII, Theorem XVI :

That the axes of the planets are less than the diameters drawn perpendicular to the axes.

... if our earth was not higher about the equator than at the poles, the sea would subside about the poles, and, rising towards the equator, would lay all things there under water.

... the earth will be higher at the equator than at the poles by 85472 feet, or  $17\frac{1}{10}$  miles. And its height at the equator will be about 19,658,600 feet, and at the poles 19,573,000 feet.

## Success of the Principia

- Immediate in Britain and the Netherlands  
*'the greatest Discovery in Nature that ever was since the World's Creation'.*

- France : de l'Hopital, Voltaire, Mme du Chatelet
- Maupertuis:

The shape of the Earth :

geodetic missions to Lapland and Peru to measure pendulum swings;

Newton correct: Earth is flatter at the poles.



## ***What shape is the Earth?***

In 1672, Mr Richer, in a voyage to Cayenna, near the equator, found that the pendulum of this clock no longer made its vibrations so frequently as in the latitude of Paris. In consequence of this it was discovered that, whereas the gravity of bodies is by so much the less powerful, as these bodies are farther removed from the centre of the earth, the region of the equator must absolutely be much more elevated than that of France; and so must be further removed from the centre; and, therefore, that the earth could not be a sphere.

Voltaire

It is evident in general that Sir Isaac Newton's Figure of a flat Spheroid, and Mr Cassini's of a long one, will give very different Distances of Places that have the same Longitude and Latitude. ... In a course of 100 Degrees Longitude, there might be a Mistake of more than two Degrees, if sailing really upon Sir Isaac Newton's Earth one should imagine himself to be upon Mr Cassini's.

And how many Ships have perished by smaller mistakes?

Pierre Maupertuis

# Maupertuis's Mission to Lapland



## Letters between Euler and Clairaut

### Clairaut to Euler : 11 Sept 1747

Pending something better, the law for all nature should be as  $1/\text{dist}^2 + \text{a small function of distances}$  detectable for the moon and almost zero for great distances ...

### Euler to Clairaut : 30 Sept 1747

- disagreed - Mercury doesn't fit the explanation

### Clairaut to Euler : 7 December 1747

Mercury can be dealt with by adding  $l^2/357d^2$  ( $d = \text{distance between earth and moon}$ ) - but this doesn't seem right - and  $1/d^4$  is too strong

### Clairaut - general announcement : 17 May 1749

Having considered anew from a new viewpoint, I have been led to reconcile observations with a force  $\propto$  the inverse square of the distance.

# The Motion of the Moon



Euler  
Clairaut  
Laplace

226 MÉCHANIQUE ANALITIQUE.

9. De cette manière la formule générale du mouvement  $\ddot{r} + \dot{\alpha} = 0$  (art. 1) sera transformée en celle-ci,  
 $\ddot{r} \dot{\varphi} + \dot{r} \dot{\vartheta} + \dot{\vartheta} \dot{\varphi} + \ddot{\vartheta} \varphi + \&c = 0$ ,

dans laquelle on aura

$$\begin{aligned} x &= d \cdot \frac{\partial T}{\partial \dot{x}} - \frac{\partial T}{\partial \dot{t}} + \frac{\partial V}{\partial t} \\ y &= d \cdot \frac{\partial T}{\partial \dot{y}} - \frac{\partial T}{\partial \dot{\varphi}} + \frac{\partial V}{\partial \varphi} \\ z &= d \cdot \frac{\partial T}{\partial \dot{z}} - \frac{\partial T}{\partial \dot{\vartheta}} + \frac{\partial V}{\partial \vartheta} \\ &\&c, \end{aligned}$$

en supposant

$$T = S \left( \frac{dx^2 + dy^2 + dz^2}{dt^2} \right) m, V = S n m,$$

$$\& dn = P dp + Q dq + R dr + \&c.$$

Si donc dans le choix des nouvelles variables  $\xi, \eta, \varphi, \&c$ , on a eu égard aux équations de condition données par la nature du système proposé, ensorte que ces variables soient maintenant tout-à-fait indépendantes les unes des autres, & que par conséquent leurs variations  $\dot{\xi}, \dot{\eta}, \dot{\varphi}, \&c$ , demeurent absolument indéterminées, on aura sur le champ les équations particulières  $\ddot{\xi} = 0, \ddot{\eta} = 0, \ddot{\varphi} = 0, \&c$ , lesquelles serviront à déterminer le mouvement du système; puisque ces équations sont en même nombre que les variables  $\xi, \eta, \varphi, \&c$ , d'où dépend la position du système à chaque instant.

Mais quoi qu'en puisse toujours ramener la question à cet état, puisqu'il ne s'agit que d'éliminer par les équations de condition, autant de variables qu'elles permettent de le faire, &c de prendre ensuite pour  $\xi, \eta, \varphi, \&c$ , les variables

CELESTIAL MECHANICS

BY THE

MARQUIS DE LA PLACE,

FORMER PROFESSOR OF MATHEMATICS IN THE COLLEGE OF ST. JAMES'S, AND MEMBER OF THE ACADEMY OF SCIENCES, OF THE SOCIETY OF FRIENDS OF THE MATHS OF NATURE, OF THE ROYAL ACADEMY OF SCIENCES AND LETTERS, OF THE ACADEMY OF MEDICINE OF PARIS, OF THE ROYAL ACADEMY OF SCIENCES AND LETTERS, OF THE ROYAL ACADEMY OF MEDICINE, PHYSICS, MORALS, ETC., MEMBER OF THE ACADEMY OF ARTS AND SCIENCES, ETC.

TRANSLATED, WITH A COMMENTARY,

NATHANIEL BOWDITCH, LL.D.

FORMER OF THE ROYAL INSTITUTE OF SURVEYORS, TELEGRAPHISTS, AND MARINERS; OF THE ASTRONOMICAL DIVISION OF LEARNED, OF THE INSTITUTIONAL DIVISION OF PHILOSOPHY, OF THE ASTRONOMICAL DIVISION OF LIFE AND PHYSICAL SCIENCES, ETC.

VOLUME I.

CHELSEA PUBLISHING COMPANY, INC.  
BRONX, N.Y.



Poincaré



LES MÉTHODES NOUVELLES  
DE LA

MÉCANIQUE CÉLESTE

PAR

J. Poincaré,

PROFESSEUR D'ASTRONOMIE À L'UNIVERSITÉ DE PARIS.

VOLUME I.

ÉDITION PREMIÈRE. — ÉDITION DES TROISIÈMES ÉDITIONS  
ÉDITION EN PLATINUM.

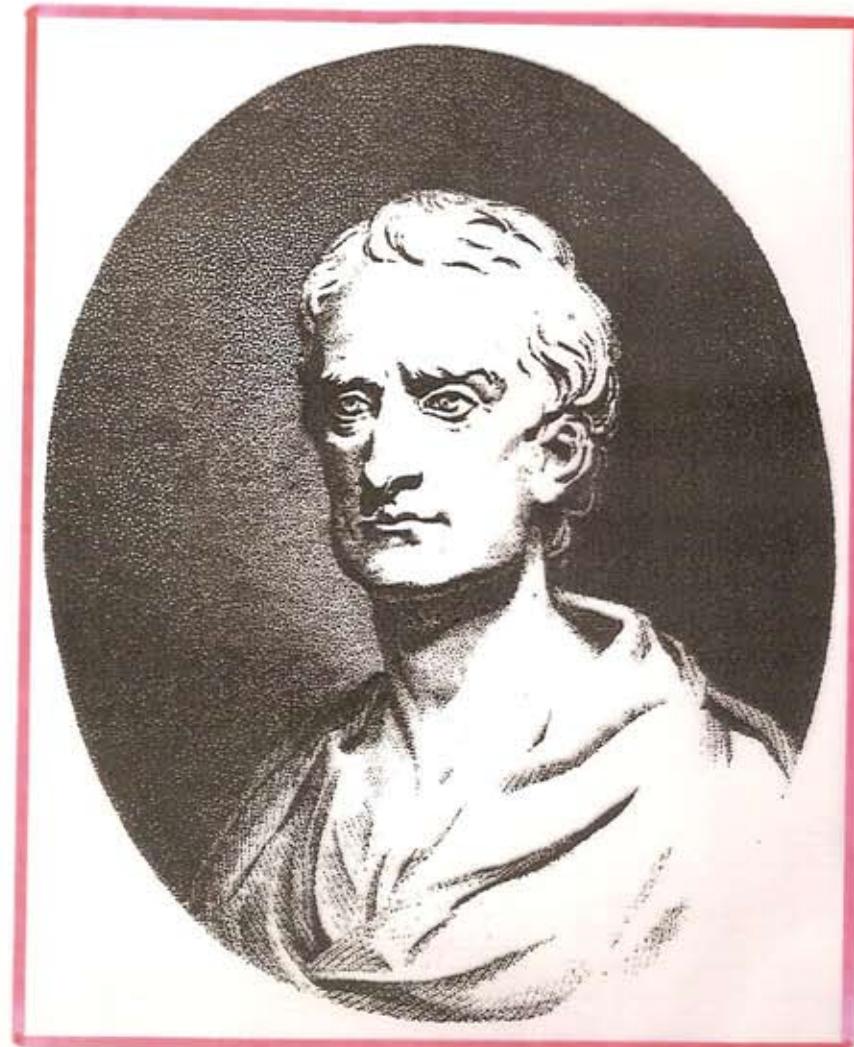
PARIS,  
LIBRAIRIE NATIONALE ET FIGUE, IMPRIMÉE ET BROSSÉE  
PAR ALBERT BOUDIN, 1890. — 1891.  
1892.

Lagrange

Nature, and Nature's Laws  
Lay hid in Night.

God said, Let Newton be !  
and All was Light.

A. POPE



LINEÆ  
TERTII ORDINIS  
NEUTONIANÆ,

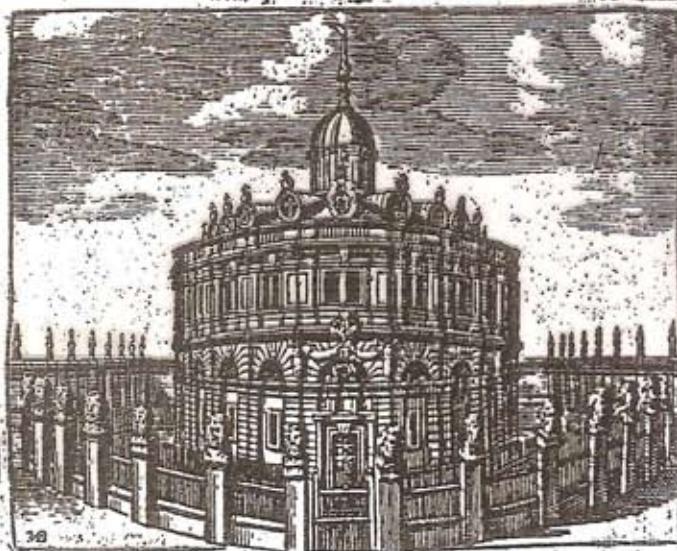
SIVE  
Illustratio Tractatus D. Neutoni  
De Enumeratione Linearum  
*Tertii Ordinis.*

Cui Subjungitur

Solutio Trium Problematum.

*R. Ball 2 m 9 coll 2500*

Authore JACOBO STIRLING, è Coll. Ball. Oxon.



OXONIÆ,

E THEATRO SHELDONIANO, Impensis Edvardi Whistler  
Bibliopolæ Oxoniensis, MDCCXVII.

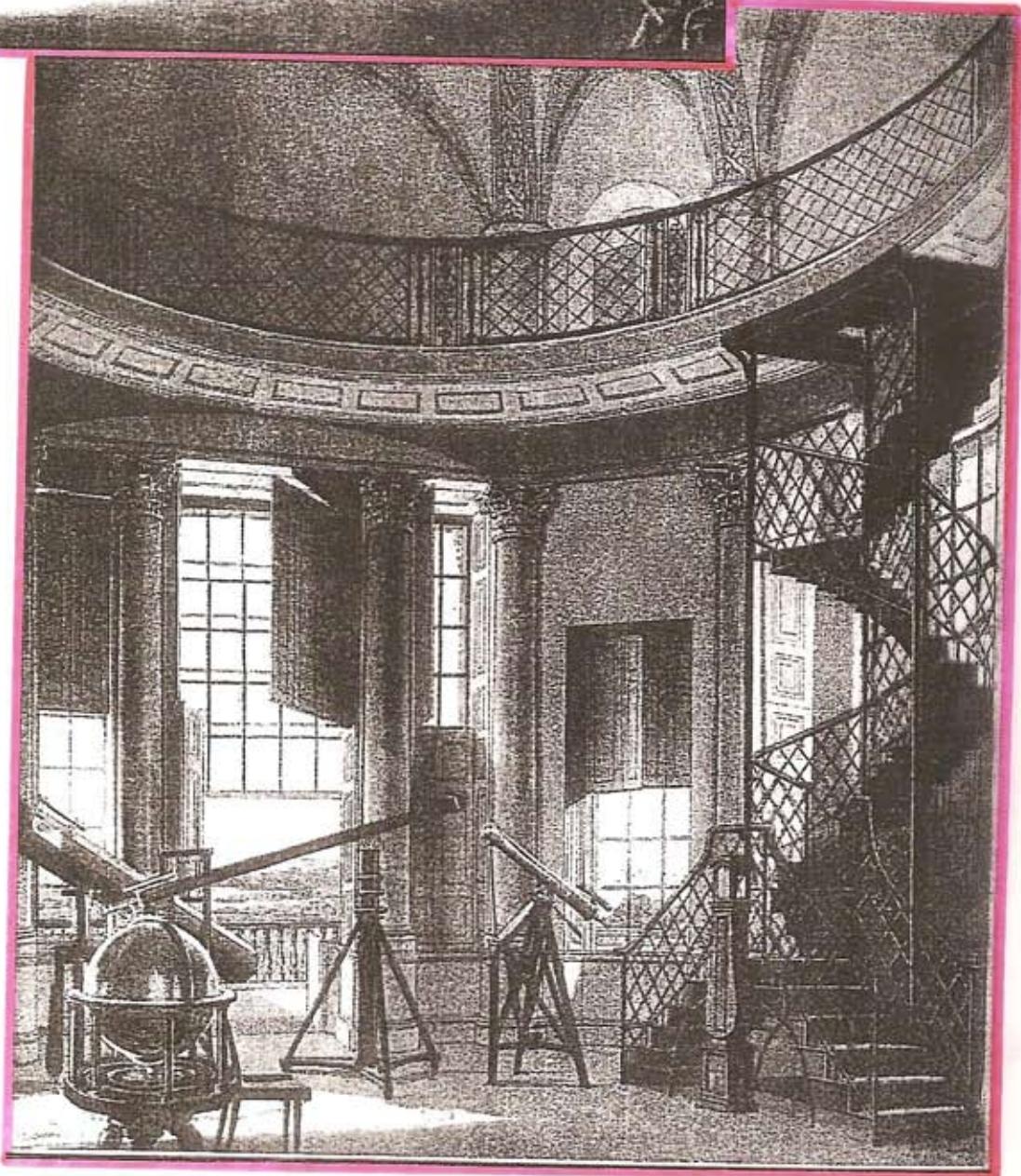
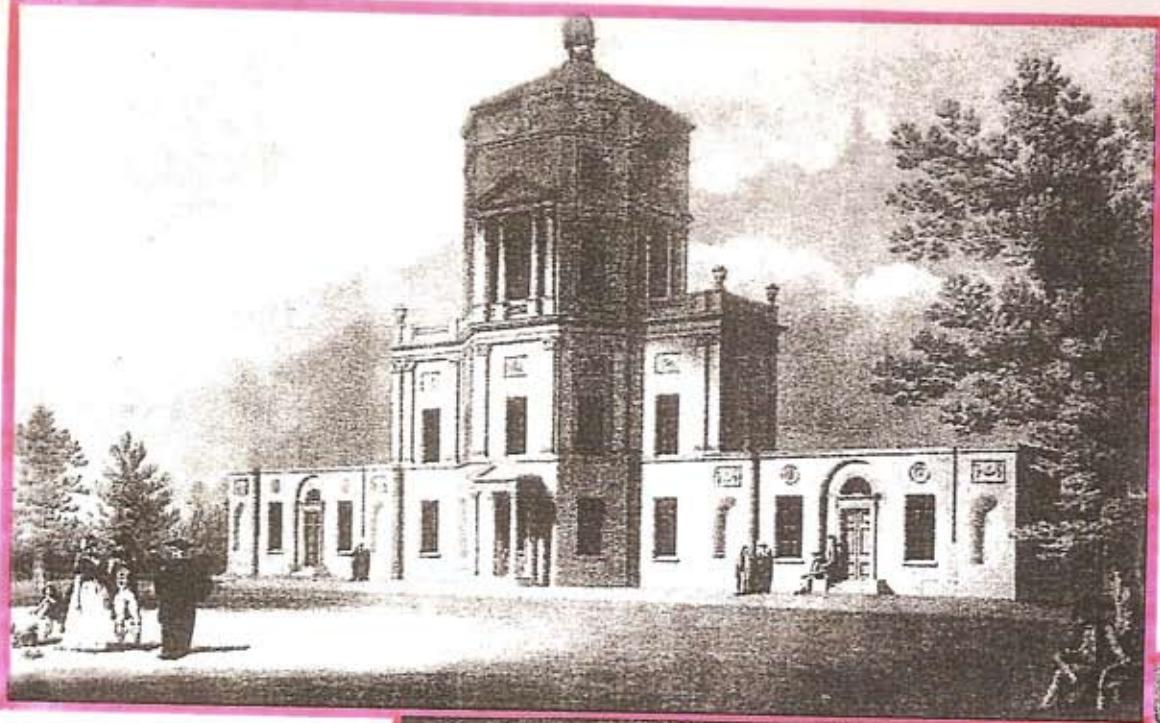
*Soc. Red. Lond.*



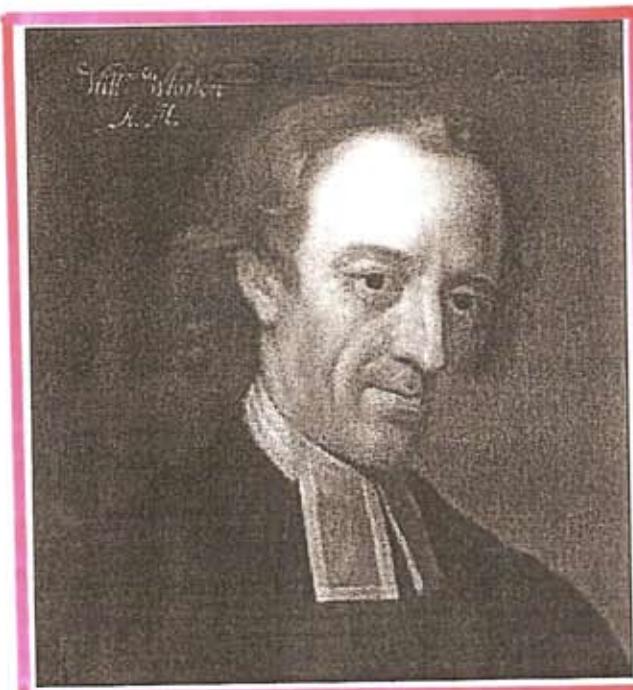
Published by H. Chapman March 1, 1796

Price Six pence

# Hornsby's Radcliffe Observatory



# William Whiston



A *Paul*

New METHOD  
For Discovering the  
**LONGITUDE**  
BOTH AT  
**SEA and LAND,**  
Humbly Proposed to the Consideration  
of the PUBLICK.

---

BY

*William Whiston, M. A.*

*Humphry Ditton, Master*

*Formerly Professor of*

*the Mathematicks in*

*the University of Cam-*

*bridge.*

*and*

*of the New Mat-*

*hematick School in*

*Cheif's Hospital, Lon-*

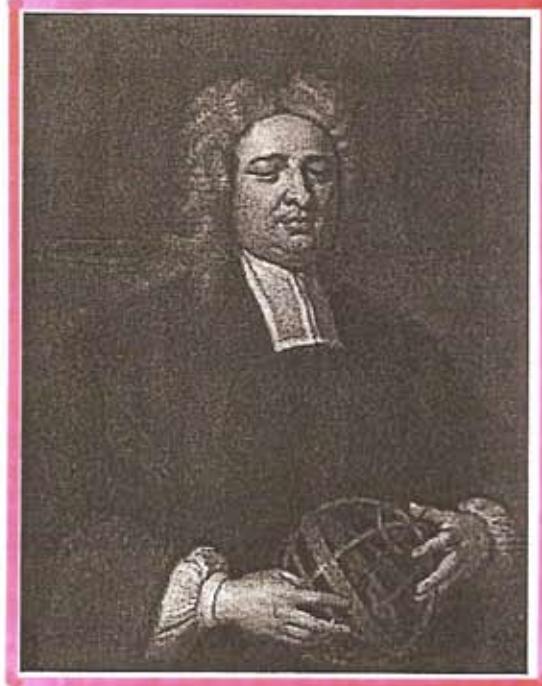
*don.*

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*L O N D O N :*  
Printed for JOHN PHILLIPS, at the *Black Bull* in *Cornhill.* 1714. July 15.



Nicholas



Saunderson

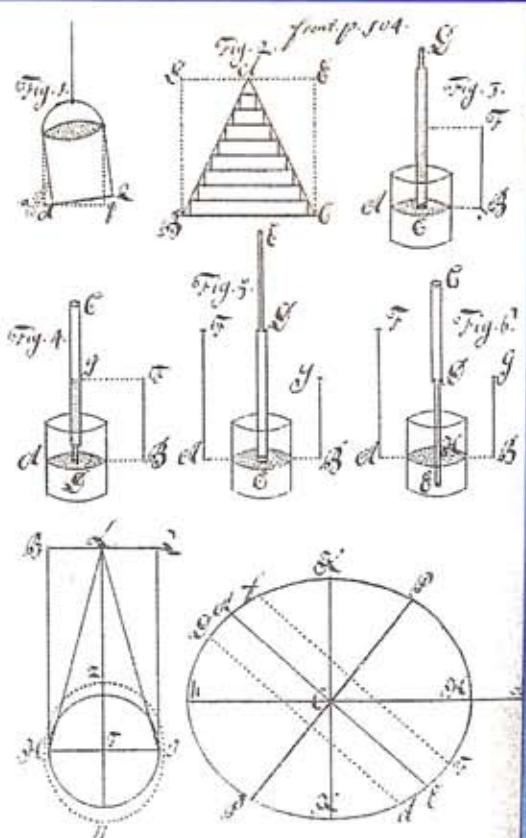
*M. Janer's Mechanicks*

1. Mechanics is that Part of Natural Philosophy, wh treats of the Nature & Laws of Motion.

2. What we have to say on this subject, shall be reduced to three Heads. Under of 1<sup>o</sup> we shall explain the Nature of Machines, concerning their Action, & Powers of Height acting contumeliously against Obstacles, & other Resistances. Under 2<sup>o</sup> we shall speak of those Laws, w<sup>t</sup> relate to the Effects of moving Forces upon such Bodies, as are at liberty to move without any Obstacle, or Resistance. In 3<sup>o</sup> we shall apply 1<sup>o</sup> & 2<sup>o</sup> to the Motion of Planets, & show of the whole Planetary System is governed by one.

*Part 1<sup>o</sup> of Motion in general.*

3. Motion is a continual shifting out of one place into another; without continuing in the same place for any time; that affection of Motion, by wh<sup>t</sup> a Body runs through a certain Space in a certain Time is called of Velocity, or Velocity of Motion. The greater the force is impressed upon a Body to make it change its Place, the greater is its Motion. (for the Effect is always proportional to its Cause) & that Motion is called of Momentum or Quantity of Motion.



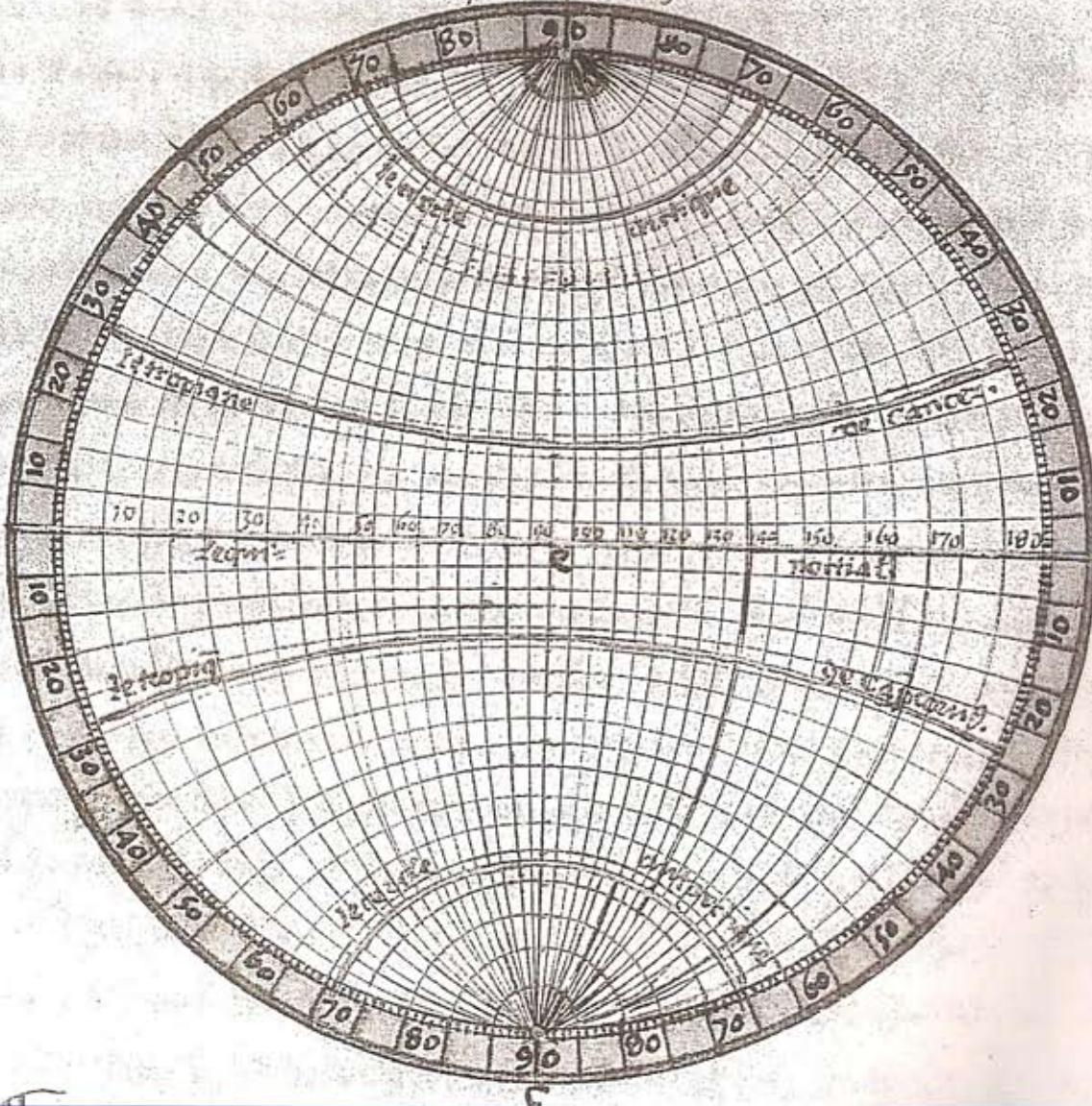
# De l'Esphere.

Septenatior.



Orient.

Ocident.



## The Ballad of Gresham College

*If to be rich and to be learn'd*

*Be every Nation's cheifest glory,*

*How much are English men concern'd,*

*Gresham to celebrate thy story*

*Who built th'Exchange t'enrich the Citty*

*And a Colledge founded for the witty.*

...

*The College will the whole world measure;*

*Which most impossible conclude,*

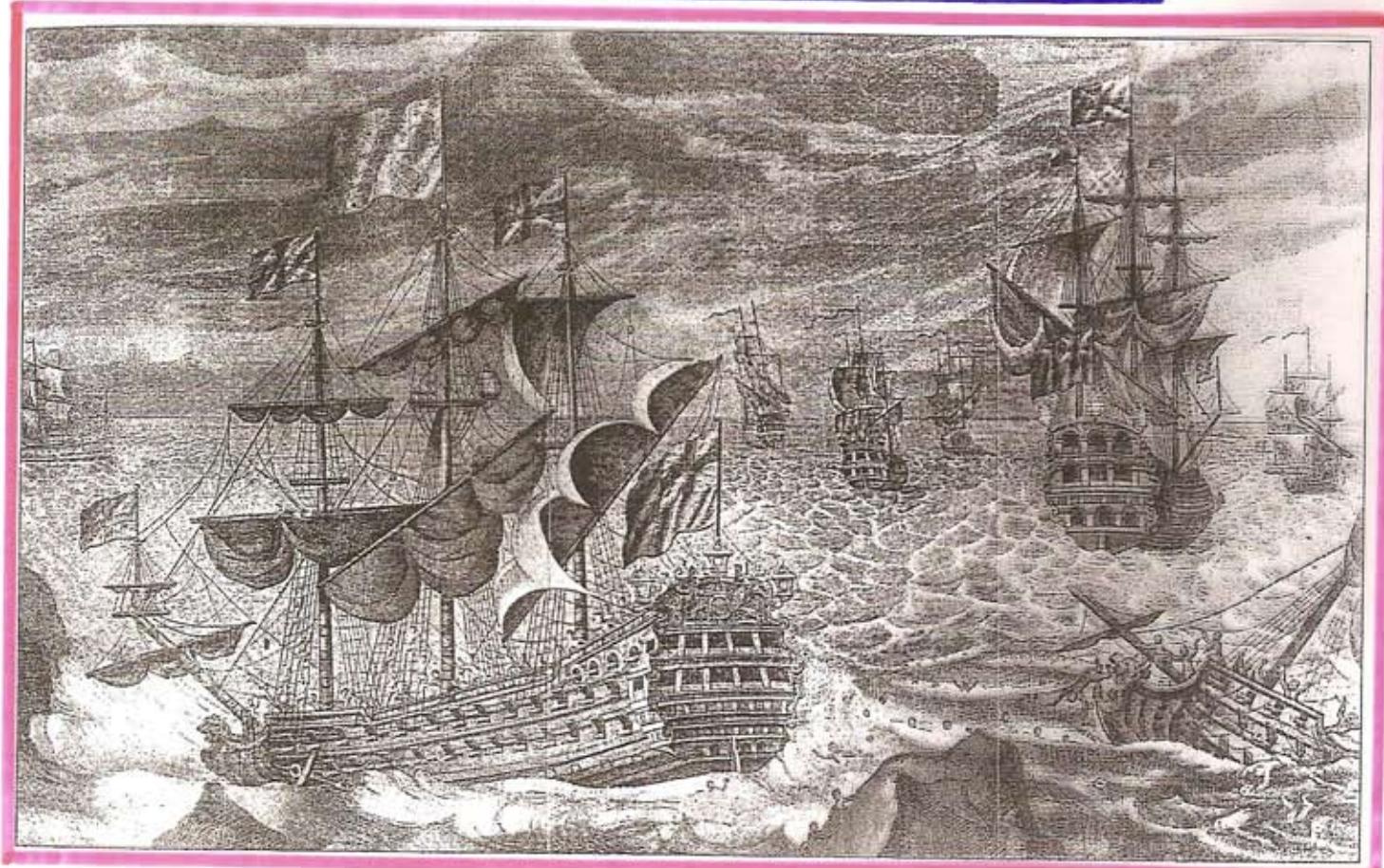
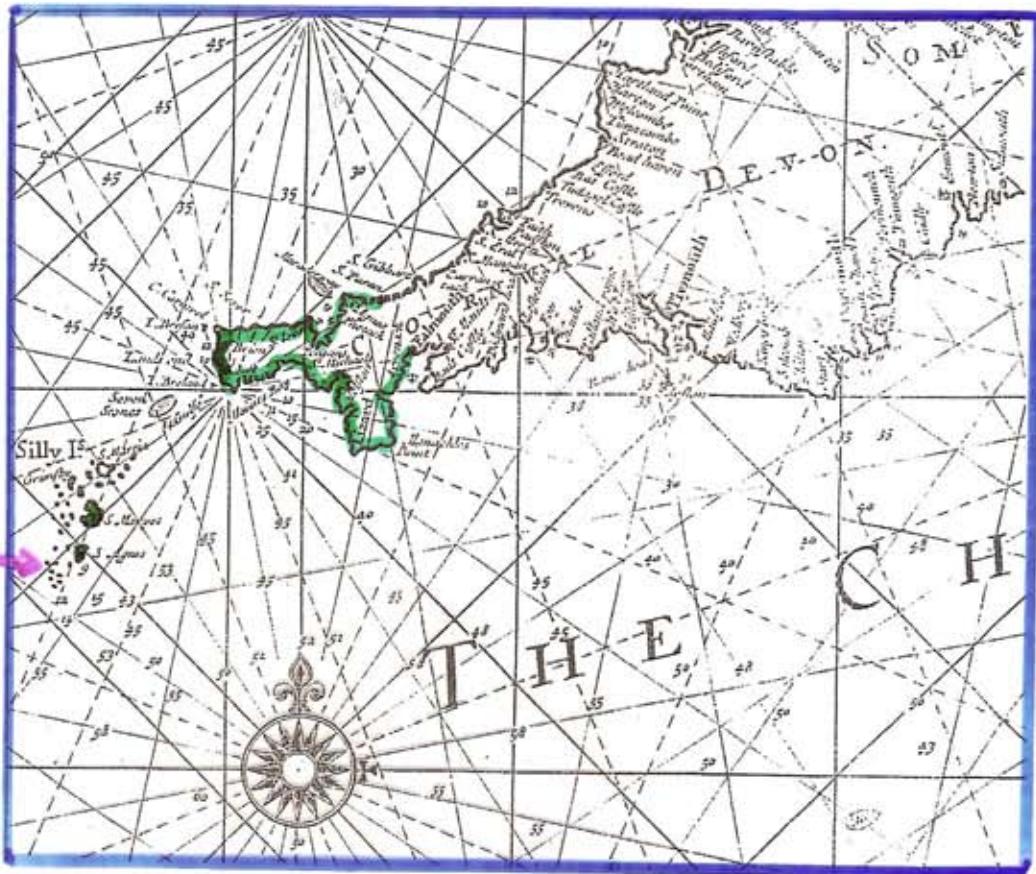
*And Navigation make a pleasure*

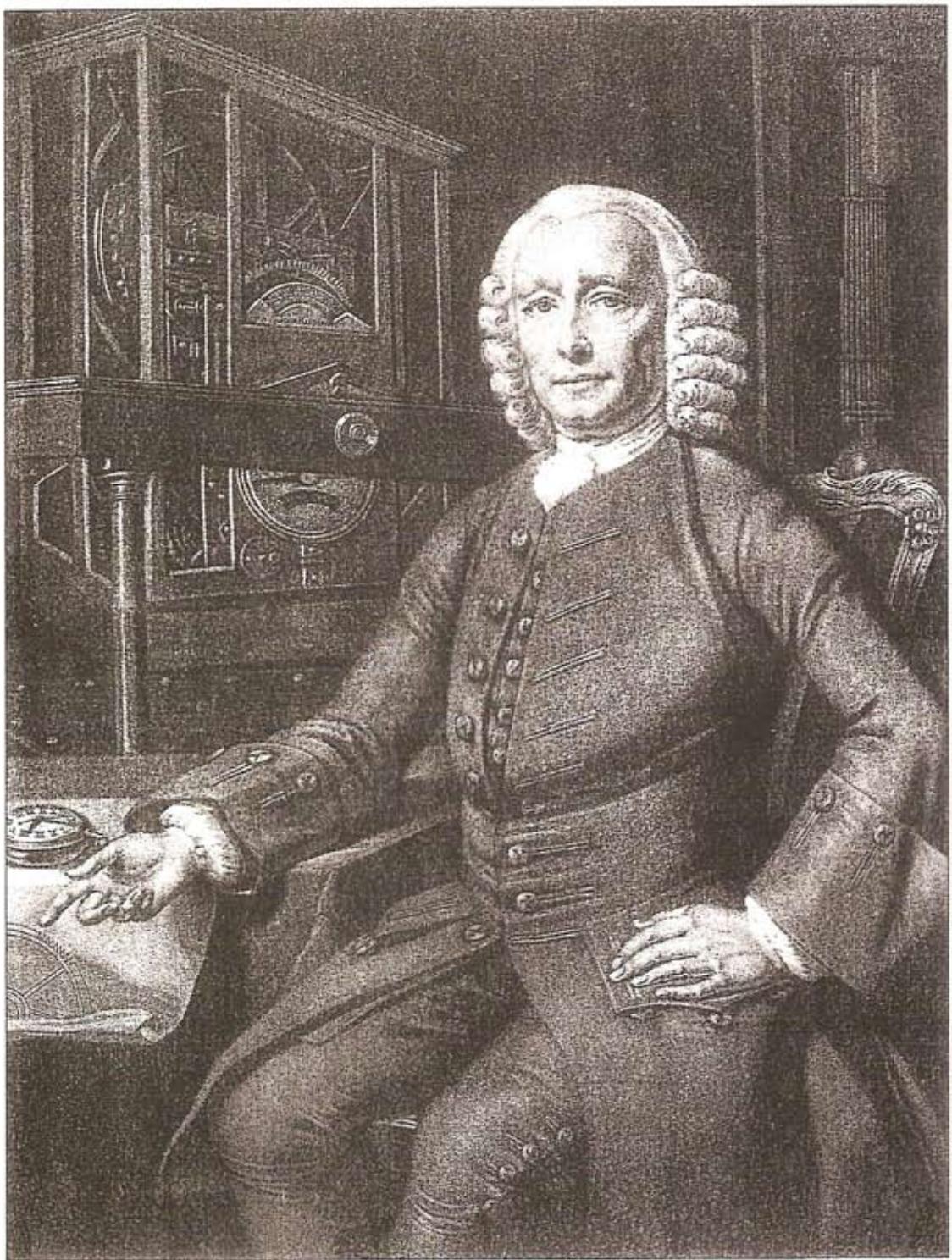
*By finding out the Longitude.*

*Every Tar shall then with ease*

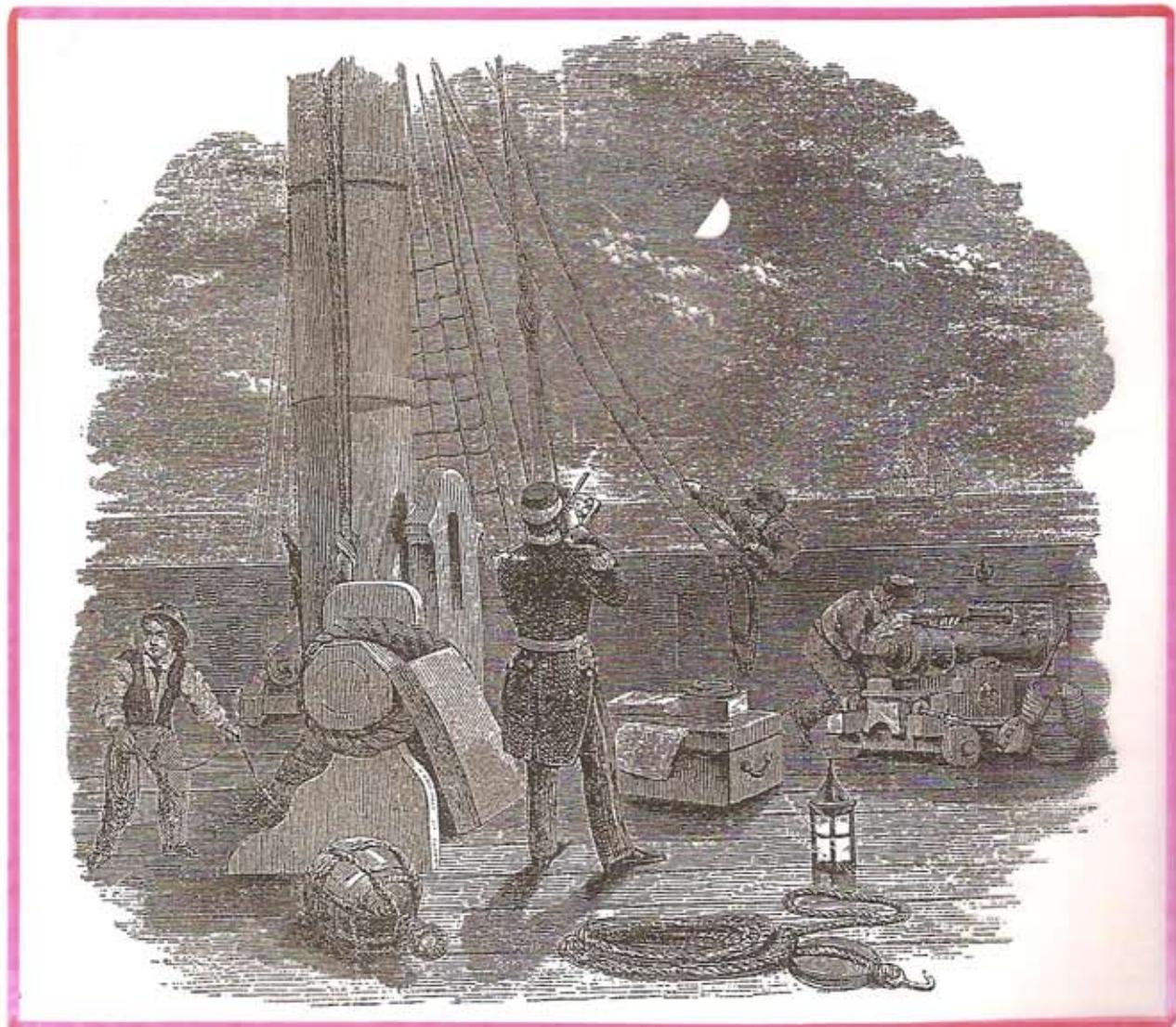
*Sayle any ship to the Antipodes.*

# The Wreck of the 'Association' (1707)





John Harrison



# Leibniz's calculus

Variables: no concept of motion

- sequences of close values

Derivatives: differences of successive  
(tangents) values - infinitely small

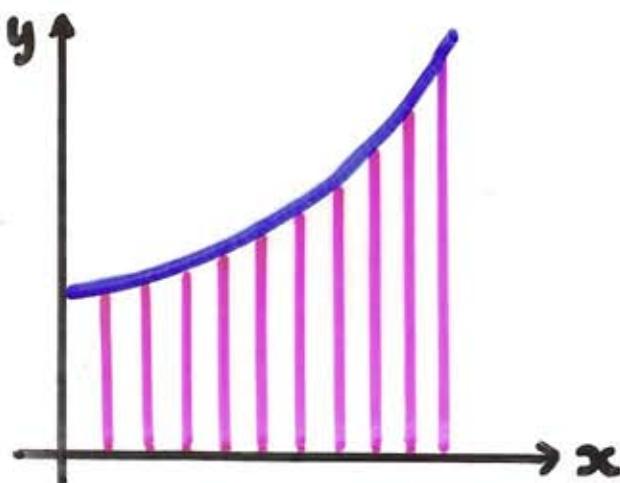
- notation  $dy/dx$

Integrals: area = sum of lines  
(areas)

notation:

all lines = omn. l

$$= \int l$$



# Jakob Bernoulli (1654-1705)



polar coordinates

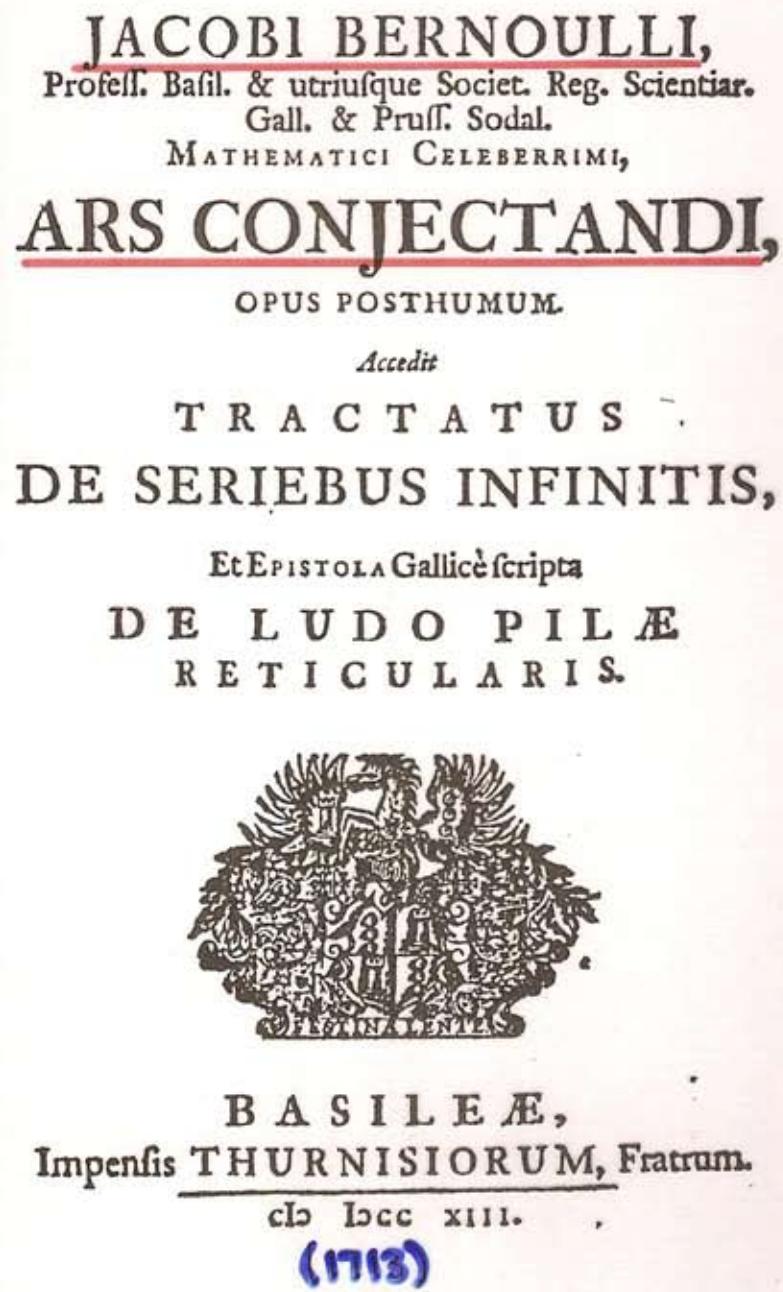
catenary

logarithmic spiral

probability ('Ars Conjectandi')

word "integral"

# Jakob Bernoulli's 'Ars Conjectandi' (1713)



probability:

'law of large numbers'

Limit theorems

binomial distribution

Combinatorics:

figurate numbers

combinations

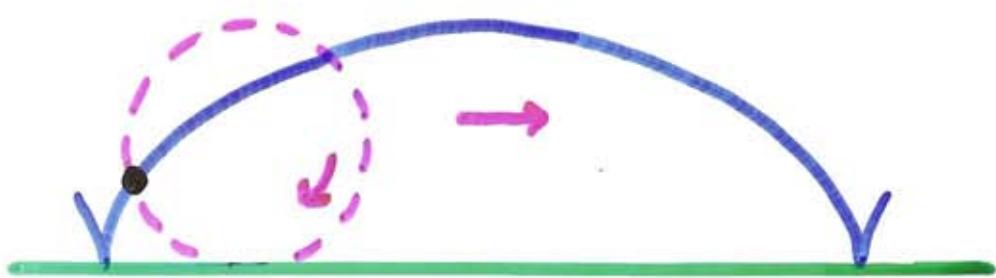
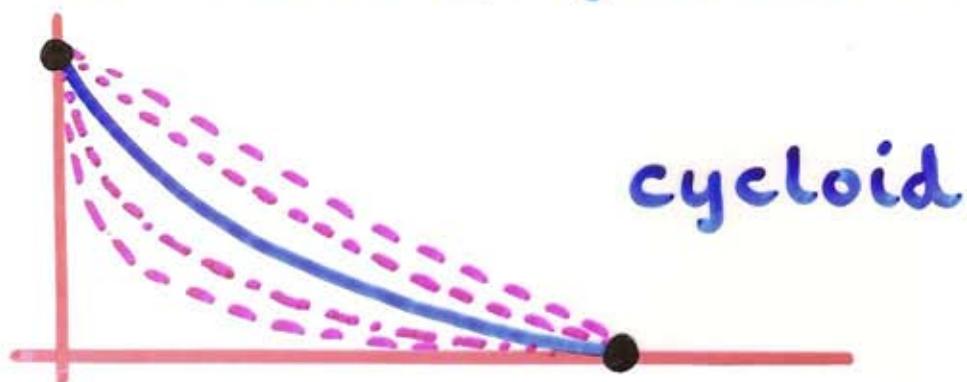
'Bernoulli numbers'

preceded by a hymn on the infinite variety of nature - this variety stems from the combinations and arrangements of its parts, and the combinatorial art helps us to enumerate these.

# Brachistochrone Problem

1697 : Johann Bernoulli :

Find the 'curve of quickest descent'



'I recognize the lion by his claws'

Johann  
Bernoulli  
(1667-1748) →

calculus of variations  
brachistochrone  
teacher of L'Hôpital, ...



Daniel  
Bernoulli  
← (1700-1782)

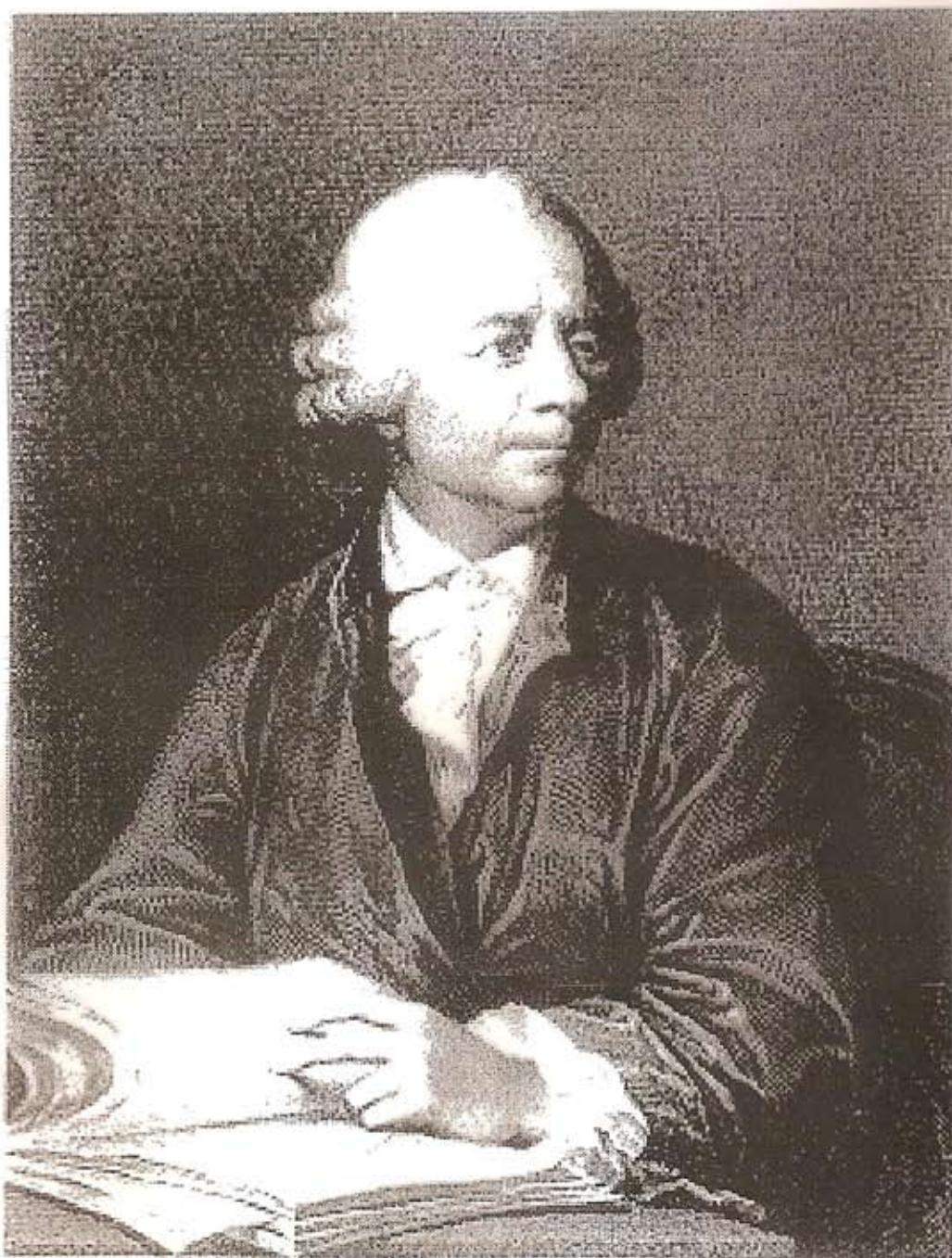
probability  
hydrodynamics

# Marquis de l'Hôpital (1661-1704)



Published the first book on the  
calculus (1696):

'Analyse des infiniment petits'



Leonhard Euler  
(1707 - 1783)

**INSTITUTIONES  
CALCULI  
DIFFERENTIALIS**

CUM EIUS VSU  
IN ANALYSI FINITORUM  
AC  
DOCTRINA SERIERUM

AUCTORE  
**LEONHARDO EULERO**

ACAD. REG. SCIENT. ET ELEG. LITT. BORUSS. DIRECTORE  
PROF. HONOR. ACAD. IMP. SCIENT. PETROP. ET ACADEMIARUM  
REGIARUM PARISINAE ET LONDINENSIS  
SOCIO.



IMPENSIS  
ACADEMIAE IMPERIALIS SCIENTIARUM  
PETROPOLOITANAE  
1755.

**INSTITUTIONVM  
CALCVLI INTEGRALIS  
VOLVMEN PRIMVM**

IN QVO METHODVS INTEGRANDI A PRIMIS PRIN-  
CIPIIIS VSQVE AD INTEGRATIONEM AEQVATIONVM DIFFE-  
RENTIALIVM PRIMI GRADVS PERTRACTATVR.

Auctore

**LEONHARDO EVLERO**

ACAD. SCIENT. BORVSSIAE DIRECTORE VICENNALI ET SOCIO  
ACAD. PETROP. PARISIN. ET LONDIN.

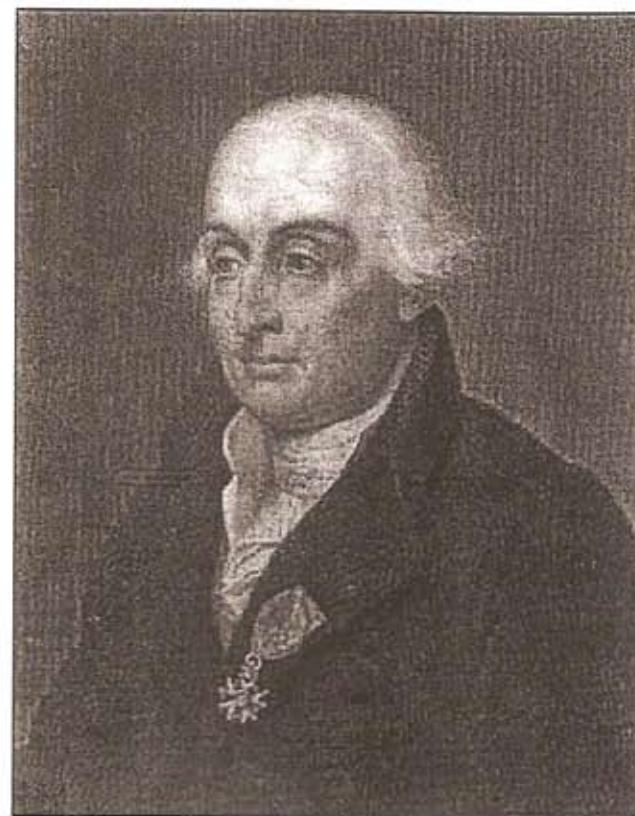
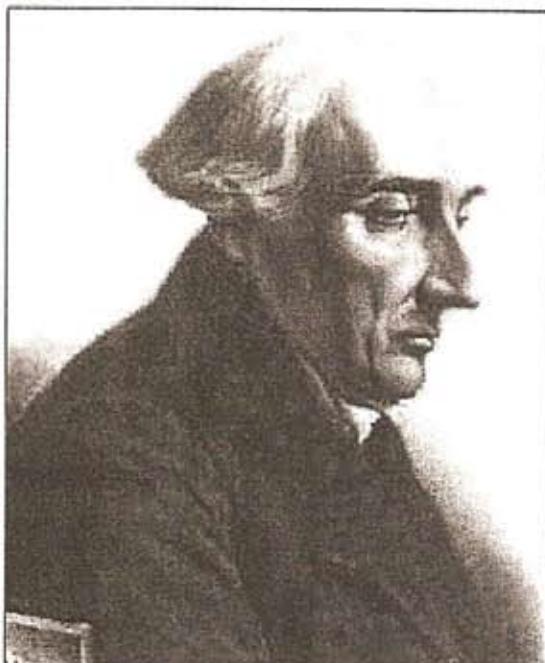


PETROPOLI

Impensis Academiae Imperialis Scientiarum

1768.

# Joseph-Louis Lagrange (1736-1813)

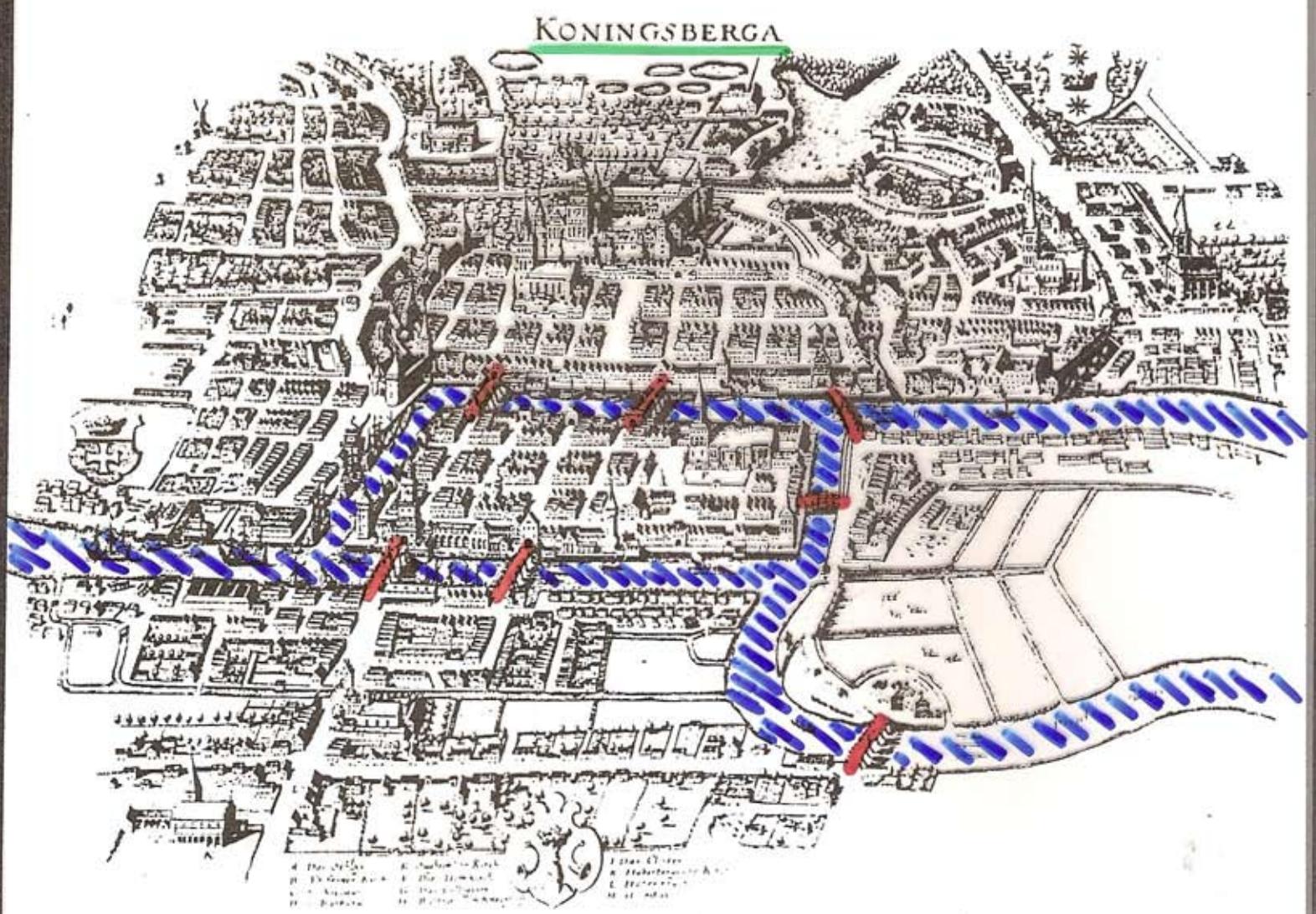


$$f(x) = a + bx + cx^2 + dx^3 + \dots$$

$$f'(x) = b + 2cx + 3dx^2 + \dots$$

$$\sin x = x - x^3/6 + x^5/120 - \dots$$

$$\text{so } [\cos x =] 1 - x^2/2 + x^4/24 - \dots$$



## SOLUTIO PROBLEMATIS AD GEOMETRIAM SITUS PERTINENTIS

Commentatio 53 indicis ENESTROEMIANI

Commentarii academiae scientiarum Petropolitanae 8 (1736), 1741, p. 128—140

1. Praeter illam geometriae partem, quae circa quantitates versatur et omni tempore summo studio est exculta, alterius partis etiamnum admodum ignotae primus mentionem fecit LEIBNITZIUS<sup>1)</sup>, quam *Geometriam situs* vocavit. Ista pars ab ipso in solo situ determinando situsque proprietatibus eruendis occupata esse statuitur; in quo negotio neque ad quantitates respiciendum neque calculo quantitatum utendum sit. Cuiusmodi autem problemata ad hanc situs geometriam pertineant et quali methodo in iis resolvendis uti oporteat, non satis est definitum. Quamobrem, cum nuper problematis cuiusdam mentio esset facta, quod quidem ad geometriam pertinere videbatur, at ita erat comparatum, ut neque determinationem quantitatum requireret neque solutionem calculi quantitatum ope admitteret, id ad geometriam situs referre haud dubitavi, praesertim quod in eius solutione solus situs in considerationem veniat, calculus vero nullius prorsus sit usus. Methodum ergo meam, quam ad huius generis problemata solvenda inveni, tanquam specimen Geometriæ situs hic exponere constitui.

2. Problema autem hoc, quod mihi satis notum esse perhibebatur, erat sequens: Regiomonti in Borussia esse insulam *A*, *der Kneiphof* dictam, flumque eam cingentem in duos dividi ramos, quemadmodum ex figura (Fig. 1) videre licet; ramos vero huius fluvii septem instructos esse pontibus *a*, *b*, *c*, *d*, *e*, *f* et *g*. Circa hos pontes iam ista proponebatur quaestio, num quis cursum ita instituere queat, ut per singulos pontes semel et non plus quam semel transeat. Hocque fieri posse, mihi dictum est, alios negare alios dubitare; neminem vero affirmare. Ego ex hoc mihi sequens maxime generale formavi problema: quaecunque sit fluvii figura et distributio in ramos atque quiunque fuerit numerus pontium, invenire, utrum per singulos pontes semel tantum transiri queat an vero secus.

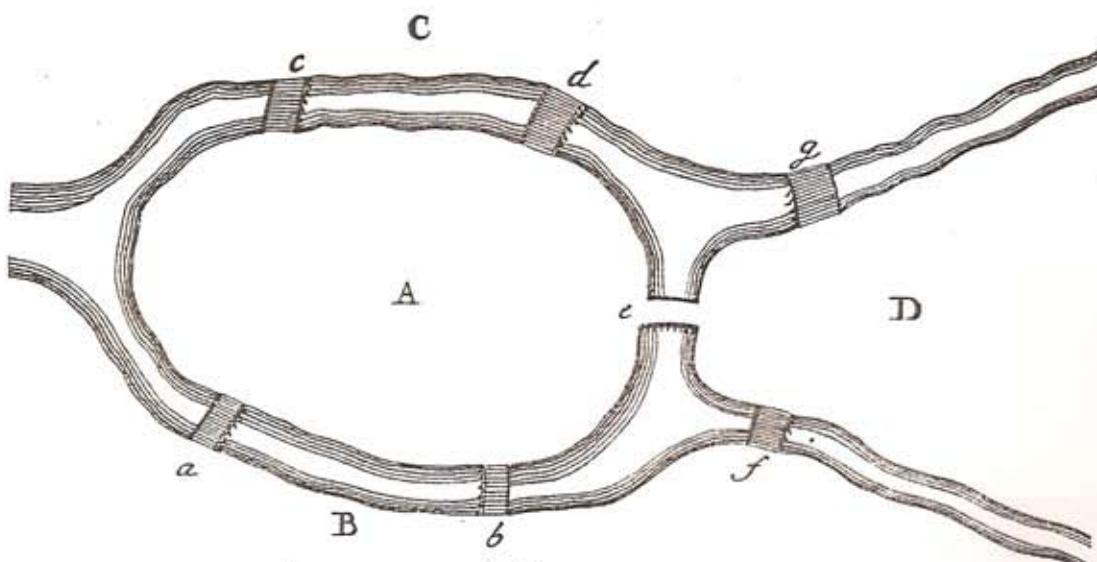


Fig. 1.

# Letter from Leonhard Euler

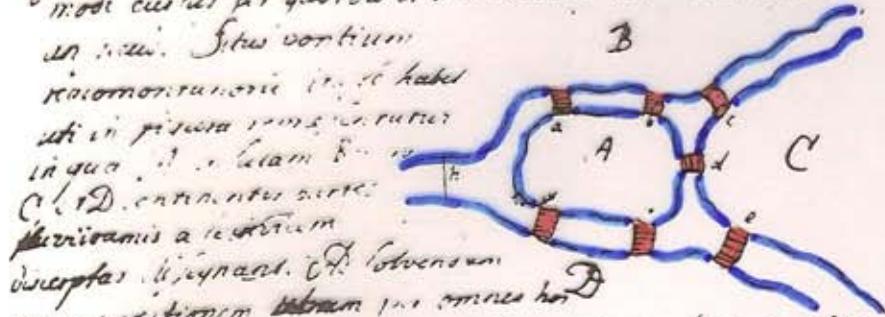
## to Giovanni Marinoni (Vienna)

13 March 1736

40

teris uasdam necessitatis. Tenui ammunicare quas ut  
benetile occupas siue de eis iudicium oportebat etiam abg  
ueri res. Questio multi aliquando proponebatur circa uulcanum  
in urbe Regiomonti: si tam fluvio potest oriturus trahito cum  
lum, quarebatur nescire que nos ingulos dorso continuo  
curvi ferri, lantulare sucat, fonsque ceruicibus minime  
reducit per lege curvam in latitudine rotundam. Hoc questione  
est ad vulcanum tam non remissum riserat, sed maxima  
contingentia, ut si uulcanus ac eam soleremur, non contra reg  
curvam existimoma. In uulcanum uulcanum non uenit nam  
ea forte ad geometram. Ita num solumque uulcanus  
existinet. Cum igitur sic de re sua meicitatus, factus adotus  
cum regulam fermecita demonstratione munera, qua in hec  
punctis questionibus statim discernere licet utrum hanc  
modi cibas per quatuor et octuaginta tres portas in latitudine  
in uulcanum. Situs portuum

Regiomontanorum est, ut habeat  
ut in figura rite indicatur  
in qua A. et lacum B. et  
C. et D. continentur uulcanum  
per uulcanum a rectitudine  
disceptas lignynas. Ut solvensum  
nunc questionem habemus in omnes horum  
temporibus, per unumquemq; uel non plus ambulari possit.  
In non, ante omnia est dividendum, non sine reverentia aqua dispergenda  
ut in hoc exemplo uero etiam responsum quas litteris A, B, C, D  
notari. Inde videtur, ut est quod portes in unamquemq; respondeant  
conuocant, seu notius utrum numerus portuum eo ducentiuncis sit  
par ut unius. Sic in nostro exemplo ad A sunt portes, ad idem  
in B. in C. in D. in quatuor, per portas expinguunt, seu numer  
portuum ad iniquas. Discutendum est tamen, quod ad questionem



19. Sunt duae insulae *A* et *B* aqua circumdatae, qua cum aqua communicent quatuor fluvii, quemadmodum figura (Fig. 3) repraesentat. Traecto porro sint super aquam insulas circumdantem et fluvios quindecim pontes *a*, *b*, *c*, *d* etc. et quaeritur, num quis cursum ita instituere queat, ut per

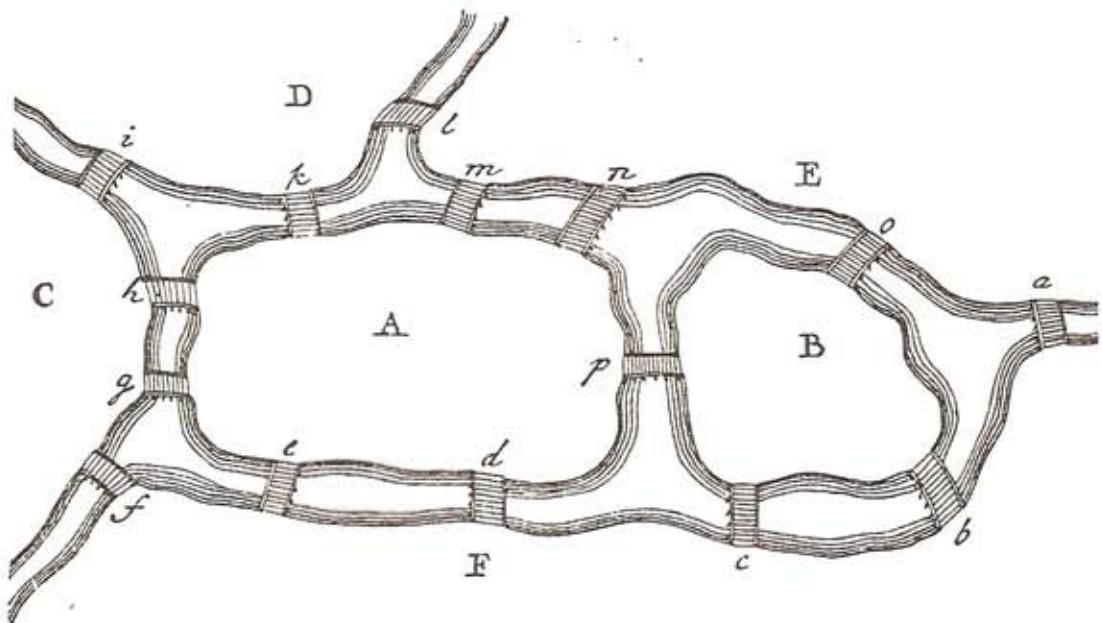


Fig. 3.

omnes pontes transeat, per nullum autem plus quam semel. Designo ergo primum omnes regiones, quae aqua a se invicem sunt separatae, litteris *A*, *B*, *C*, *D*, *E*, *F*, cuiusmodi ergo sunt sex regiones. Dein numerum pontium 15 unitate augeo et summam 16 sequenti operationi praefigo.

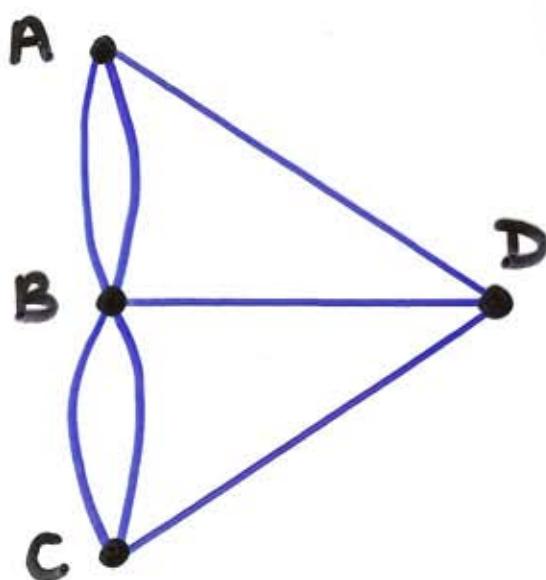
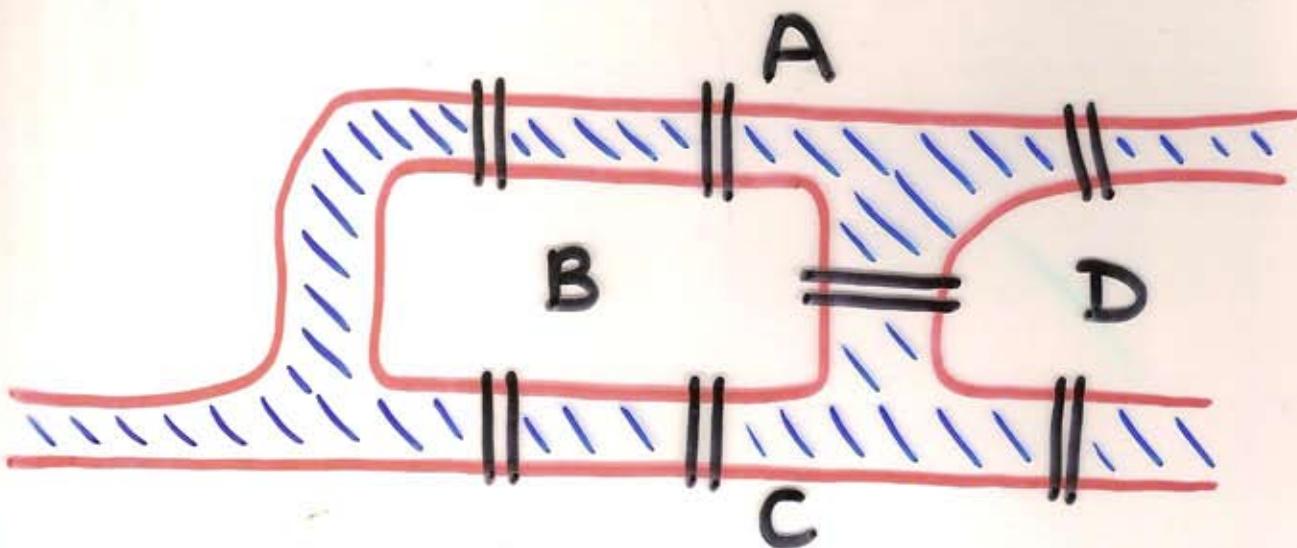
		16
<i>A*</i> ,	8	4
<i>B*</i> ,	4	2
<i>C*</i> ,	4	2
<i>D</i> ,	3	2
<i>E</i> ,	5	3
<i>F*</i> ,	6	3
		16

20. Casu ergo quounque proposito statim facilime poterit cognosci, utrum transitus per omnes pontes semel institui queat an non, ope huius regulae:

- *Si fuerint plures duabus regiones, ad quas ducentium pontium numerus est impar, tum certo affirmari potest talem transitum non dari.*
- *Si autem ad duas tantum regiones ducentium pontium numerus est impar, tunc transitus fieri poterit, si modo cursus in altera harum regionum incipiatur.*
- *Si denique nulla omnino fuerit regio, ad quam pontes numero impares conductant, tum transitus desiderato modo institui poterit, in quacunque regione ambulandi initium ponatur.*

Hac igitur data regula problemati proposito plenissime satisfit.

# The BRIDGES of KÖNIGSBERG



Solved by EULER  
in 1736

whenever you enter a vertex,

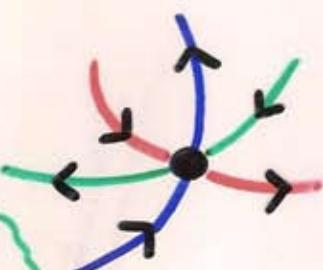
you must be able to leave it

So 0 or 2 odd degrees.

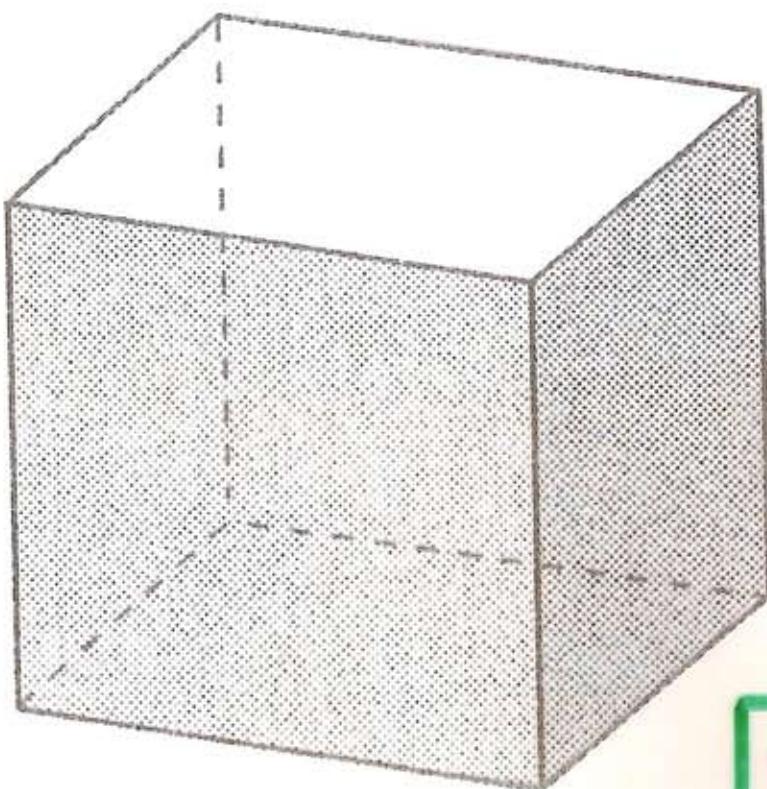
even  
degree

Königsberg has degrees 3, 3, 3, 5,

and so is impossible.



## Euler's Polyhedron Formula



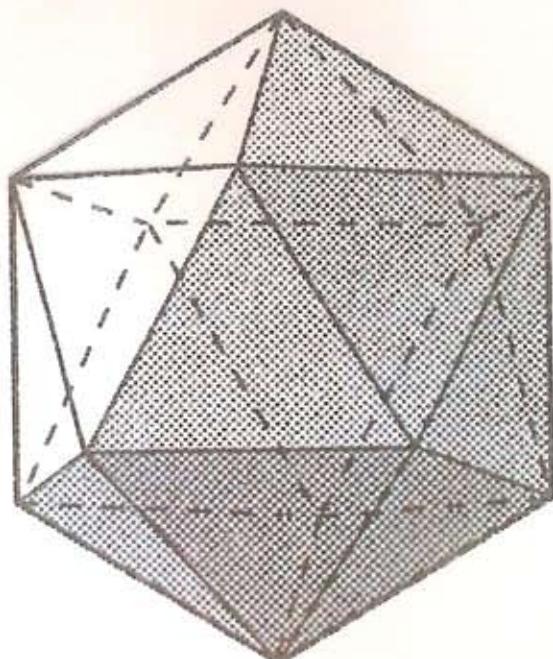
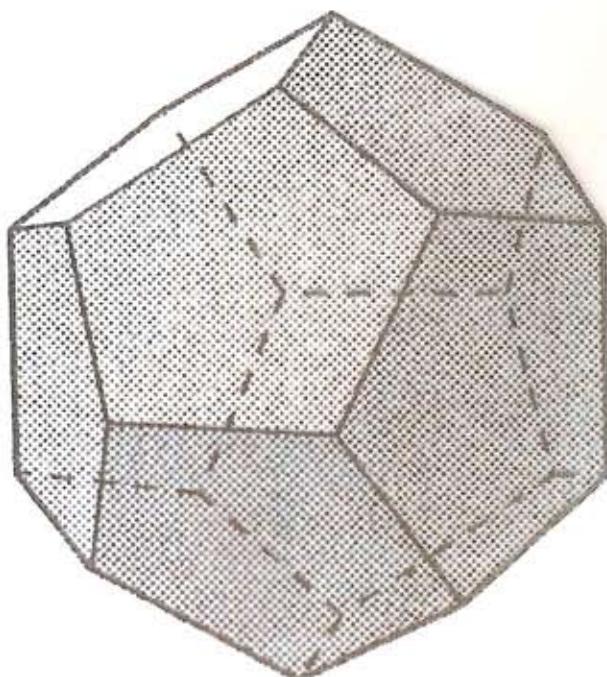
8 vertices

6 faces

12 edges

$$8 + 6 = 12 + 2$$

$$V + F = E + 2$$



$$20 + 12 = 30 + 2$$

$$12 + 20 = 30 + 2$$

# Euler on Polyhedra

Berlin d. 14. November 1750.

Neulich kam mir in Sinn die allgemeinen Eigenschaften der Körper, welche hedris planis eingeschlossen sind, zu bestimmen, weil kein Zweifel ist, dass sich in denselben nicht eben dergleichen allgemeine Eigenschaften finden sollten, als in den figuris planis rectilineis, deren Eigenschaften darin bestehen, dass 1. in einer jeglichen figura plana der numerus laterum dem numero angulorum gleich ist, hernach 2. dass die summa angulorum omnium gleich ist bis tot rectis quot sunt latera, demtis quatuor. Wie aber in den figuris planis nur latera und anguli zu betrachten vorkommen, so müssen bei den Körpern mehr Stücke in Betracht gezogen werden, als

- I. die hedrae, deren Anzahl sey  $= H$ ;
- II. die anguli solidi, deren Anzahl sey  $= S$ ;
- III. die Fügungen, wo zwey hedrae secundum latera zusammenkommen, so ich aus Mangel eines recipirten Worts, *acies* nenne, deren Anzahl sey  $= A$ ;
- IV. die latera singularum hedrarum, quorum omnium simul sumtorum numerus sit  $= L$ ;
- V. die anguli plani singularum hedrarum, quorum omnium numerus sit  $= P$ .

1. Bei diesen fünf Stücken ist nun erstlich klar, dass  $P = L$ , weil in allen hedris der numerus angulorum  $=$  numero laterum.

2. Ist auch immer  $A = \frac{1}{2}L$ , oder  $A = \frac{1}{2}P$ , weil immer zwey latera diversarum hedrarum zusammenkommen, um eine aciem zu formiren.

3. Dahero ist der numerus laterum seu angulorum planorum omnium hedrarum corpus includentium allzeit par.

4. Semper est vel  $L = 3H$  vel  $L > 3H$  } at est  $P = L$ .  
5. Semper est vel  $P = 3S$  vel  $P > 3S$  }

Dieses ist klar, weil keine hedra aus weniger als drey Seiten, und kein angulus solidus aus weniger als drey angulis planis bestehen kann. Folgende Proposition aber kann ich nicht recht rigorose demonstriren:

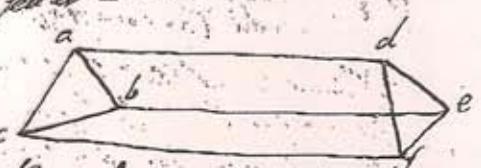
6. In omni solido hedris planis inclusio aggregatum ex numero hedrarum et numero angulorum solidorum binario superat numerum acierum, seu est  $H + S = A + 2$ , seu  $H + S = \frac{1}{2}L + 2 = \frac{1}{2}P + 2$ .

# Euler's Polyhedron Letter (1750)

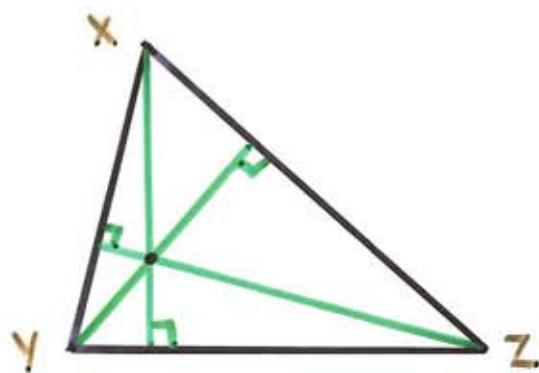
2.  $\text{G}P = A = \frac{1}{2}d$  unde  $A = \frac{1}{2}P$ . huius in plurim latera  
 decagonum  $\text{G}P$  summa longitudo, in aciem  $\text{G}P$  formorum.  
 3. Igitur  $P$  de numeris lateriorum seu angulorum planorum omnium  
 hedrorum corporis incidentium aliquid par.  
 4. Semper autem vel  $L = 3H$  vel  $L > 3H$  et atque  $P = d$ .  
 5. Semper autem vel  $P = 3S$  vel  $P > 3S$  et atque  $P = d$ .  
 Igitur  $H$  sit huius Hedronum et longitudo eius dupla, et si  
 angulus solidus et longitudo eius et angulus planus  $\text{G}P$  huius Hedronum  
 figurae propositione ab eo inveniatur, et si  $\text{G}P$  non rigore demonstratur  
 6. In aliis solidis hedrorum planis videlicet aggregatum ex numero hedronum  
 et numero angulorum solidorum binario separat numerum eum  
 seu est  $\text{G}P + S = A + 2$  seu  $\text{G}P + S = \frac{1}{2}d + 2 = \frac{1}{2}P + 2$ .  
 7. Impossibile est ut sit  $A + 6 > 3H$  vel  $A + 6 > 3S$   
 8. Impossibile est ut sit  $\text{G}P + 2.7 = S$  vel  $S + 2.7 = \text{G}P$ .  
 9. Hoc illud formari potest solidum cuius omnis hedra sit triangularis  
 laterum, nec cuyus omnis angulus solidi ex sex planbris solidis angulis planis  
 sit conficiens.  
 10. Summa omnium angulorum planorum, qui in ambito solidi conjugatae  
 accipiunt, tot angulis rectis equivalens, quae sunt unitates in  $\frac{1}{2}A - \frac{1}{2}\text{G}P$ .  
 11. Summa omnium angulorum planorum conjugatae quater tot angulis  
 rectis, quae sunt anguli solidi dentis octo, seu est  $\frac{1}{2}AS - 8$  rectis.

Exemplo sit prima triangulana ubi aut  
 1. numerus Hedronum  $H = 5$   
 2. numerus ang. soli:  $S = 6$

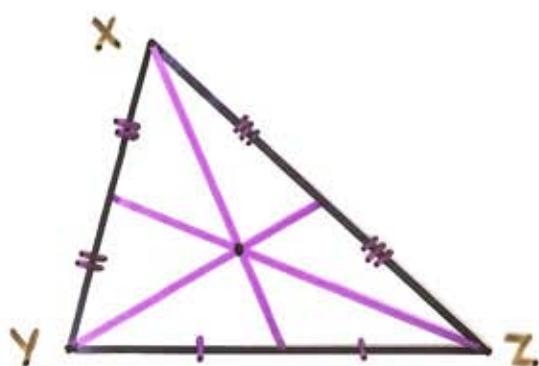
3. numerus alterius (ab, ac, bc, ad, bd, cd)  $A = 9$   
 4. numerus laterum et angulorum planorum  $L = P = 18$ . Triplidit enim corporis  
 decolor triangulorum et trilaterorum quadrilaterorum, unde  $L = P = 2.3 + 3.4 = 18$ .  
 Propterea in Theor. 6:  $\text{G}P + S$  (ii)  $= A + 2$  (ii)  
 summa summa omnium angulorum planorum (ad) in figura  $D = 4$  rectis, et  $D$  sum  
 longitudo  $\square = 12$  rectis, et  $\frac{1}{2}S$  rectis  $= \frac{1}{2}(A-H) = AS - 8$  rectis.



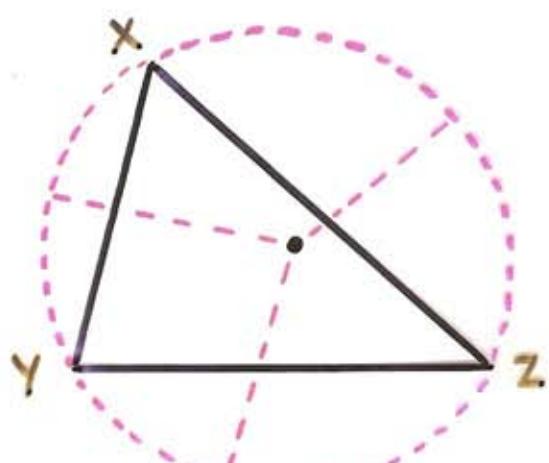
# Geometry of a Triangle



orthocentre O

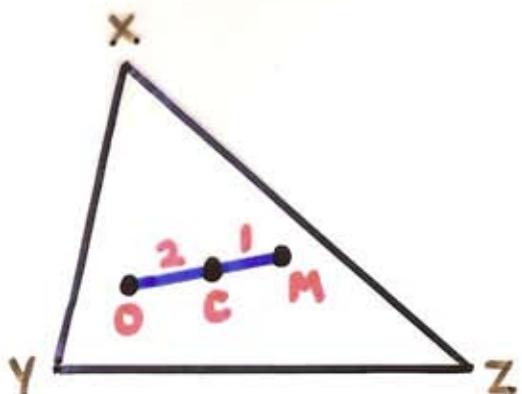


centroid C



circumcentre M

Euler Line  
( $OC = 2CM$ )



## Fermat Prime Numbers

Fermat :  $F_n = 2^{2^n} + 1$  is prime (?)

$$F_0 = 2^1 + 1 = 3, \quad F_1 = 2^2 + 1 = 5,$$

$$F_2 = 2^4 + 1 = 17, \quad F_3 = 2^8 + 1 = 257,$$

$$F_4 = 2^{16} + 1 = 65,537$$

$$F_5 = 4,294,967,297 ? \quad \leftarrow 2^{32} + 1$$

Euler :  $F_5$  is divisible by 641

Proof :  $641 = 5^4 + 2^4 = (5 \times 2^7) + 1.$

$$\begin{aligned} \text{So } 2^{32} + 1 &= 2^{28}(5^4 + 2^4) - (5 \cdot 2^7)^4 + 1 \\ &= 2^{28} \cdot 641 - (641 - 1)^4 + 1 \\ &= 641 \times K, \text{ so } 641 \mid 2^{32} + 1. \end{aligned}$$

# Euclid Book I, Prop. I

On a given straight line  
to construct an equilateral triangle

Let AB be the line.

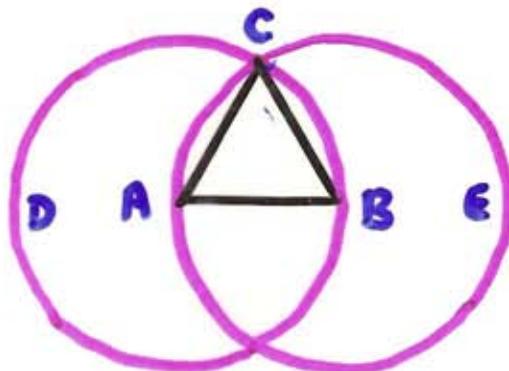
With centre A and  
distance AB, draw the  
circle BCD. [Post. 3]

With centre B and distance BA,  
draw the circle ACE. [Post. 3]

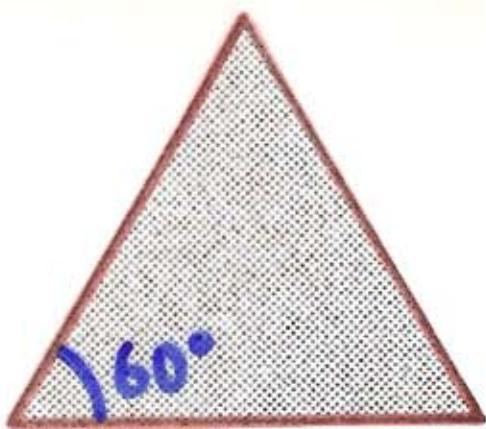
Join AC and BC. [Post. 1]

Then the triangle ABC is equilateral.

Proof . . .



# Regular Polygons



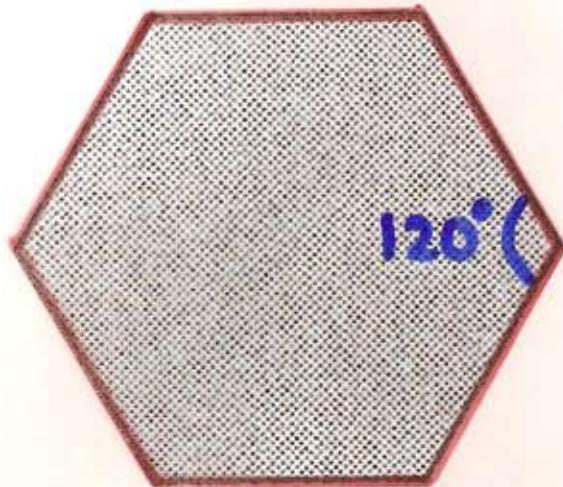
triangle



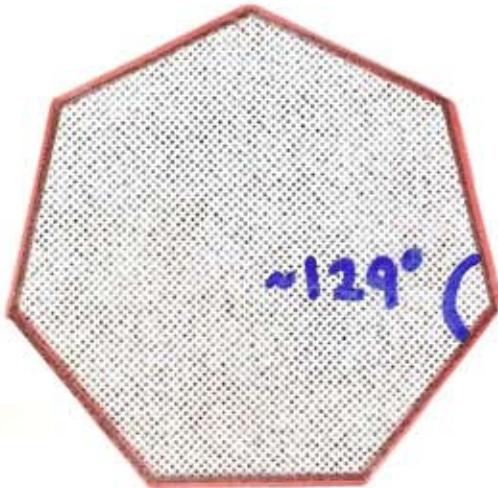
square



pentagon



hexagon



heptagon



octagon

# Constructing Polygons

Gauss: A regular polygon with  $n$  sides can be constructed with straight-edge and compasses if and only if  $n$  has the form

$$n = 2^k \times p_1 \times p_2 \times \dots$$

↖ distinct Fermat primes

3, 4, 5, 6, 8, 10, 12, 15, 16,

17, 20, 24, 30, 32, 34, 40, 48,

51, 60, 64, 68, 80, 85, 96,

102, ..., 257, ..., 65537, ...